December 8th, 2024

Dr. Dan Romanyk, PhD, P.Eng

University of Alberta - Mechanical Engineering Department

RE: Final Design Report for MecE 360 Project

Dear Dr. Romanyk:

This report outlines the final design and analysis of the transmission system developed by the Birdie Boys team for a luxury golf cart, specifically for use on private golf courses in Edmonton, AB.

The design objectives focused on creating a three shaft, four gear, and 3 bearing set system that ensures smooth power transfer from the HPEVS AC-9 48V electric motor to the wheels. Key considerations included achieving a gear ratio between 7.5:1 and 10:1, optimizing gear and shaft design, and adhering to AGMA standards.

Our work delivered the selection of ideal gear train, shaft design, and bearings and provided the final design overview.

Please review the detailed report and reach out to the Team Contact, Hyunseok Jang, at hyunseok@ualberta.ca for any questions, concerns, or needed clarification.

We appreciate the opportunity to work on this project.

Sincerely, Hyunseok (Team Contact)

Group 12, The Birdie Boys

12/08/2024

Final Report:

Transmission Design for Electrically Powered Golf Cart

Tahir Anjum, Ben Cote, Cole Pilgrim, Hyunseok Jang, William Platero

BIRDIE BOYS GROUP 12

ABSTRACT

This report outlines Birdie Boys completed analysis for their proposed design of a transmission for a luxury golf cart. The design consists of three shafts: input, idler, and output. There is one helical gear on both the input and output shaft, while the idler shaft carries two helical gears. Overall, the gears provide a 7.5:1 speed reduction from the input to output shaft. This report focuses on the analysis completed throughout the design process and the finalized design selection.

AISI 4140 steel was chosen for all gears materials, sourced from the KHK helical gear catalog. The calculation of gear safety factors adheres to AGMA standard 2001-D04, which requires determining the correction factors for bending/contact strengths and factors for bending/contact stresses. The final gear train selection must have a yield bending safety factor above 1.2 and contact safety factor above 1.15, which was an internally set criteria based on the gear manufacturer's gear safety factor calculation. Safety factors were considered to provide a reliable transmission to best prevent gear teeth fatigue failure. Moreover, the bore diameter of the selected gears must meet the minimum diameter determined from the shaft analysis.

AISI 4140 oil-quenched steel was selected as the optimal shaft material. In accordance with AGMA recommended DE Elliptic criteria, which evaluates shaft failure, minimum shaft diameters were calculated at the gear locations. Next, choose the first viable gear bore diameter available to assess MECE 360 Inc. given deflection and twist criteria and determine if increasing the diameter is necessary. The finalized shaft diameters at the gear locations are 30mm, 32mm, and 40mm for the input, idler, and output shaft respectively. Additional analysis confirmed that the angle of twist, linear deflection, and angular deflection at gears and bearings meet required limits.

Birdie Boys aimed to select appropriate single-row ball bearings from the SKF catalogue while adhering to constraints on bore diameter and performance requirements. Bearings were evaluated based on dynamic and static loading criteria, considering shaft bending and twisting constraints, life expectancy targets, and axial-radial load relationships. Iterative calculations identified initial bearing models, but due to bore diameter limitations, larger conservative alternatives were selected: model 6305 for the input shaft, model 6306 for the idler shaft, and model 6307 for the output shaft. The selected bearings meet the required load capacities and dimensional constraints.

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1. Introduction

Birdie Boys have been tasked with designing a transmission system for an electrically powered vehicle, specifically a luxury golf cart intended for use at private golf courses in the Edmonton area. This report summarizes and describes the complete analysis used to reach the transmission design as well as the final design decisions. The design consists of a three shaft, four gear system which transmits power from the HPEVS AC-9 48V Electric Motor to the drive system. The primary objective of this report is providing analysis and determining final selections for gears, shafts, and bearings. The analysis was executed for gears, shafts, and bearings for an optimal design that can withstand operational loads, ensure reliability and efficient power transmission. Birdie Boys have selected the KHK gear catalog to choose high-quality gears that meet AGMA standards. Helical gears were selected for their quieter operation and smoother meshing compared to spur gears. The shafts were custom machined to ensure compatibility with the selected gears. Bearings were selected from SKF catalog to ensure compatibility with the recommended shaft diameter as well as manufacturability of the assembly. The initial target linear velocity of the golf cart was 30 km/h which required an output of 348 RPM, but restrictions were imposed on the design due to necessity of meeting minimum shaft diameter as well as limited variety of helical gears on the market. Considering the restrictions, the new target linear velocity of the golf cart is in the range of 25 - 35 km/h.

2. Design Methodology

The design process will be explained in this section. This includes the fundamental design specifications and assumptions.

2.1. Design Specifications and Assumptions

As an overview on the transmission assembly and how they would look like, figure 1 is provided below. See Appendix D for the full drawing package.

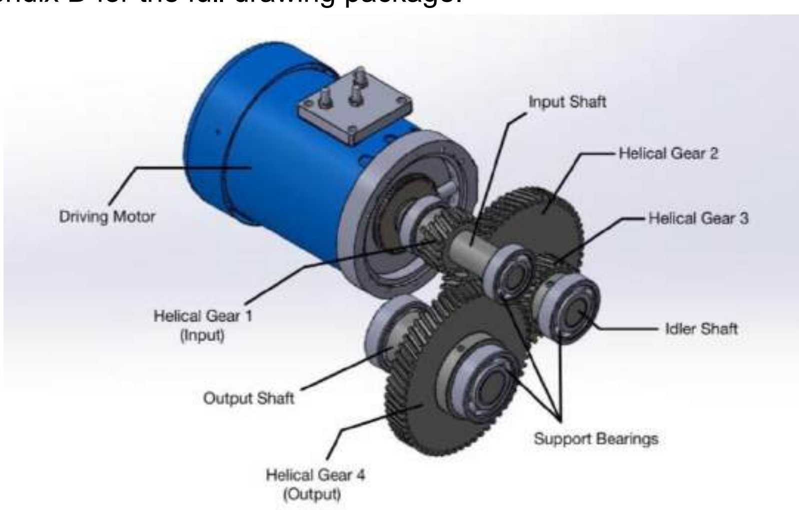

Figure 1 *- Final transmission layout.*

An existing electric golf cart in the market was used as a reference for the client's golf cart design [1]. The initial plan for the output speed was 30km/h but this was revised due to the limited amount of options of helical gears available in the market. Therefore, the output speed was revised to the range of 25-35 km/h, which lies in the acceptable range of typical golf cart speeds being 19-40 km/h [2]. The engine chosen for the vehicle is HPEVS AC-9 48V Electric Motor. Considering the maximum torque and RPM of the motor, the ideal gear ratio fell into the range of about 7.5:1 to 10:1 which aligns with the goal of 25-35 km/h (check Appendix C - C7 for detailed calculations). Following are the detailed specifications assumptions of the overall design, which support the design analysis in subsequent sections:

1. Motor Specs (See Appendix B - Figure 1)
    a. Maximum horsepower: 23.3hp at 3000 RPM
    b. Ideal Torque: 41.3 lb•ft (56 N•m) at 3000 RPM
2. Cart specs
    a. Total mass of cart (Including passengers, cargo, transmission): 600kg
        i. Cart mass: 350kg
        ii. Carrying capacity: 250kg
        iii. Tire Size: 18 x 8.50 - 8
3. Weather condition
    a. Temperature: Maximum of 40°C
    b. Weather: Rainy conditions for extreme condition (conservative) analysis
4. Due to the importance of creating a luxurious consumer experience, only helical gears were used because of the smooth and quiet nature compared to other types of gears
5. Conservative life cycle of 16.632×10^7 for input shaft and input pinion (Appendix C - C1 for derivation of life cycle assumption)
6. Assumed maximum incline
    a. 15 degrees
7. Required output torque on rear axle shaft: Greater than 124.4 Nm (Appendix C - C6)
    a. This required torque assumes a non-slip condition (tire isn't screeching). Maximum torque from the rear axle occurs when maximum static friction is achieved - at maximum incline (15 degrees) and maximum coefficient of friction (1.0).
    b. This was derived for a conservative analysis of the load on transmission, as all other vehicle driving conditions will require less torque than this maximum torque value
8. Operating temperature
    a. Assume a cooling system will be installed to ensure the gearbox transmission temperature is below the maximum operating temperature of 220 degrees Fahrenheit [3].
9. Meet AGMA standards
    a. Selected designs (Gear sets and shafts) must adhere to AGMA standard ANSI/AGMA 2001--D04 and AGMA 908-B89, to meet the AGMA requirements

for operating strength, durability, and precision. Any design that doesn't satisfy AGMA standards will automatically be eliminated

10. Acceptable gear ratio and corresponding maximum cart speed
    a. Gear ratio range: 7.5:1 - 10:1
    b. This ratio was calculated based on assuming the acceptable output speed in range of 25 km/h and 35 km/h (Check Appendix C - C7 for calculation)

2.2. Design Analysis Approach

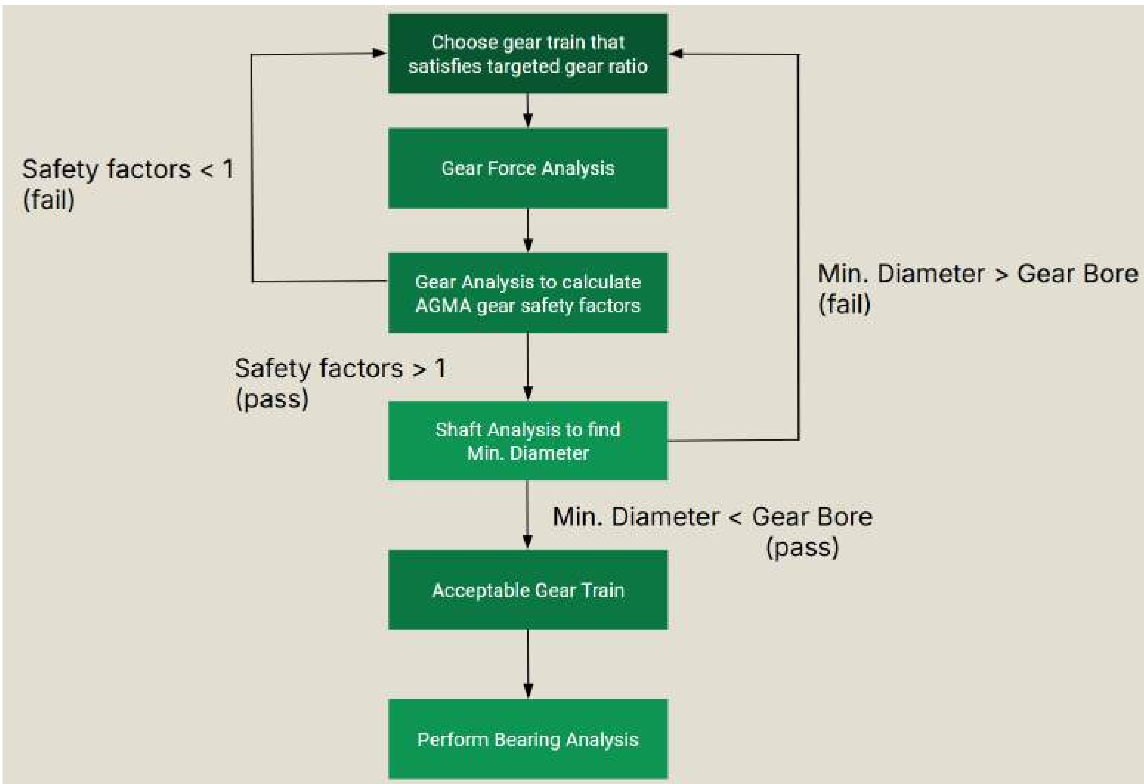

Figure 2 - Design analysis approach flow chart.

3. Analysis

3.1. Required Gear Ratio and Calculation

First, an ideal gear ratio to achieve the desired output speed range was calculated. The calculation resulted in the targeted gear ratio of 7.5:1 to 10:1 (Appendix C - C7). However, the torque also had to be considered to finalize the ideal gear ratio range, which resulted in the range of 2.2:1 to 8.3:1 (See Appendix C - C6 for detailed calculation). However, the gear ratio is still allowed to exceed the targeted 8.3:1, as it would just be an overdesign that would result in the tire to screech at maximum torque on the steepest incline with the maximum coefficient of static friction. Therefore, the ideal gear ratio is finalized as 7.5:1 to 8.3:1, though a range of 8.3:1 to 10:1 remains acceptable.

3.2. Gears

3.2.1. Gear Analysis Introduction

KHK Gears was selected as the gear vendor as it had the most options available [11]. After determining the required gear ratio for the transmission design, key criteria for the selected gear sets were identified:

1. AGMA bending safety factors (n_b) must exceed 1.2 and contact safety factors (n_c) must exceed 1.15 in accordance with the gear manufacturer website [12].
   a. Bending and contact stresses were calculated using the AGMA 2001-D04 equations, along with geometric factors and properties from the AGMA 908-B89.
   b. Bending and Contact Safety Factor Equations:

$$n_b = \frac{S_{fb}}{\sigma_b} \quad \text{(Eq. 1)}$$

$$n_c = \left(\frac{S_{fc}}{\sigma_c}\right)^2 \quad \text{(Eq. 2)}$$

   c. Strengths are corrected to account for factors that affect the performance and durability of the gears, such as number of cycles, operating temperature, and material properties
2. Additionally, the bore diameters of the available gears must be equal to or larger than the minimum shaft diameter at the gear locations to prevent excessive shaft bending or twisting.

Safety factors provide a measure of a design's reliability by comparing the corrected material strength to the applied stresses. If the applied bending stress exceeds the corrected material bending strength, the safety factor for bending will drop below 1. This indicates that the component is overstressed and at significant risk of fatigue failure due to bending. The same logic applies to contact safety factor. A safety factor greater than 1 indicates that the design has sufficient strength to withstand the applied stresses, ensuring reliability under the specified operating conditions. However, It's always best to attempt to achieve a higher safety factor to ensure minimize the risk of the gear failure. During a long term operation, the gear with the safety factor closest to 1 will fail first.

To begin the analysis and verify whether the selected gear train met these criteria, the gear material was initially assumed to be AISI 4140 steel, the strongest material available in the catalog. This choice allowed for potential downgrading to a less expensive material if AISI 4140 proved to be overly robust for the transmission's requirements.

3.2.2. Gear Analysis Procedure and Assumptions

Some key assumptions were made before conducting gear analysis based on design assumptions and in accordance to AGMA 2001-D04:

1. Assume lifespan of 166.632 million cycles for gear on input shaft

    a. Idler and output shaft gear cycle would decrease by the respective gear mesh ratio (See calculation in Appendix C - C4 → gear SMath).
2. Assume all gears have the same hardness (all gears are the same material).
3. Assume operating temperature is less than 250°F.
4. Assume reliability of 99%.

Following is the gear analysis procedure (See Appendix C - C4 for detailed execution):

1. Select Gear Train: Choose a gear train that meets the desired gear ratio and aligns with design requirements.
2. Identify Forces: Calculate axial, tangential, and radial forces, as well as moments induced by the gears, based on the transmission design. (Check appendix C - C3 for example calculation)
3. Determine Bending/Contact Stress Factors: Using AGMA 2001-D04 standards, calculate the bending and contact stress factors (e.g., application factor (K_a) of 1.25, reliability factor (K_r) of 1 for 99% reliability, etc.) based on gear geometry, forces, and assumptions.
4. Evaluate Bending and Contact Stresses: Calculate the bending and contact stresses using the determined factors.
5. Strength Assessment:
    a. Determine uncorrected bending and contact strengths from AGMA standard (material-specific properties).
    b. Apply correction factors for life span, temperature, materials, and operating conditions to adjust these strengths.
6. Safety Factor Verification: Ensure the calculated safety factors (strength-to-stress ratios) exceed 1.2 for bending and 1.15 for contact to confirm feasibility.
7. Shaft Analysis:
    a. Calculate the minimum shaft diameter needed at gear locations to handle forces without excessive bending or twisting.
8. Bore Diameter Check:
    a. If gear bore diameters are too small for the calculated shaft diameter, restart with a new gear train as there are not many options for helical gears with bigger bore diameter in catalog.
    b. If bore diameters are bigger, the design is acceptable (smaller helical gear bore diameters are usually available in catalog).
9. Optimize Design: In consideration of the design decision matrix (See Appendix A - Table 2) that emphasizes the importance of compact size and durability, if safety factors are not met or better optimized designs for size or safety could be considered, repeat Steps 1–8.

3.2.3. Results

Detailed overview of the gear analysis result is outlined in Appendix A - Table 3.

The first iteration of the gear train, the gear ratio was 7.5:1, which lied right in the predetermined ideal gear ratio from section 3.1. Looking at the safety factor, the gear on the idler shaft is likely to fail from contact because its contact safety factor of 1.1555 is smaller than all the other safety factors of gears and pinions in the same set. Although this gear train passed the safety factor criteria and met the targeted gear ratio, the input pinion bore diameter of 20mm was smaller than the suggested shaft diameter of 30mm, which meant new gear train had to be selected instead. Moreover, it was determined that only AISI 4140, which was the strongest steel in the catalog, will be considered for the gear materials, because AISI 1045 steel would certainly result in a contact safety factor of less than 1.15.

The second iteration of the gear train resulted in the gear ratio of 8:1, which also meets the ideal gear ratio. Looking at the safety factor, gear 2 will most likely fail first from bending due to a safety factor of 1.3183. This gear train passed both criteria outlined in 3.2.1 and also met the ideal gear ratio. However, considering the emphasis of compact size and high reliability from the design decision matrix, a gear train that consists of gears with smaller pitch diameter and bigger face width was explored.

The third gear train resulted in the gear ratio of 7.5:1. Looking at the safety factor, gear 3 is most likely to fail due to contact, having the smallest safety factor of 1.3651. This gear train also passed both criteria and met the ideal gear ratio. The smallest safety factor of 1.3651 from this iteration is greater than the smallest safety factors from the previous iterations, and also provides a more compact design compared to the second iteration as the chosen gears have the smaller pitch diameter, making it the final selection.

Table 1 - *Final gear train selection*

Iteration	Gear	Gear Type	Number of Teeth	Pitch Diameter (mm)	Bore Diameter (mm)	Material	Bending Safety Factor	Contact Safety Factor	Gear Ratio of the Train	Output speed of the set (km/h)	Output Torque (N m)
3	1	Helical (Right)	20	60	30	4140 Steel	3.1211	2.5265	7.5:1	34.472	420 Nm
	2	Helical (Left)	60	180	32	4140 Steel	2.8855	5.486	…		
	3	Helical (Left)	24	72	32	4140 Steel	1.3885	1.3651	…		
	4	Helical (Right)	60	180	40	4140 Steel	1.5571	3.7817	…		

3.3. Shafts

3.3.1. Shaft Analysis Introduction

Shaft analysis was conducted upon verifying all the gear safety factors were met. Shaft steps were incorporated into the design to prevent axial movement of gears and bearings. Keys and keyways were used to prevent slippage of gears on the shaft and were selected in accordance to ANSI-B17.1 [15] standard which governs geometry of keys. To account for the stress concentration by shaft steps and keyways, the design uses fillets to reduce stress contraction factor and improve the fatigue life.

After preliminary research on common golf transmission shaft materials, AISI 4140 steel [10] was chosen for its high yield strength, durability, and fatigue resistance, making it ideal for high-torque applications. These design considerations ensure all shafts within the transmission operate as intended.

Key design objective is to prevent failure due to operation loads, including bending, axial forces, torque, and fatigue. Moreover, design efficiency must be optimized through iterations by minimizing shaft length for better overall compactness and reduction of weight. Detailed shaft analysis calculations can be found in Appendix C - C9 through C15.

In order to pass the shaft analysis, the following criteria must be met:

1. Strength Failure (DE Elliptic Criteria)
    a. First Cycle Yield:

$$\sigma'_{max} < \frac{S_y}{n} \quad \text{(Eq. 3)}$$

    b. Fatigue Failure:

$$\left(\frac{n\sigma'_a}{S_e}\right)^2 + \left(\frac{n\sigma'_m}{S_y}\right)^2 < 1 \quad \text{(Eq. 4)}$$

2. Shaft Twist ≤ 3 deg/m
3. Linear Deflection at gears ≤ 0.127 mm
4. Angular Deflection at gears ≤ 0.03 deg (0.0005 rad)
5. Angular Deflection at bearings ≤ 0.004 rad

3.3.2. Assumptions

Some key assumptions were made to conduct shaft analysis:

1. Assume the operating temperature is doesn't exceed 220 °F
2. Assume lifespan $1.6632 \times 10^8 \geq 10^6$ cycles therefore, infinite life (see appendix C - C1)
3. Assume 95% reliability → designing for moderate load conditions, high-quality materials, ensure durability under controlled, low-stress golf course environments
4. Assume constant torque, but will fluctuate between 0 (when not in use) and operating torque
5. Assume forces on shaft are fully reversed, due to shaft rotation causing a critical element to switch between compression and tension of equal magnitudes

3.3.3. Procedure

1. Create diagrams and FBDs. Use the calculated gear forces and resultant moments acting on the shaft to calculate the support reactions.
2. Once the forces are calculated, use singularity functions to find shear force and bending moments.
3. After considering the SCF for the keyway, calculate the mean and alternating torque and bending moment on the shaft.
4. Assume a minimum diameter of 0.11-2" which will be checked later.
5. Calculate the corrected endurance strength using correction factors. Use the assumptions to get the respective values for correction factors.
6. Use DE Elliptic failure criteria (AGMA guidelines) to iteratively solve for the minimum diameter at the gear locations. Utilize MATLAB for the iterations (Appendix C-10)
7. Check there is no first cycle failure → consider startup torque (maximum torque)
8. Check the assumptions with the new minimum diameter and ensure they are valid.
9. Use the singularity functions to solve for slope and deflection.
10. Check the evaluation criteria for deflection and twist as outlined in the introduction.
11. If any of the criteria is not met, assume a new minimum diameter and try again until the criteria is met.
12. Considering the design decision matrix (Appendix A - Table 2), which emphasizes compactness, the length of the shaft was optimized by positioning the gears between the shaft shoulders and bearings. See the following figure for schematic.

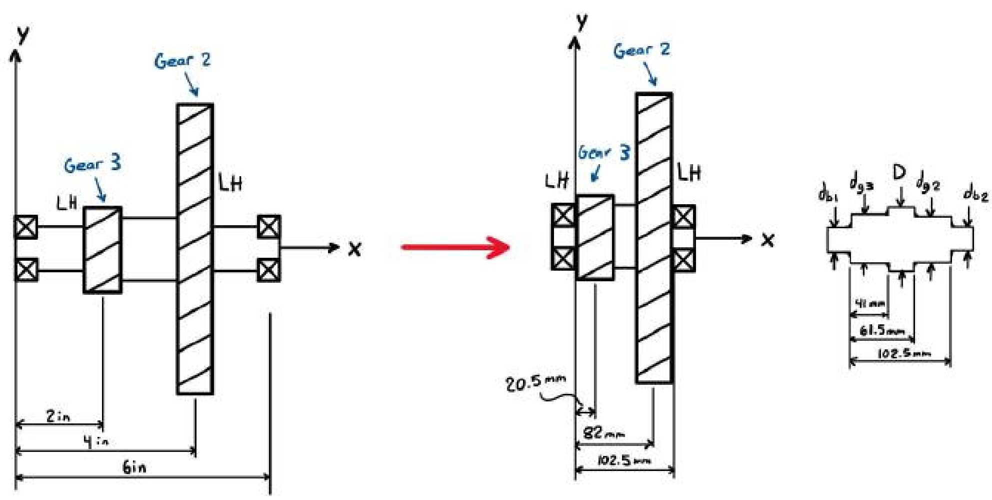

Figure 3 - Schematic of changing original shaft length to optimized shaft length.

3.3.4. Results

Given that all the gear lengths are 41 mm and the gears are spaced apart by 20.5 mm (half of the gear length), the length in between the bearings for all the shafts is 102.5 mm.

Using re-iteration, the input shaft passed all of the above criteria and it was concluded that the minimum shaft diameter at the pinion is 30 mm while the minimum shaft diameter at bearing 1 and 2 is 25 mm. Additionally, the fatigue and yield safety factors were recalculated to be 8.80 and 17.02 respectively for this shaft diameter. It can be noted that the obtained safety factors are both greater than the shaft analysis design safety factor of 1.872, indicating the shaft is safe for use. Note, such a high safety factor is due to upsizing to the only available gear bore diameter. The computed minimum diameter from the DE Elliptic failure criteria gives 0.821" (20.9 mm) which is significantly smaller than the finalized shaft diameter of 30 mm.

For the idler shaft, a minimum diameter around both gears of 32 mm and a minimum diameter around both bearings of 30 mm has been specified which meets all required conditions and defined evaluation criteria. Additionally, the fatigue and yield safety factors were recalculated to be 3.49 and 6.89, respectively for this shaft diameter.

On the output shaft, a minimum diameter at the gear of 40 mm and a minimum diameter around both bearings of 35 mm has been specified. The fatigue and yield safety factors were recalculated to be 2.96 and 5.77, respectively for this shaft diameter.

3.3.5. Key Selection

Keys play a critical role in preventing tangential gear slippage on the shafts, ensuring reliable and efficient power transmission. Since keys don't require any design customization, McMASTER-CARR was selected as a vendor for keystock [13]. The chosen material was 1045 Carbon Steel because it is a cost effective choice with sufficient strength. There was no need to use stainless steel for the keys as the transmission is fully covered in its housing and does not require corrosion resistance in wet environments.

The procedure for key calculations are as follows:

1. Select key size according to the ANSI standard, which is dependent on shaft diameter.
2. Calculate the bending and shear stresses on the key.
3. Calculate the resulting Von Mises stress.
4. Compare the Von Mises stress with yield stress of 1045 Carbon Steel.
5. Ensure that all keys pass the stress-strength comparison.

For detailed key failure calculations see Appendix C - C15.

3.4. Bearing

3.4.1. Bearing Introduction

Upon completion of the shaft analysis, bearing analysis had to be carried out to finalize the transmission design. Due to the precalculated shaft diameters and geometry (See Appendix B - Figure 4) as well as the demand from MECE 360 Inc., checklist for the bearing selection was as follows:

1. Bearing bore diameter must be smaller than the gear's bore diameter (30mm for input shaft, 32mm for idler, 40mm for output). Gear is assembled to the shaft before the bearing, so the shaft diameter at the bearing must be designed smaller than the shaft diameter.
2. Bearing bore diameter must be close to the gear bore diameter (but still smaller) which would likely satisfy the shaft deflection and twist requirements and minimize the stress concentration of the shaft.
3. Bearing cannot be customized and has to be chosen from the bearing catalogue
4. Single row ball bearings will be considered to enable calculations using equivalent dynamic load table (See Appendix B - Figure 9)

3.4.2. Bearing Analysis Assumptions and Procedure

Assumptions:

1. All selected bearings have inner rotating rings (Rotation Factor, V = 1)
2. Operating temperature of the transmission never exceeds 240 degrees fahrenheit as per design specification, to avoid damage to lubrication. [14, p. 37]
3. Life expectancy of 166.32 million cycles for input shaft bearings, 55.44 million for idler, 22.176 million for output, as per gear analysis

Procedure (See appendix C-C16 for detailed execution of this procedure):

1. Take the reaction forces calculated from gear/shaft analysis to calculate dynamic loading
2. Reference SKF bearing catalogue (Series 634 - 6324 M) to find the corresponding static loading
3. Referring to the equivalent dynamic bearing load table, if the axial load divided by the radial load at bearing is greater than the "e" value, interpolate the X and Y factors. If less than "e", no need to consider the axial load
4. Using the X and Y factors, calculate the equivalent dynamic bearing load, F_{eff}
5. Treating F_{eff} as the new radial load, calculate the new dynamic load and check its corresponding static load and bearing model
6. Repeat steps 3-5 until the iterations converge to the same bearing model
7. Ensure the bearing's bore diameter is close to but smaller than the gear bore diameter
8. If the bore diameter is too small, select the bearing with a bigger diameter from the catalogue. Bearings with a greater diameter from the same catalogue can carry even greater loads so it would be a conservative design.

3.4.3. Results

The bearing analysis was conducted for a single row ball bearing on SKF bearing catalogue, series 634 - 6324M. For the bearings on the input shaft, the iterations converged to bearing model 6300 and 6303 for left and right bearing respectively (left bearing is on the opposite side of the motor, right bearing on the motor side). However, these two bearings had the bore diameter of 10mm and 17mm respectively, which was significantly smaller than the gear bore diameter of 30mm. Therefore, bearing 6305 was chosen as it had a greater diameter of 25mm, which was also smaller than the input shaft gear bore diameter of 30mm.

For the bearings on the idler shaft, the iterations converged to bearing model 6303 and 6301 for left and right bearing respectively. However, these two bearings had the bore diameter of 17mm and 11mm respectively, this was also significantly smaller than the gear bore diameter of 32mm. Therefore, bearing 6306 was chosen as it had a diameter of 30mm, which was the closest to the idler shaft gear bore diameter.

For the bearings on the output shaft, the iterations converged to bearing model 6303 and 6302 for left and right bearing respectively. However, these two bearings had the bore diameter of 17mm and 15mm respectively, this was also significantly smaller than the suggested minimum diameter of 40mm. Therefore, bearing 6307 was chosen as it had the closest diameter of 35mm.

4. Design Compliance

All initial specification requirements were met, with the exception of the initially set maximum speed of 30 km/h. The transmission would require a speed reduction of 8.6:1 or higher in order to meet this maximum speed. Due to a lack of available helical gears with high teeth count that fit the criteria of gear bore diameter greater than minimum shaft diameter, the gear ratio had to be adjusted. This meant that the maximum speed of 30 km/h had to be adjusted to the range of 25 - 35 km/h. Although the team initially imposed a specification of 30km/h, it remained within the acceptable range of a mainstream golf cart speed, making it a justifiable change.

5. Project Management

5.1. Time Management

Appendix A - Table 1 shows the finalized GANTT chart used throughout the entire project. Overall a majority of the projected time allocation remained accurate. However, there were small deviations during heavy exam preparation periods, when more time was spent on the project in the 1-2 weeks leading up to deliverables dates. Despite the deviations, Birdie Boys were able to complete all required tasks on time while meeting all the goals and criterias set for the transmission design project. The chart below shows the time spent on each aspect of the design process.

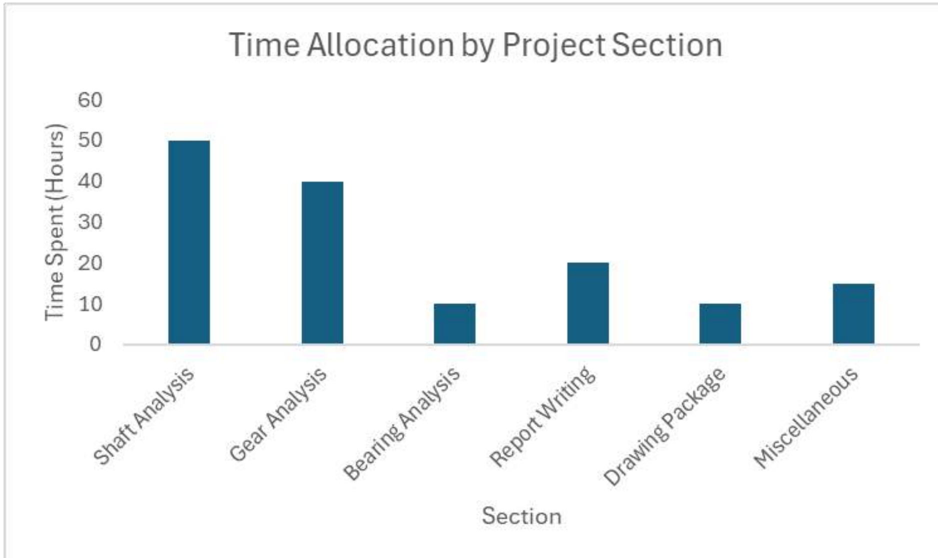

Figure 4 - Time Allocation by Project Section

5.2. Meeting Minutes

Appendix E shows the detailed meeting minutes that were taken and used throughout the design process. Meeting minutes were mostly used to keep track of the tasks and deadlines for each member, and the plans for subsequent meetings.

6. Other Considerations

Time constraints required selecting a design early on and committing to it, limiting the exploration of alternatives. Potential improvements include incorporating additional shafts and gears to achieve the exact target speed of 30 km/h, and considering energy-efficient transmission designs. Furthermore, optimizing the connection to the golf cart's axle to minimize energy loss during power transfer could enhance performance.

7. Conclusion

This report provides an in-depth analysis of the procedure conducted and presents the final design for the luxury golf cart transmission. Following material selection, the gear, shaft, and bearing analysis were carried out. Gear analysis ensures the selection of optimal gear sets in accordance with the design decision matrix that are compliant to AGMA standards. Shaft analysis ensures designs capable of withstanding operating conditions in compliance with AGMA standards. Finally, bearing analysis ensures it is strong enough to withstand operational forces.

While the initial criteria were met, some areas showed room for improvement. For instance, gear 3 had the lowest bending and contact safety factors of 1.36 and 1.39 respectively, suggesting a potential risk of failure as the manufacturer recommended guidelines for bending and contact safety factor of 1.2 and 1.15, respectively. Enhancing these safety factors, without modifying the design, would involve selecting stronger materials. However, material availability from manufacturers limited alternatives that met all design constraints.

In summary, the design fulfills all fundamental requirements. However, there is room for improvement, particularly in aligning with the design decision matrix's focus on reliability, durability, and size. Refining key gear specifications such as pitch diameter, number of teeth, and material could significantly enhance the design's overall performance.

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Appendix A (Tables)

***Table 1 -** GANTT chart used to track progress throughout the design process.*

PROJECT TITLE	Golf Cart Transmission Design	COMPANY NAME	Birdie Boys
PROJECT MEMBERS	Ben Cote, Cole Pilgrim, Hyunseok Jang, Tahir Anjum, William Plate	DATE	9/4/2024

TASK TITLE	TASK OWNER	START DATE	DUE DATE	DURATION	PCT OF TASK COMPLETE
Letter of Intent (LOI)	Sept 13, 2024				
Basic Background Research	ALL	9/4/2024	9/9/2024	6	100%
Scope of Vehicle	ALL	9/9/2024	9/11/2024	3	100%
Scenario and Design Constraints	ALL	9/9/2024	9/12/2024	4	100%
Team Name	ALL	9/12/2024	9/12/2024	1	100%
Logo	William	9/12/2024	9/12/2024	1	100%
Concept Report (CR) - Progress Report 1	Sept 27, 2024				
Assumptions & Group-Imposed Requirements	ALL	9/16/2024	9/18/2024	3	100%
Project Expectations	ALL	9/16/2024	9/18/2024	3	100%
Gantt Chart	Ben	9/18/2024	9/20/2024	3	100%
Legal/Standard Requirement Research	Hyunseok	9/18/2024	9/20/2024	3	100%
Summary Timeline (Table)	Ben	9/23/2024	9/24/2024	2	100%
Selection of Motor	ALL	9/23/2024	9/24/2024	2	100%
Design Specification Matrix	Hyunseok & Cole	9/19/2024	9/24/2024	6	100%
Write Progress Report 1	ALL	9/23/2024	9/25/2024	3	100%
Review Report 1	ALL	9/27/2024	9/27/2024	1	100%
Analytical Report (AR) - Progress Report 2	Nov 8, 2024				
Determining Necessary Analyses	ALL	10/1/2024	10/4/2024	4	100%
Group-to-Group Peer Review of PR 1	ALL	10/7/2024	10/7/2024	1	100%
Assumptions, Schematics, FBD's	Ben & Hyunseok	10/7/2024	10/11/2024	5	100%
Gear Analysis	Will & Hyunseok	10/14/2024	10/18/2024	5	100%
Shaft Analysis	Ben	10/21/2024	10/25/2024	5	100%
Bearing Analysis	Hyunseok	10/28/2024	10/30/2024	3	100%
Solid Modeling	William	10/21/2024	11/1/2024	11	100%
Determining Future Analysis	ALL	10/30/2024	11/1/2024	2	100%
Write Progress Report 2	ALL	11/1/2024	11/6/2024	6	100%
Review Report 2	ALL	11/8/2024	11/8/2024	1	100%
Final Report	Dec 9, 2024				
Group-to-Group Peer Review of PR 2	ALL	11/18/2024	11/18/2024	1	100%
Final Analysis	Ben & Hyunseok	11/18/2024	11/22/2024	5	100%
Finalize Solid Models & Drawings	William	11/18/2024	11/22/2024	5	100%
Discuss Design (design compliance)	ALL	11/25/2024	11/27/2024	3	100%
Final Timeline & Hours Spent	Ben	11/27/2024	11/27/2024	1	100%
Write Cover Letter	Ben & Cole	11/27/2024	11/29/2024	3	100%
Write Final Report	ALL	11/29/2024	12/4/2024	6	100%
Prepare Team Oral Presentation	ALL	12/2/2024	12/6/2024	5	100%
Review Final Report	ALL	12/5/2024	12/6/2024	2	100%
Final Team Presentation (Oral Presentation)	ALL	12/7/2024	12/7/2024	1	100%
Final Peer Review of Group Members	ALL	12/9/2024	12/9/2024	1	100%

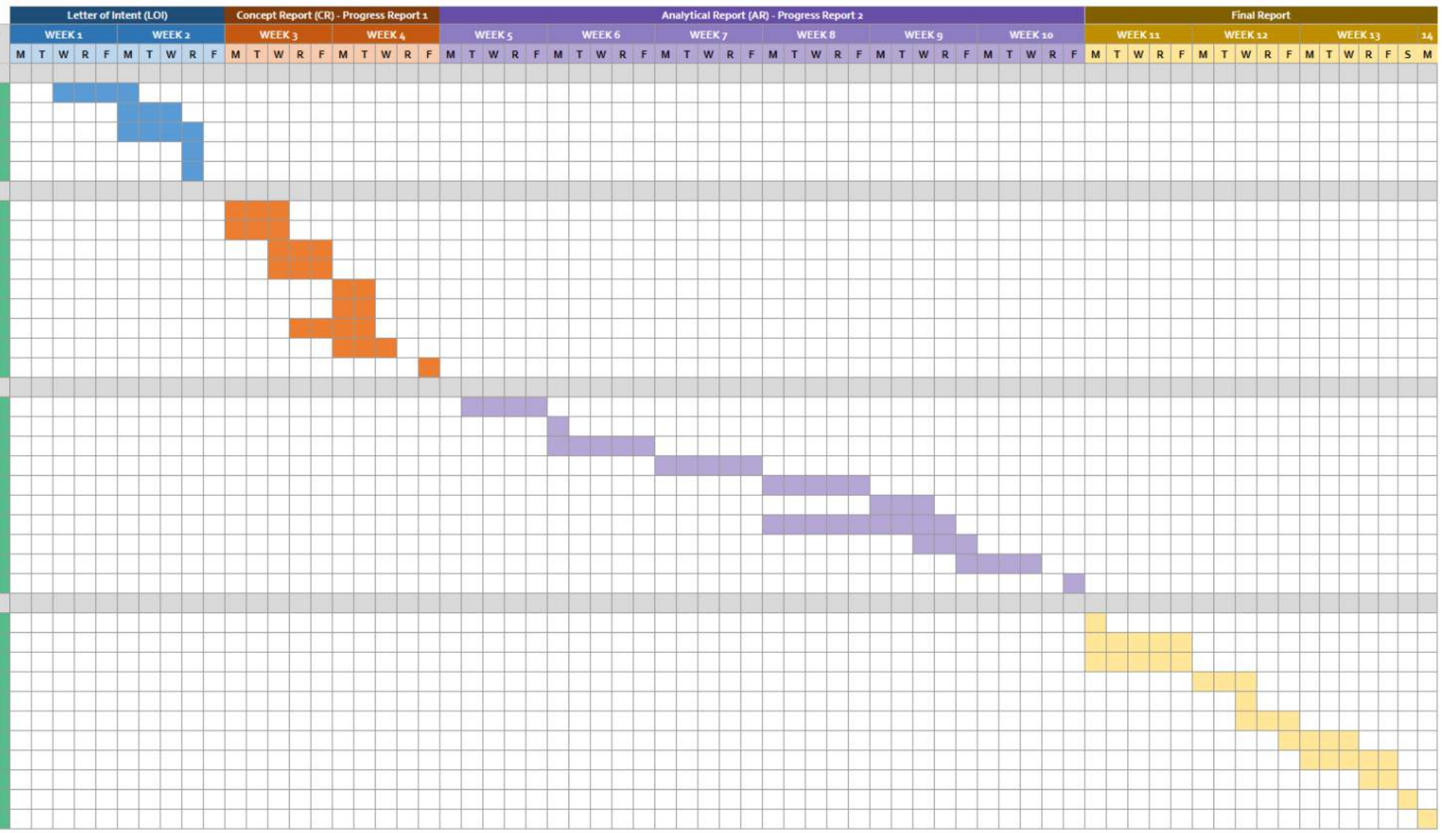


Table 2 *- Decision Matrix Applied to Gear Set and Shaft Selection (0-30% is not very important, 40-70% important to an extent, 80-100% very important).*

Factors	Description	Weighting (/100%)
Size	Compact (Smaller pitch diameter of gears, and smaller diameter and length of shafts and if possible) and lighter gear sets and shafts are preferred. It should easily fit within the wheelbase of 165cm X 135 cm, which is not difficult to achieve considering gear and shaft dimensions will be well within this dimension. Hence this factor was assigned the lowest weighting. This weight is increased from the previous design matrix since minimizing the transmission size can also be preferred due to its contribution to improved mileage.	60%
Durability	Should withstand 16.632×10^7 input cycles without failure (output cycle will be less than the input cycle due to the gear ratio). This is computed based on the assumption of 3 years of operation, for 2 hours a day, 7 days a week, for 5 and a half months (May until mid October). This life cycle assumes that a luxury golf course will replace outdated golf carts frequently (hence only 3 years of use) and that the golf cart will only be in active operation for a maximum of 2 hours during 10 hours of playing time per day. Since the transmission failing would result in dissatisfaction of customers as well as higher cost for warranty and maintenance, this was assigned higher weighting than the size and manufacturability factor.	70%
Operating Environment	The transmission box gets as hot as 175-220 Fahrenheit during operation. The gears and shafts should withstand this temperature. Also, gear and shaft material should preferably be water & corrosion resistant. This is goes hand-in-hand with durability, and therefore assigned the same weight	70%
User Experience	Selected gears are assessed based on the noise level they produce. The gear set characteristics that reduce the noise level can be found in the following [16]. The shaft with less deflection will lower the chance of gear failure and reduce the risk of noise and vibration [17]. Since the design emphasizes a high-end driving experience (less vibration and noise of the cart), the weighting on this is assigned higher than the other categories.	80%

Manufacturability	Gear sets and shaft designs that would cause less complexity while resulting in less cost of production is preferred. However, due to our main customer being the luxury golf course, the cost and final price of the golf cart is less of a concern as long as the vehicle provides a superior driving experience. Therefore, it was assigned the lowest weight compared to other factors.	50%
Power Output	Should ensure that the gear ratio allows the tires to reach a maximum angular velocity of 220.458 - 464.143 RPM which corresponds to a linear velocity 19 - 40 km/h. Due to our motor's optimal output of 3000 RPM, we will be ideally using a gear reduction of 6.46:1 to 13.6:1 to reach our optimal speed. This is a necessary condition for the vehicle to operate according to the design purpose, and hence was assigned highest weighting. Whichever set that provides the speed closest to our optimal speed of 30km/h will be assigned a higher score. For the shaft, power dissipation can be reduced if we use a lighter shaft that has less bending and twisting.	100%

Table 3 *- Gear combinations (sets) Comparison*

Iteration	Gear	Gear Type	Number of Teeth	Pitch Diameter (mm)	Bore Diameter (mm)	Module (mm)	Material	Bending Safety Factor	Contact Safety Factor	Gear Ratio of the Train	Output speed of the set (km/h)	Output Torque (N m)
1	1	Helical (Right)	20	60	20	3	4140 Steel	3.1013	2.3529	7.5:1	34.472	420 Nm
	2	Helical (Left)	50	150	30	3	4140 Steel	2.8164	4.1716	…		
	3	Helical (Left)	20	60	20	3	4140 Steel	1.3269	1.1555	…		
	4	Helical (Right)	60	180	30	3	4140 Steel	1.5334	3.9203	…		
2	1	Helical (Right)	30	60	30	2	4140 Steel	1.4706	1.6893	8:1	32.318	448 Nm
	2	Helical (Left)	100	200	35	2	4140 Steel	1.3183	4.1242	…		
	3	Helical (Left)	25	75	35	3	4140 Steel	1.3191	1.3394	…		
	4	Helical (Right)	60	180	50	3	4140 Steel	1.4707	3.5456	…		
3	1	Helical (Right)	20	60	30	3	4140 Steel	3.1211	2.5265	7.5:1	34.472	420 Nm
	2	Helical (Left)	60	180	32	3	4140 Steel	2.8855	5.486	…		
	3	Helical (Left)	24	72	32	3	4140 Steel	1.3885	1.3651	…		
	4	Helical (Right)	60	180	40	3	4140 Steel	1.5571	3.7817	…		

Table 4 *- Values Used in Gear Analysis Process*

Gear Factors	Values
$K_a = C_a$ (Application Factor)	1.25
K_b (Rim Thickness Factor)	1
C_H (Hardness Factor)	1
K_I (Idler Factor)	1
$K_T = C_T$ (Temperature Factor)	1
C_f (Surface Finish)	1
$K_s = C_s$ (Size Factor)	1
$K_{mp} = C_{mp}$ (Load Distribution Factor)	1.6
K_{Lp} (Bending Life Factor)	0.9132
C_{Lp} (Contact Life Factor)	0.8543
$K_R = C_R$ (Reliability Factor)	1.00
C_p (Elastic Coefficient) [MPa^0.5]	1.91
S'_{fbp} (AGMA Uncorrected Bending Stress) [MPa]	230
S'_{fcp} (AGMA Uncorrected Contact Stress) [MPa]	1100

Appendix B (Figures)

Figure 1 *- HPEVS AC-9 48V Electric Motor Performance Graph:*

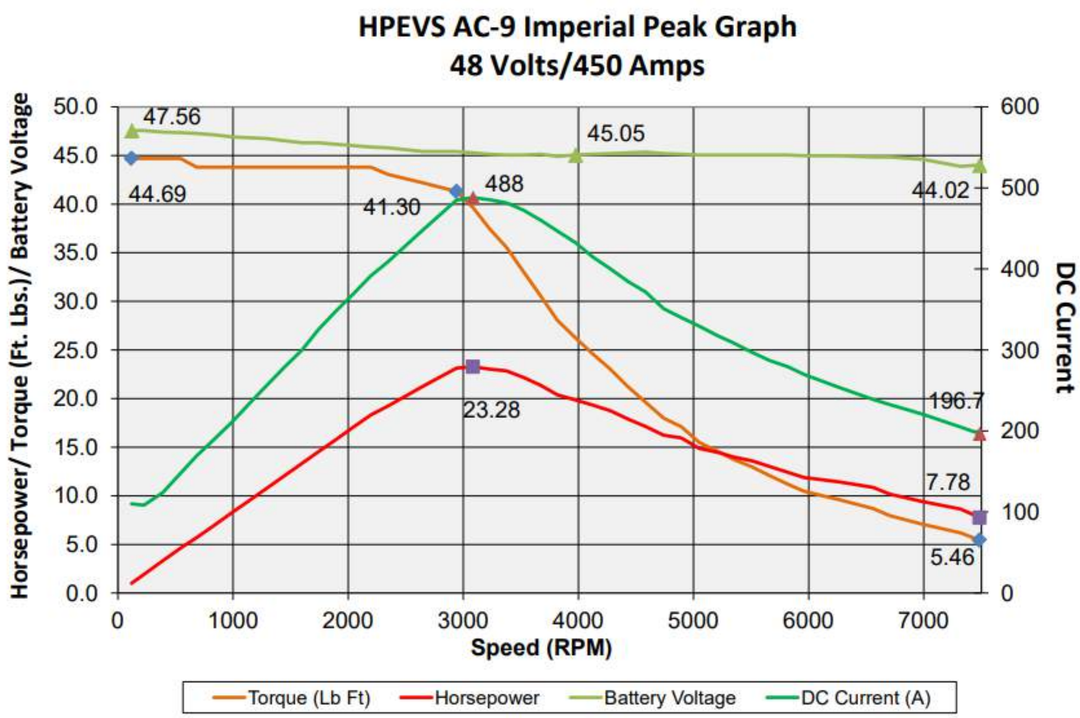


Figure 2 and 3 *- Life Factor graphs MecE360 Lecture 13:*

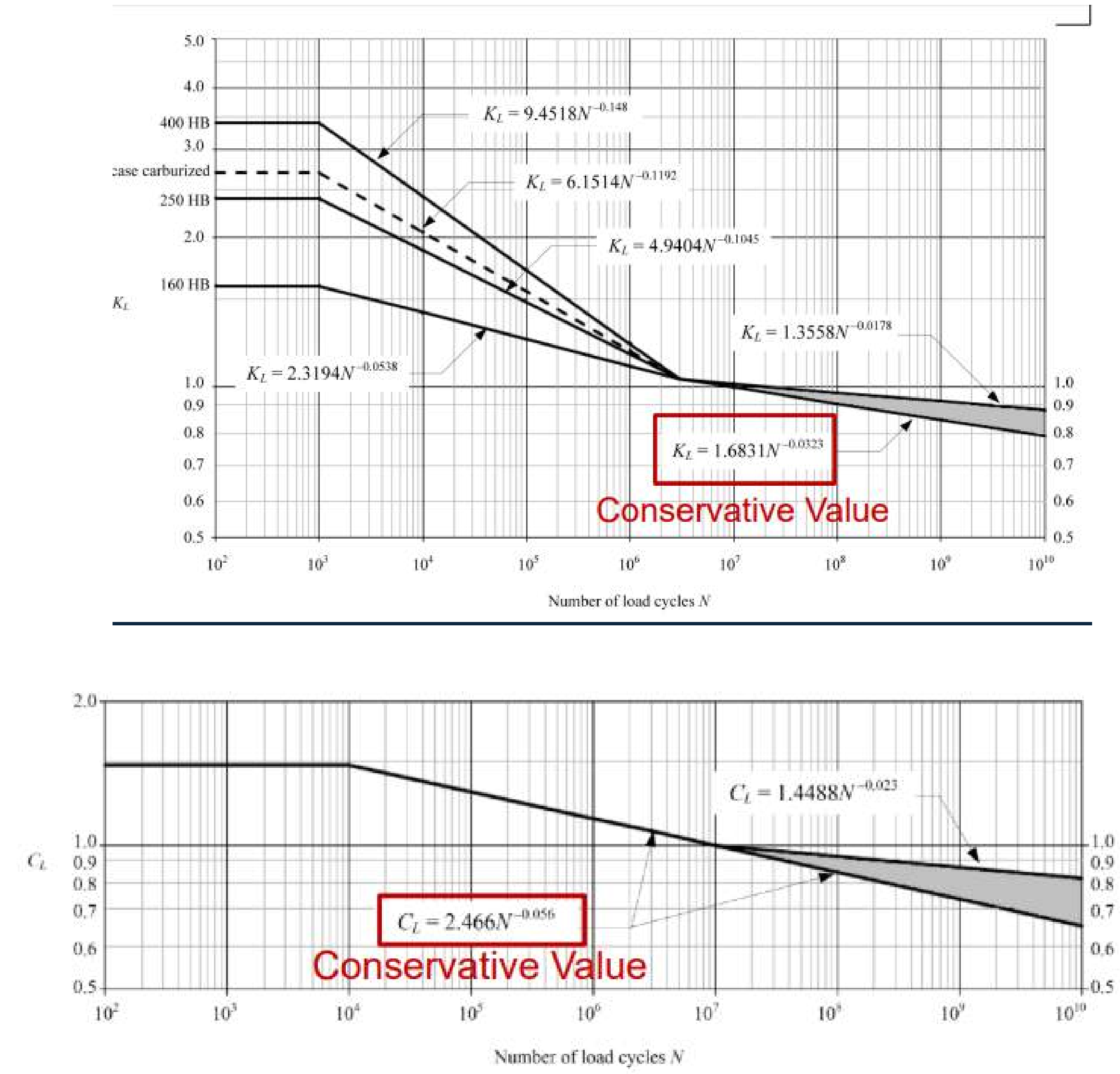


Figure 4 *- Idler Shaft Schematic*

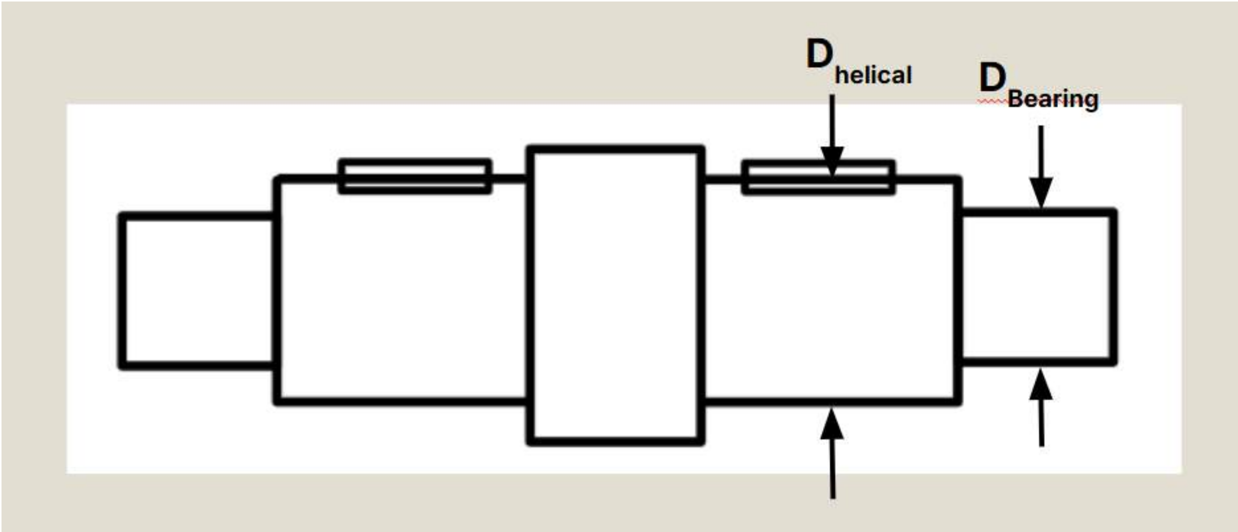
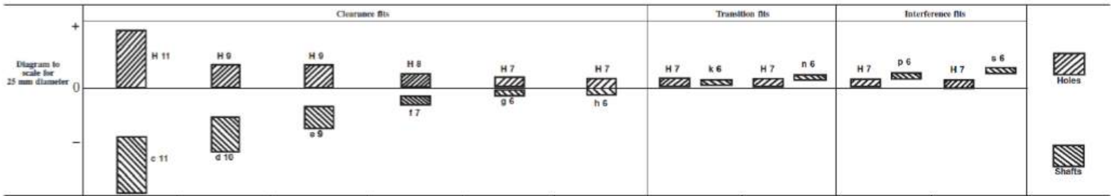


Figure 5 - This is a snapshot of the tolerance table provided by MecE 265 inc.

Table 12.1 Description of fits for circular objects

Fit	ISO Symbol		Description
	Hole Basis	Shaft Basis	
Clearance Fits	H11/c11	C11/h11	Loose fit. Wide tolerance
	H9/d9	D9/h9	Free running. Not when accuracy is important.
	H8/f7	F8/h7	Close running fit.
	H7/g6	G7/h6	Sliding fit. Not meant for the two parts running against each other but suitable for sliding adjustments.
	H7/h6	H7/h6	Snug fit but easy assembly
Transition Fits	H7/k6	K7/h6	Accurate location with some interference
	H7/n6	N7/h6	Use when larger interference is permissible
Interference Fits	H7/p6	P7/h6	For rigidity and correct alignment but not for power transmission
	H7/s6	S7/h6	Medium drive fit for ordinary steel parts. The tightest fit for cast iron parts
	H7/u6	U7/h6	High interference. Shrink fit recommended. Force fit for heavy parts only

Figure 6 - *This is a snapshot of the tolerance table provided by MecE 265 inc.*



Nominal sizes		Clearance fits												Transition fits				Interference fits				Nominal sizes	
		Tolerance		Tolerance		Tolerance		Tolerance		Tolerance		Tolerance		Tolerance		Tolerance		Tolerance		Tolerance			
Over	To	H11	c11	H9	d10	H9	e9	H8	f7	H7	g6	H7	h6	H7	k6	H7	n6	H7	p6	H7	s6	Over	To
mm	mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	mm	mm
—	3	+60 0	−60 −120	+25 0	−20 −60	+25 0	−14 −39	+14 0	−6 −16	+10 0	−2 −8	+10 0	−6 0	+10 0	+6 +0	+10 0	+10 +4	+10 0	+12 +6	+10 0	+20 +14	—	3
3	6	+75 0	−70 −145	+30 0	−30 −78	+30 0	−20 −50	+18 0	−10 −28	+12 0	−4 −12	+12 0	−8 0	+12 0	+9 +1	+12 0	+16 +8	+12 0	+20 +12	+12 0	+27 +19	3	6
6	10	+90 0	−80 −170	+36 0	−40 −98	+36 0	−25 −61	+22 0	−13 −28	+15 0	−5 −14	+15 0	−9 0	+15 0	+10 +1	+15 0	+19 +10	+15 0	+24 +15	+15 0	+32 +23	6	10
10	18	+110 0	−95 −205	+43 0	−50 −120	+43 0	−32 −75	+27 0	−16 −34	+18 0	−6 −17	+18 0	−11 0	+18 0	+12 +1	+18 0	+23 +12	+18 0	+29 +18	+18 0	+39 +28	10	18
18	30	+130 0	−110 −240	+52 0	−65 −149	+52 0	−40 −92	+33 0	−20 −41	+21 0	−7 −20	+21 0	−13 0	+21 0	+15 +2	+21 0	+28 +15	+21 0	+35 +22	+21 0	+48 +35	18	30
30	40	+160 0	−120 −280	+62 0	−80 −180	+62 0	−50 −112	+39 0	−25 −50	+25 0	−9 −25	+25 0	−16 0	+25 0	+18 +2	+25 0	+33 +17	+25 0	+42 +26	+25 0	+59 +43	30	40
40	50	+160 0	−130 −290																			40	50
50	65	+190 0	−140 −330	+74 0	−100 −220	−74 0	−60 −134	+46 0	−30 −60	+30 0	−10 −29	+30 0	−19 0	+30 0	+21 +2	+30 0	+39 +20	+30 0	+51 +32	+30 0	+72 +53	50	65
65	80	+190 0	−150 −340																	+30 0	+78 +59	65	80
80	100	+220 0	−170 −390	+87 +0	−120 −260	−87 −0	−72 −159	+54 0	−36 −71	+35 0	−12 −34	+35 0	−22 0	+35 0	+25 +3	+35 0	+45 +23	+35 0	+59 +37	+35 0	+93 +71	80	100
100	120	+220 0	−180 −400																	+35 +0	+101 +79	100	120
120	140	+250 0	−200 −450	+100 0	145 305	+100 0	−84 −185	+63 0	−43 −83	−40 0	−14 −39	+40 0	−25 0	+40 0	+28 +3	+40 0	+52 +27	−40 0	+68 +43	+40 0	+117 +92	120	140
140	160	+250 0	−210 −460																	+40 0	+125 +100	140	160
160	180	+250 0	−230 −480																	+40 0	+133 +108	160	180
180	200	+290 0	−240 530	+115 0	−170 −355	+115 0	−100 −215	+72 0	−50 −96	+46 0	−15 −44	+46 0	−29 0	+46 0	+33 +4	+46 0	+60 +31	+46 0	+79 +50	+46 0	+151 +122	180	200
200	225	+290 0	260 550																	+46 0	+159 +130	200	225
225	250	+290 0	280 570																	+46 0	+169 +140	225	250
250	280	+320 0	300 620	+130 0	−190 −400	+130 0	−110 −240	+81 0	−56 −108	+52 0	−17 −49	+52 0	+32 0	+52 0	−36 +4	+52 0	+66 +34	+52 0	+88 +56	+52 0	+190 +158	250	280
280	315	+320 0	330 650																	+52 0	+202 +170	280	315
315	355	+360 0	360 −720	+140 0	−210 −440	+140 0	−125 −265	+89 0	−62 −119	+57 0	−18 −54	+57 0	−36 0	+57 0	+40 +4	+57 0	+73 +37	+57 0	+98 +62	+57 0	+226 +190	315	355
355	400	+360 0	400 760																	+57 0	+244 +208	355	400
400	450	+400 0	440 840	+155 0	−230 −480	+155 0	−135 −290	+97 0	−68 −131	+63 0	−20 −60	+63 0	−40 0	+63 0	+45 +5	+63 0	+80 +40	+63 0	+108 +68	+63 0	+272 +232	400	450
450	500	+400 0	480 880																	+63 0	+292 +252	450	500

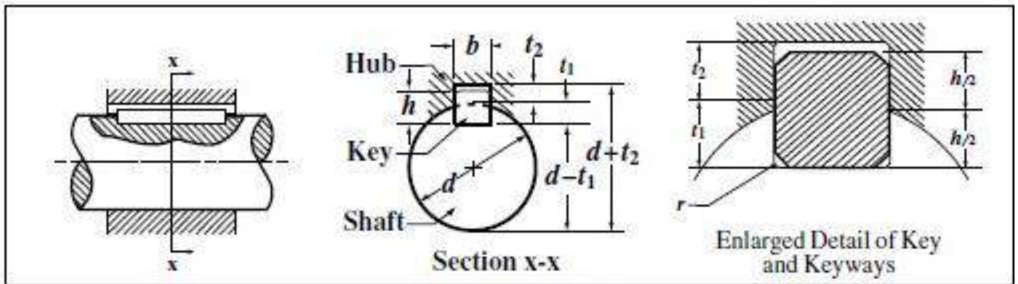
Table 12.2 Selected ISO Fits : Hole Bases

Figure 7 - *This is a snapshot of the tolerance table provided by MecE 265 inc.*

		Clearance fits												Transition fits				Interference fits					
Nominal sizes		Tolerance		Tolerance		Tolerance		Tolerance		Tolerance		Tolerance		Tolerance		Tolerance		Tolerance		Tolerance		Nominal sizes	
Over	To	h11	C11	h9	D10	h9	E9	h7	F8	h6	G7	h6	H7	h6	K7	h6	N7	h6	P7	h6	S7	Over	To
mm	mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	0.001 mm	mm	mm
–	3	0 – 60	+ 120 + 60	0 – 25	+ 60 + 20	0 – 25	+ 39 + 14	0 – 10	+ 20 + 6	0 – 6	+ 12 + 2	0 – 6	+ 10 0	0 – 6	0 – 10	0 – 6	– 4 – 14	0 – 6	– 6 – 16	0 – 6	– 14 – 24	–	3
3	6	0 – 75	+ 145 + 70	0 – 30	+ 78 + 30	0 – 30	+ 50 + 20	0 – 12	+ 28 + 10	0 – 8	+ 16 + 4	0 – 8	+ 12 0	0 – 8	+ 3 – 9	0 – 8	– 4 – 16	0 – 8	– 8 – 20	0 – 8	– 15 – 27	3	6
6	10	0 – 90	+ 170 + 80	0 – 36	+ 98 + 40	0 – 36	+ 61 + 25	0 – 15	+ 35 + 13	0 – 9	+ 20 + 5	0 – 9	+ 15 0	0 – 9	+ 5 – 10	0 – 9	– 4 – 19	0 – 9	– 9 – 24	0 – 9	– 17 – 32	6	10
10	18	0 – 110	+ 205 + 95	0 – 43	+ 120 + 50	0 – 43	+ 75 + 32	0 – 18	+ 43 + 16	0 – 11	+ 24 + 6	0 – 11	+ 18 0	0 – 11	+ 6 – 12	0 – 11	– 5 – 23	0 – 11	– 11 – 29	0 – 11	– 21 – 39	10	18
18	30	0 – 130	+ 240 + 110	0 – 52	+ 149 + 65	0 – 52	+ 92 + 40	0 – 21	+ 53 + 20	0 – 13	+ 28 + 7	0 – 13	+ 21 0	0 – 13	+ 6 – 15	0 – 13	– 7 – 28	0 – 13	– 14 – 35	0 – 13	– 27 – 48	18	30
30	40	0 – 160	+ 280 + 120	0 – 62	+ 180 + 80	0 – 62	+ 112 + 50	0 – 25	+ 64 + 25	0 – 16	+ 34 + 9	0 – 16	+ 25 0	0 – 16	+ 7 – 18	0 – 16	– 8 – 33	0 – 16	– 17 – 42	0 – 16	– 34 – 59	30	40
40	50	0 – 160	+ 290 + 130																			40	50
50	65	0 – 190	+ 330 + 140	0 – 74	+ 220 + 100	0 – 74	+ 134 + 60	0 – 30	+ 76 + 30	0 – 19	+ 40 + 10	0 – 19	+ 30 0	0 – 19	+ 9 – 21	0 – 19	– 9 – 39	0 – 19	– 21 – 51	0 – 19	– 42 – 72	50	65
65	80	0 – 190	+ 340 + 150																	0 – 19	– 48 – 78	65	80
80	100	0 – 220	+ 390 + 170	0 – 87	+ 260 + 120	0 – 87	+ 159 + 72	0 – 35	+ 90 + 36	0 – 22	+ 47 + 12	0 – 22	+ 35 0	0 – 22	+ 10 – 25	0 – 22	– 10 – 45	0 – 22	– 24 – 59	0 – 22	– 58 – 93	80	100
100	120	0 – 220	+ 400 + 180																	0 – 22	– 66 – 101	100	120
120	140	0 – 250	+ 450 + 200	0 – 100	+ 305 + 145	0 – 100	+ 185 + 85	0 – 40	+ 106 + 43	0 – 25	+ 54 + 14	0 – 25	+ 40 0	0 – 25	+ 12 – 28	0 – 25	– 12 – 52	0 – 25	– 28 – 68	0 – 25	– 77 – 117	120	140
140	160	0 – 250	+ 460 + 210																	0 – 25	– 85 – 125	140	160
160	180	0 – 250	+ 480 + 230																	0 – 25	– 93 – 133	160	180
180	200	0 – 290	+ 530 + 240	0 – 115	+ 355 + 170	0 – 115	+ 215 + 100	0 – 46	+ 122 + 50	0 – 29	+ 61 + 15	0 – 29	+ 46 0	0 – 29	+ 13 – 33	0 – 29	– 14 – 60	0 – 29	– 33 – 79	0 – 29	– 105 – 151	180	200
200	225	0 – 290	+ 550 + 260																	0 – 29	– 113 – 159	200	225
225	250	0 – 290	+ 570 + 280																	0 – 29	– 123 – 169	225	250
250	280	0 – 320	+ 620 + 300	0 – 130	+ 400 + 190	0 – 130	+ 240 + 110	0 – 52	+ 137 + 56	0 – 32	+ 62 + 17	0 – 32	+ 52 0	0 – 32	+ 16 – 36	0 – 32	– 14 – 66	0 – 32	– 36 – 88	0 – 32	– 138 – 190	250	280
280	315	0 – 320	+ 650 + 330																	0 – 32	– 150 – 202	280	315
315	355	0 – 360	+ 720 + 360	0 – 140	+ 440 + 210	0 – 140	+ 265 + 125	0 – 57	+ 151 + 62	0 – 36	+ 75 + 18	0 – 36	+ 57 0	0 – 36	+ 17 – 40	0 – 36	– 16 – 73	0 – 36	– 41 – 98	0 – 36	– 169 – 226	315	355
355	400	0 – 360	+ 760 + 400																	0 – 36	– 187 – 244	355	400
400	450	0 – 400	+ 840 + 440	0 – 155	+ 480 + 230	0 – 155	+ 290 + 135	0 – 63	+ 165 + 68	0 – 40	+ 83 + 20	0 – 40	+ 63 0	0 – 40	+ 18 – 45	0 – 40	– 17 – 80	0 – 40	– 45 – 108	0 – 40	– 209 – 272	400	450
450	500	0 – 400	+ 880 + 480																	0 – 40	– 229 – 292	450	500

Table 12.3 Selected ISO Fits : Shaft Bases

Figure 8 - *This is a snapshot of the tolerance table for key and key hub provided by MecE 265 inc.*



Shaft: Nominal Diameter d, Over	Shaft: Nominal Diameter d, Up to and Incl	Key: Size, $b \times h$	Keyway Width b: Nominal	Free Fit: Shaft (H9)	Free Fit: Hub (D10)	Normal Fit: Shaft (N9)	Normal Fit: Hub (J_S9)[a]	Close Fit: Shaft and Hub (P9)	Depth Shaft t_1: Nominal	Depth Shaft t_1: Tolerance	Depth Hub t_2: Nominal	Depth Hub t_2: Tolerance	Radius r: Max.	Radius r: Min.
Keyways for Square Parallel Keys														
6	8	2×2	2	+0.025	+0.060	−0.004	+0.012	−0.006	1.2	+0.1 / 0	1	+0.1 / 0	0.16	0.08
8	10	3×3	3	0	+0.020	−0.029	−0.012	−0.031	1.8		1.4		0.16	0.08
10	12	4×4	4	+0.030 / 0	+0.078 / +0.030	0 / −0.030	+0.015 / −0.015	−0.012 / −0.042	2.5		1.8		0.16	0.08
12	17	5×5	5						3		2.3		0.25	0.16
17	22	6×6	6						3.5		2.8		0.25	0.16
Keyways for Rectangular Parallel Keys														
22	30	8×7	8	+0.036	+0.098	0	+0.018	−0.015	4	+0.2 / 0	3.3	+0.2 / 0	0.25	0.16
30	38	10×8	10	0	+0.040	−0.036	−0.018	−0.051	5		3.3		0.40	0.25
38	44	12×8	12	+0.043	+0.120 / +0.050	0 / −0.043	+0.021 / −0.021	−0.018 / −0.061	5		3.3		0.40	0.25
44	50	14×9	14						5.5		3.8		0.40	0.25
50	58	16×10	16						6		4.3		0.40	0.25
58	65	18×11	18						7		4.4		0.40	0.25
65	75	20×12	20	+0.052	+0.149 / +0.065	0 / −0.052	+0.026 / −0.026	−0.022 / −0.074	7.5		4.9		0.60	0.40
75	85	22×14	22						9		5.4		0.60	0.40
85	95	25×14	25						9		5.4		0.60	0.40
95	110	28×16	28						10		6.4		0.60	0.40
110	130	32×18	32	+0.062	+0.180 / −0.080	0 / −0.062	+0.031 / −0.031	−0.026 / −0.088	11		7.4		0.60	0.40
130	150	36×20	36						12	+0.3 / 0	8.4	+0.3 / 0	1.00	0.70
150	170	40×22	40						13		9.4		1.00	0.70
170	200	45×25	45						15		10.4		1.00	0.70
200	230	50×28	50						17		11.4		1.00	0.70
230	260	56×32	56	+0.074	+0.220 / +0.100	0 / −0.074	+0.037 / −0.037	−0.032 / −0.106	20		12.4		1.60	1.20
260	290	63×32	63						20		12.4		1.60	1.20
290	330	70×36	70						22		14.4		1.60	1.20
330	380	80×40	80						25		15.4		2.50	2.00
380	440	90×45	90	+0.087	+0.260	0	+0.043	−0.037	28		17.4		2.50	2.00
440	500	100×50	100	0	+0.120	−0.087	−0.043	−0.124	31		19.5		2.50	2.00

[a] Tolerance limits J_S9 are quoted from BS 4500, "ISO Limits and Fits," to three significant figures.

All dimensions in millimeters.

Figure 9 - Equivalent Dynamic Load Table from MecE360 Inc. Topic 16

Bearing Type			In Relation to the Load the Inner Ring is		Single Row Bearings 1)		Double Row Bearings 2)				e
			Rotating	Stationary	$\frac{F_a}{VF_r} > e$		$\frac{F_a}{VF_r} \leq e$		$\frac{F_a}{VF_r} > e$		
			V	V	X	Y	X	Y	X	Y	
3) Radial Contact Groove Ball Bearings	4) $\frac{F_a}{C_0}$	5) $\frac{F_a}{i Z D_w^2}$									
	0.014	25	1	1.2	0.56	2.30	1	0	0.56	2.30	0.19
	0.028	50				1.99				1.99	0.22
	0.056	100				1.71				1.71	0.26
	0.084	150				1.55				1.55	0.28
	0.11	200				1.45				1.45	0.30
	0.17	300				1.31				1.31	0.34
	0.28	500				1.15				1.15	0.38
	0.42	750				1.04				1.04	0.42
	0.56	1000				1.00				1.00	0.44
20°			1	1.2	0.43	1.00	1	1.09	0.70	1.63	0.57
25°					0.41	0.87		0.92	0.67	1.44	0.68
30°					0.39	0.76		0.78	0.63	1.24	0.80
35°					0.37	0.66		0.66	0.60	1.07	0.95
40°					0.35	0.57		0.55	0.57	0.93	1.14
Self-Aligning Ball Bearings			1	1	0.40	0.4 cot α	1	0.42 cot α	0.65	0.65 cot α	1.5 tan α
Self-Aligning and Tapered Roller Bearings			1	1.2	0.40	0.4 cot α	1	0.45 cot α	0.67	0.67 cot α	1.5 tan α

Appendix C (Calculations)

***C1 -** Lifetime Cycles.*

Lifetime Cycles

The purpose of calculating lifetime cycles is to determine the life factor in gear and bearing analysis, as well as to assess whether in infinite life or finite life range for shaft analysis.

Input values and Assumptions

Input shaft speed: $RPM_{in} := 3000\ \text{rpm}$

Operating Time Assumptions:
- running for 2 hrs a day (120min)
- 7 days a week
- 4 weeks a month
- May to mid Oct (~5.5 months)
- 3 year life

Number of Teeth

$N_1 := 20\ \text{teeth}$

$N_2 := 60\ \text{teeth}$

$N_3 := 24\ \text{teeth}$

$N_4 := 60\ \text{teeth}$

Input Shaft Cycles:

$$RPM_{in} \frac{1\ \text{cycles}}{\text{rev}} \frac{60\ \text{min}}{\text{hr}} \frac{2\ \text{hr}}{\text{day}} \frac{7\ \text{day}}{\text{week}} \frac{4\ \text{week}}{\text{month}} \frac{5.5\ \text{month}}{\text{yr}} \cdot 3\ \text{yr} = 1.6632 \cdot 10^{8}\ \text{cycles}$$

$$InputLife := 1.6632 \cdot 10^{8}\ \text{cycles}$$

Idler Shaft Cycles:

$$IdlerLife := InputLife \cdot \frac{N_1}{N_2} = 5.544 \cdot 10^{7}\ \text{cycles}$$

Output Shaft Cycles:

$$OutputLife := IdlerLife \cdot \frac{N_3}{N_4} = 2.2176 \cdot 10^{7}\ \text{cycles}$$

C2 - *Factor coefficients and uncorrected strengths for gear analysis*

- Application factor: Since the electric vehicle is being used in relatively smooth terrain, the gears will experience moderate shock. Therefore, a value of 1.25 was assigned.
- Rim thickness factor: The gears have a sufficient rim thickness in proportion to the tooth height, and therefore is 1.
- Hardness factor: All gears are made of the same material. Therefore the hardness factor is 1.
- Idler factor: Transmission design does not contain an idler gear, therefore the idler factor is 1.
- Temperature Factor: it is expected that our gearbox will run at a temperature below 250 Fahrenheit. Therefore the temperature factor is 1.
- Surface finish factor: Due to the polished/ground finish of our gears, the surface finish is assigned a value of 1.
- Size factor: No appropriate value/relations have been developed for the gears so the size factor is given a value of 1.
- Load distribution factor: The catalog of helical gears do not exceed a face width of 50mm therefore the load distribution factor is given to be 1.6.
- Bending life and contact life factors: Number of cycles of 16.632×10^7 were considered in the calculation. Both factors gave a value of 1 (detailed calculation in Appendix C - C1).
- Reliability Factor: In accordance with AGMA standards, it has been set at 1.00, which corresponds to a target reliability of 99%. While this level of reliability aligns with industry standards and ensures dependable performance, it is important to note that the application of a luxury golf cart does not require the same extreme reliability considerations as safety-critical systems like airplanes.
- Elastic coefficient: chosen AISI 4140 Chromoly steel grade 2 as the material for our golf cart transmission system because it offers an ideal balance between strength, wear resistance, and durability without over-engineering.
- Uncorrected bending stress and contact stress: Our application, supporting a maximum load of 600 kg, doesn’t require the extreme surface hardness or core strength found in applications like hydraulic systems, which endure much higher torque and stress. It was determined the value of uncorrected bending strength to be 380 MPa and uncorrected contact stress to be 1310 MPa based on the gear hardness provided by AGMA table.

C3 - *Gear force Calculation*

Gear force Calculation

Given for Motor:

$RPM_{in} := 3000\ rpm$ $T_{run} := 41.3\ lbf\ ft$ $T_{max} := 44.69\ lbf\ ft$ (max torque on startup)

Gear Analysis Summary: Pressure Angle $\phi := 20\ °$ Helix Angle $\psi := 21.5\ °$

Number of Teeth	Module	Pitch Diameter
$N_1 := 30\ teeth$	$m_1 := 2\ \frac{mm}{teeth}$	$d_{p1} := m_1 \cdot N_1 = 60\ mm$
$N_2 := 100\ teeth$	$m_2 := 2\ \frac{mm}{teeth}$	$d_{p2} := m_2 \cdot N_2 = 200\ mm$
$N_3 := 25\ teeth$	$m_3 := 3\ \frac{mm}{teeth}$	$d_{p3} := m_3 \cdot N_3 = 75\ mm$
$N_4 := 60\ teeth$	$m_4 := 3\ \frac{mm}{teeth}$	$d_{p4} := m_4 \cdot N_4 = 180\ mm$

Find the forces exerted on the input shaft (gears 1):

Transverse Presure Angle $\phi_t := \text{atan}\left(\frac{\tan(\phi)}{\cos(\psi)}\right) = 21.365\ °$

Loads on Gear 1

$$W_{t1} := \frac{T}{0.5 \cdot d_{p1}} = 419.608\ lbf$$

$$W_{r1} := W_{t1} \cdot \tan(\phi_t) = 164.1465\ lbf$$

$$W_{a1} := W_{t1} \cdot \tan(\psi) = 165.288\ lbf$$

$$W_1 := \frac{W_{t1}}{\cos(\psi) \cdot \cos(\phi)} = 479.9324\ lbf$$

C4 - Gear Analysis Calculations

ITERATION 1

Input Pinion and Gear Pair

Fundamental specifications

Subscript of g is for gear and p is for pinion in this 1st gear pair

Torque Input

$T_{in} := 56\ N\,m$

Input Pinion and Gear Specification

$F_p := 25\ mm$ $\quad F_g := 25\ mm$ $\quad m := 3\ mm$ $\quad T_p := T_{in}$

$N_p := 20$ $\quad N_g := 50$ $\quad p_d := \frac{1}{m} = 333.3333 \cdot \frac{1}{m}$

$a_p := 3\ mm$ $\quad a_g := 3\ mm$ $\quad \Psi := 21.5\ deg$

$d_p := N_p \cdot m$ $\quad d_g := N_g \cdot m$ $\quad \phi := 20\ deg$

$$\phi_t := atan\left(\frac{\tan(\phi)}{\cos(\Psi)}\right) = 21.365\ deg$$

$$\omega_g := \omega_{input} \cdot \frac{N_p}{N_g} = 125.6637\ \frac{rad}{s}$$

$$T_g := \frac{T_p \cdot \omega_{input}}{\omega_g} = 140\ N\,m$$

Input Shaft Speed

$$\omega_{input} := 3000\ rpm = 314.1593\ \frac{rad}{s}$$

Define m_n and Z (if table is not used to find I value)

**value of addendum is used here

Number of cycles (lifespan) of pinion and gear until failure

$$NC_p := \left(16.632 \cdot 10^7\right)$$

$$NC_g := NC_p \cdot \frac{N_p}{N_g} = 6.6528 \cdot 10^7$$

Force Input:

$$r_p := \frac{d_p}{2} = 0.03\ m$$

$$r_g := \frac{d_g}{2} = 0.075\ m$$

$$W_{tp} := \left(\frac{T_p}{r_p}\right) = 1866.6667\ N$$

$$W_{tg} := \frac{T_g}{r_g} = 1866.6667\ N$$

Geometry Factor, J and I

Table 1: AGMA 908-B89 Geometry Factors for Spur and Helical Gears

I AND *J* FACTORS FOR:[1]

20.0 DEG. PRESSURE ANGLE
20.0 DEG. HELIX ANGLE
0.250 TOOL EDGE RADIUS
EQUAL ADDENDUM ($x_1 = x_2 = 0$)

2.250 WHOLE DEPTH FACTOR
0.024 TOOTH THINNING FOR BACKLASH
LOADED AT TIP

GEAR TEETH		PINION TEETH 12 P	12 G	14 P	14 G	17 P	17 G	21 P	21 G	26 P	26 G	35 P	35 G	55 P	55 G	135 P	135 G
12	I																
	J	U	U														
14	I																
	J	U	U	U	U												
17	I					0.125											
	J	U	U	U	U	0.44	0.44										
21	I					0.140		0.129									
	J	U	U	U	U	0.45	0.46	0.47	0.47								
26	I					0.156		0.145		0.133							
	J	U	U	U	U	0.45	0.49	0.48	0.49	0.50	0.50						
35	I					0.177		0.167		0.155		0.138					
	J	U	U	U	U	0.46	0.51	0.49	0.52	0.51	0.53	0.54	0.54				
55	I					0.205		0.197		0.188		0.172		0.144			
	J	U	U	U	U	0.47	0.54	0.50	0.55	0.52	0.56	0.55	0.57	0.58	0.58		
135	I					0.245		0.242		0.238		0.229		0.209		0.151	
	J	U	U	U	U	0.48	0.58	0.51	0.59	0.54	0.60	0.57	0.61	0.60	0.62	0.64	0.64

ITERATION 1

Geometric Factors Derived from python code (

$I_p := 0.18787500000000001$

$J_p := 0.49$

$J_g := 0.54$

Bending and Contact Stress Factors for Input Pinion

Subscript ending with p indicates that the factor only applies to pinion stress calculations

Application Factor (Electric, moderate shock)

$K_a := 1.25$

$C_a := K_a$

Figure 1 - Application factor [1]

Driving machine	Driven machine: Uniform	Driven machine: Moderate shock	Driven machine: Heavy shock
Uniform (electric, turbine)	1.00	1.25	1.75+
Light shock (multi-cylinder engine)	1.25	1.50	2.00+
Medium Shock (single cylinder engine)	1.50	1.75	2.25+

Size Factor

$K_s := 1$

$C_s := K_s$

Rim Thickness Factor (Solid disk gears, no rim)

$K_B := 1$

Hardness Factor (all gears have the same material)

$C_H := 1$

Idler Factor

$K_I := 1$

None of the gears are Idler gears. Either pinion or a gear for all gears

Temperature Factor (less than 250F)

$K_T := 1$

$C_T := 1$

Surface Finish

$C_f := 1$

Dynamic Factor

$Q_{vp} := 11$

Gear quality#11 from Gear Manufacturer

$V_{tp} := \omega_{input} \cdot r_p$

$$B_p := \frac{\left(12 - Q_{vp}\right)^{\frac{2}{3}}}{4}$$

$$A_p := 50 + 56 \cdot \left(1 - B_p\right)$$

$$K_{vp} := \left(\frac{A_p}{\left(A_p + \sqrt{200 \cdot V_{tp} \frac{s}{m}}\right)}\right)^{B_p} = 0.9079$$

$C_{vp} := K_{vp}$

Load Distribution Factor

$K_{mp} := 1.6$

****Face width smaller than 2 inch**

$C_{mp} := K_{mp}$

Bending Life Factor

$$K_{Lp} := 1.6831 \cdot \left(NC_p\right)^{-0.0323} = 0.9132$$

Refer to Figure 2 and 3 in appendix A for the equations

Contact Life Factor

$$C_{Lp} := 2.466 \cdot \left(NC_p\right)^{-0.056} = 0.8543$$

Reliability Factor (99% reliability, following AGMA standard 2001-D04)

$K_R := 1$

$C_R := K_R$

Figure 2 - *Reliability Factor* [2]

Reliability (%)	K_R
90	0.85
99	1.00
99.9	1.25
99.99	1.50

Elastic Coefficient (Gear material is steel):

$C_p := 191 \cdot \sqrt{MPa}$

Figure 3 - Elastic coefficient factor [3]

Pinion material	E_p psi (MPa)	Gear material: Steel	Gear material: Cast Iron	Gear material: Aluminum Bronze
Steel	30x10^6 (2 x10^5)	2300 (191)	2100 (174)	1950 (162)
Cast Iron	22x10^6 (1.5 x10^5)	2100 (174)	1960 (163)	1850 (156)
Aluminum Bronze	17.5x10^6 (1.2 x10^5)	1950 (162)	1850 (156)	1750 (145)

Figure 3 - Load Distribution factor [0]

Face width, F, inch (mm)	K_m
<2 (50)	1.6
6 (150)	1.7
9 (250)	1.8
≥20 (500)	2.0

[0] p.40, MEC E 360 Lecture 13

[1] p.31, MEC E 360 Lecture 13

[2] p.43, MEC E 360 Lecture 13

[3] p.48, MEC E 360 Lecture 13

ITERATION 1

Bending and Contact Stress calculation for Input Pinion

Subscript ending with p indicates pinion

Bending Stress Calculations for input pinion

$$\sigma_{bp} := \frac{W_{tp}}{F_p \cdot J_p \cdot m} \cdot \frac{K_a \cdot K_{mp}}{K_{vp}} \cdot K_S \cdot K_B \cdot K_I = 1.1189 \cdot 10^8 \text{ Pa}$$

Contact Stress Calculation for Pinion

$$\sigma_{cp} := C_p \cdot \sqrt{\frac{W_{tp}}{F_p \cdot I_p \cdot d_p} \cdot \frac{C_a \cdot C_{mp}}{C_{vp}} \cdot C_S \cdot C_f} = 729.6035 \text{ MPa}$$

Bending and Contact Strength calculation for Input Pinion

Subscript ending with p indicates pinion

Uncorrected Bending Stress (depends on the material)

$$S_{fbp'} := 380 \text{ MPa}$$

Uncorrected Contact Stress (depends on the material)

$$S_{fcp'} := 1310 \text{ MPa}$$

****Minimum surface hardness is 50 HRC from manufacturer, grade 2**

Corrected Bending Strength (depends on the material)

$$S_{fbp} := \frac{K_{Lp}}{K_T \cdot K_R} \cdot S_{fbp'} = 347.022 \text{ MPa}$$

Corrected Contact Strength (depends on the material)

$$S_{fcp} := \frac{C_{Lp} \cdot C_H}{C_T \cdot C_R} \cdot S_{fcp'} = 1119.1582 \text{ MPa}$$

Figure 4 - *Uncorrected contact strength from* [4]

Material designation	Heat treatment	Minimum surface hardness[1)]	Allowable contact stress number[2)], s_{ac} lb/in^2		
			Grade 1	Grade 2	Grade 3
Steel[3)]	Through hardened[4)]	see figure 8	see figure 8	see figure 8	--
	Flame[5)] or induction hardened[5)]	50 HRC	170 000	190 000	--
		54 HRC	175 000	195 000	--
	Carburized and hardened[5)]	see table 9	180 000	225 000	275 000
	Nitrided[5)] (through hardened steels)	83.5 HR15N	150 000	163 000	175 000
		84.5 HR15N	155 000	168 000	180 000
2.5% Chrome (no aluminum)	Nitrided[5)]	87.5 HR15N	155 000	172 000	189 000
Nitralloy 135M	Nitrided[5)]	90.0 HR15N	170 000	183 000	195 000
Nitralloy N	Nitrided[5)]	90.0 HR15N	172 000	188 000	205 000
2.5% Chrome (no aluminum)	Nitrided[5)]	90.0 HR15N	176 000	196 000	216 000

NOTES
[1)] Hardness to be equivalent to that at the start of active profile in the center of the face width.
[2)] See tables 7 through 10 for major metallurgical factors for each stress grade of steel gears.
[3)] The steel selected must be compatible with the heat treatment process selected and hardness required.
[4)] These materials must be annealed or normalized as a minimum.
[5)] The allowable stress numbers indicated may be used with the case depths prescribed in 16.1.

Figure 5 - *Uncorrected bending strength* [5]

Material	AGMA Class	Material designation	Heat treatment	Minimum surface hardness	Bending strength	fatigue
					Psi x 10^3	MPa
Steel					- Min value Grade 1 - Max value Grade 2	
	A1-A5		Through hardened	≤ 180HB	25-33	170-230
			Through hardened	240 HB	31-41	210-280
			Through hardened	300 HB	36-47	250-325
			Through hardened	360 HB	40-52	280-360
			Through hardened	400 HB	42-56	290-390
			Flame or induction hardened	Type A pattern 50-54 HRC	45-55	310-380
			Carburized and case hardened	55-64 HRC	55-75	380-520
		AISI 4140	Nitrided	84.6HR15N	34-45	230-310
		AISI 4340	Nitrided	83.5HR15N	36-48	250-325
Cast Iron						
	20	Class 20	As cast		5	35
	30	Class 30	As cast	175 HB	8	69
	40	Class 40	As cast	200 HB	13	90

Safety factor calculation for Input Pinion

Subscript ending with p indicates pinion

Bending Safety Factor:

$$n_{bp} := \frac{S_{fbp}}{\sigma_{bp}} = 3.1013$$

Contact Safety Factor:

$$n_{cp} := \left(\frac{S_{fcp}}{\sigma_{cp}}\right)^2 = 2.3529$$

*****Choose 380MPa as uncorrected bending strength as all gears teeth hardness is 50 to 60 HRC. Take the average of 55 HRC**

Input Pinion is done ---

[4] p.23, AGMA Standard 2001-D04
[5] p.49, MECE 360 Lecture 13

ITERATION 1

Bending and Contact Stress Factors for Gear

Subscript ending with g indicates that the factor only applies to gear stress calculations

Application Factor (multi-cylinder moderate shock)

$K_a := 1.25$

$C_a := K_a$

Rim Thickness Factor (Solid disk gears)

$K_B := 1$

Hardness Factor (all gears same hardness)

$C_H := 1$

Idler Factor

$K_I := 1$

Temperature Factor (less than 250F)

$K_T := 1$

$C_T := 1$

Surface Finish

$C_f := 1$

Dynamic Factor

$Q_{vg} := 11$

$V_{tg} := \omega_g \cdot r_g$

$$B_g := \frac{\left(12 - Q_{vg}\right)^{\frac{2}{3}}}{4}$$

$A_g := 50 + 56 \cdot \left(1 - B_g\right)$

$$K_{vg} := \left(\frac{A_g}{\left(A_g + \sqrt{200 \cdot V_{tp} \frac{s}{m}}\right)}\right)^{B_g}$$

$C_{vg} := K_{vg}$

Size Factor

$K_S := 1$

$C_S := K_S$

Load Distribution Factor

$K_{mg} := 1.6$

$C_{mg} := K_{mg}$

Bending Life Factor

$K_{Lg} := 1.6831 \cdot \left(NC_g\right)^{-0.0323}$

Contact Life Factor

$C_{Lg} := 2.466 \cdot \left(NC_g\right)^{-0.056}$

Reliability Factor (99% reliability)

$K_R := 1.25$

$C_R := K_R$

Elastic Coefficient:

$C_p := 191 \cdot \sqrt{MPa}$

Geometry Factor, I (maybe just use the table)

$I_g := I_p$

Bending and Contact Stress calculation for Gear

Subscript ending with g indicates gear

Bending Stress Calculations for gear

$$\sigma_{bg} := \frac{W_{tg}}{F_g \cdot J_g \cdot m} \cdot \frac{K_a \cdot K_{mg}}{K_{vg}} \cdot K_S \cdot K_B \cdot K_I = 1.0153 \cdot 10^8 \ Pa$$

Contact Stress Calculation for Pinion

$$\sigma_{cg} := C_p \cdot \sqrt{\frac{W_{tg}}{F_g \cdot I_g \cdot d_g} \cdot \frac{C_a \cdot C_{mg}}{C_{vg}} \cdot C_S \cdot C_f} = 461.4418 \ MPa$$

ITERATION 1

Bending and Contact Strength calculation for Input Pinion

Subscript ending with p indicates pinion

Uncorrected Bending Stress (depends on the material)

$$S_{fbg'} := S_{fbp'}$$

Uncorrected Contact Stress (depends on the material)

$$S_{fcg'} := S_{fcp'}$$

Corrected Bending Stress (depends on the material)

$$S_{fbg} := \frac{K_{Lg}}{K_T \cdot K_R} \cdot S_{fbg'} = 285.9568 \text{ MPa}$$

Corrected Contact Stress (depends on the material)

$$S_{fcg} := \frac{C_{Lg} \cdot C_H}{C_T \cdot C_R} \cdot S_{fcg'} = 942.4669 \text{ MPa}$$

Safety factor calculation for Gear

Subscript ending with g indicates gear

Bending Safety Factor:

$$n_{bg} := \frac{S_{fbg}}{\sigma_{bg}} = 2.8164$$

Contact Safety Factor:

$$n_{cg} := \left(\frac{S_{fcg}}{\sigma_{cg}}\right)^2 = 4.1716$$

ITERATION 1

Idler Pinion and Output Gear Pair

Fundamental specifications

Subscript of g is for gear and p is for pinion in this 1st gear pair

Input Pinion and Gear Specification

$N_p := 20$ $N_g := 60$

$F_p := 25\ mm$ $F_g := 25\ mm$ $m := 3\ mm$

$d_p := N_p \cdot m$ $d_g := N_g \cdot m$ $p_d := \frac{1}{m} = 333.3333 \cdot \frac{1}{m}$

$a_p := 3\ mm$ $a_g := 3\ mm$ $\Psi := 21.5\ deg$

$\phi := 20\ deg$

$$\phi_t := atan\left(\frac{\tan(\phi)}{\cos(\Psi)}\right) = 21.365\ deg$$

$$\omega_g := \omega_{input} \cdot \frac{N_p}{N_g} = 41.8879\ \frac{rad}{s}$$

$$T_g := \frac{T_p \cdot r_g}{r_p} = 420\ N\ m$$

Input Shaft Speed

$\omega_{input} := \omega_g = 125.6637\ Hz$

Torque Input (idler shaft torque)

$T_{in} := T_g = 140\ N\ m$

$T_p := T_{in} = 140\ N\ m$

Define m_n and Z (if table is not used to find I value)

**value of addendum is used here

Number of cycles (lifespan) of pinion and gear until failure

$NC_p := NC_g$

$$NC_g := NC_p \cdot \frac{N_p}{N_g} = 2.2176 \cdot 10^7$$

Force Input:

$$r_p := \frac{d_p}{2} = 0.03\ m$$

$$r_g := \frac{d_g}{2} = 0.09\ m$$

$$W_{tp} := \left(\frac{T_p}{r_p}\right) = 4666.6667\ N$$

$$W_{tg} := \frac{T_g}{r_g} = 4666.6667\ N$$

Geometry Factor, J and I

Table 1: AGMA 908-B89 Geometry Factors for Spur and Helical Gears

I AND *J* FACTORS FOR:[1]

20.0 DEG. PRESSURE ANGLE
20.0 DEG. HELIX ANGLE
0.250 TOOL EDGE RADIUS
EQUAL ADDENDUM ($x_1 = x_2 = 0$)

2.250 WHOLE DEPTH FACTOR
0.024 TOOTH THINNING FOR BACKLASH
LOADED AT TIP

		PINION TEETH															
GEAR		12		14		17		21		26		35		55		135	
TEETH		P	G	P	G	P	G	P	G	P	G	P	G	P	G	P	G
12	I																
	J	U	U														
14	I																
	J	U	U	U	U												
17	I					0.125											
	J	U	U	U	U	0.44	0.44										
21	I					0.140		0.129									
	J	U	U	U	U	0.45	0.46	0.47	0.47								
26	I					0.156		0.145		0.133							
	J	U	U	U	U	0.45	0.49	0.48	0.49	0.50	0.50						
35	I					0.177		0.167		0.155		0.138					
	J	U	U	U	U	0.46	0.51	0.49	0.52	0.51	0.53	0.54	0.54				
55	I					0.205		0.197		0.188		0.172		0.144			
	J	U	U	U	U	0.47	0.54	0.50	0.55	0.52	0.56	0.55	0.57	0.58	0.58		
135	I					0.245		0.242		0.238		0.229		0.209		0.151	
	J	U	U	U	U	0.48	0.58	0.51	0.59	0.54	0.60	0.57	0.61	0.60	0.62	0.64	0.64

ITERATION 1

Geometric Factors Derived from python code (

$I_p := 0.201734375$

$J_p := 0.493125$

$J_g := 0.55$

Bending and Contact Stress Factors for Input Pinion

Subscript ending with p indicates that the factor only applies to pinion stress calculations

Application Factor (multi-cylinder moderate shock)

$K_a := 1.25$

$C_a := K_a$

Rim Thickness Factor (Solid disk gears)

$K_B := 1$

Hardness Factor (all gears same hardness)

$C_H := 1$

Idler Factor

$K_I := 1$

Temperature Factor (less than 250F)

$K_T := 1$

$C_T := 1$

Surface Finish

$C_f := 1$

Dynamic Factor

$Q_{vp} := 11$

$V_{tp} := \omega_{input} \cdot r_p$

$$B_p := \frac{\left(12 - Q_{vp}\right)^{\frac{2}{3}}}{4}$$

$A_p := 50 + 56 \cdot \left(1 - B_p\right)$

$$K_{vp} := \left(\frac{A_p}{\left(A_p + \sqrt{200 \cdot V_{tp} \frac{s}{m}}\right)}\right)^{B_p} = 0.9368$$

$C_{vp} := K_{vp}$

Size Factor

$K_S := 1$

$C_S := K_S$

Load Distribution Factor

$K_{mp} := 1.6$

$C_{mp} := K_{mp}$

Bending Life Factor

$K_{Lp} := 1.6831 \cdot \left(NC_p\right)^{-0.0323} = 0.9406$

Contact Life Factor

$C_{Lp} := 2.466 \cdot \left(NC_p\right)^{-0.056} = 0.8993$

Reliability Factor (99% reliability)

$K_R := 1$

$C_R := K_R$

Reliability (%)	K_R
90	0.85
99	1.00
99.9	1.25
99.99	1.50

Elastic Coefficient:

$$C_p := 191 \cdot \sqrt{MPa} = 1.91 \cdot 10^{5} \frac{kg^{\frac{1}{2}}}{m^{\frac{1}{2}} s}$$

ITERATION 1

Bending and Contact Stress calculation for Input Pinion

Subscript ending with p indicates pinion

Bending Stress Calculations for input pinion

$$\sigma_{bp} := \frac{W_{tp}}{F_p \cdot J_p \cdot m} \cdot \frac{K_a \cdot K_{mp}}{K_{vp}} \cdot K_S \cdot K_B \cdot K_I = 2.6939 \cdot 10^{8}\ \text{Pa}$$

Contact Stress Calculation for Pinion

$$\sigma_{cp} := C_p \cdot \sqrt{\frac{W_{tp}}{F_p \cdot I_p \cdot d_p} \cdot \frac{C_a \cdot C_{mp}}{C_{vp}} \cdot C_S \cdot C_f} = 1095.9606\ \text{MPa}$$

Bending and Contact Strength calculation for Input Pinion

Subscript ending with p indicates pinion

Uncorrected Bending Strength (depends on the material)

$$S_{fbp'} := 380\ \text{MPa}$$

Uncorrected Contact Strength (depends on the material)

$$S_{fcp'} := S_{fcp'}$$

Corrected Bending Strength (depends on the material)

$$S_{fbp} := \frac{K_{Lp}}{K_T \cdot K_R} \cdot S_{fbp'} = 357.446\ \text{MPa}$$

Corrected Contact Strength (depends on the material)

$$S_{fcp} := \frac{C_{Lp} \cdot C_H}{C_T \cdot C_R} \cdot S_{fcp'} = 1178.0836\ \text{MPa}$$

Safety factor calculation for Input Pinion

Subscript ending with p indicates pinion

Bending Safety Factor:

$$n_{bp} := \frac{S_{fbp}}{\sigma_{bp}} = 1.3269$$

Contact Safety Factor:

$$n_{cp} := \left(\frac{S_{fcp}}{\sigma_{cp}}\right)^2 = 1.1555$$

Input Pinion is done

ITERATION 1

Bending and Contact Stress Factors for Gear

Subscript ending with g indicates that the factor only applies to gear stress calculations

Application Factor (multi-cylinder moderate shock)

$K_a := 1.25$

$C_a := K_a$

Rim Thickness Factor (Solid disk gears)

$K_B := 1$

Hardness Factor (all gears same hardness)

$C_H := 1$

Idler Factor

$K_I := 1$

Temperature Factor (less than 250F)

$K_T := 1$

$C_T := 1$

Surface Finish

$C_f := 1$

Dynamic Factor

$Q_{vg} := 11$

$V_{tg} := \omega_g \cdot r_g$

$$B_g := \frac{\left(12 - Q_{vg}\right)^{\frac{2}{3}}}{4}$$

$A_g := 50 + 56 \cdot \left(1 - B_g\right)$

$$K_{vg} := \left(\frac{A_g}{\left(A_g + \sqrt{200 \cdot V_{tp} \frac{s}{m}}\right)}\right)^{B_g}$$

$C_{vg} := K_{vg}$

Size Factor

$K_s := 1$

$C_s := K_s$

Load Distribution Factor

$K_{mg} := 1.6$

$C_{mg} := K_{mg}$

Bending Life Factor

$K_{Lg} := 1.6831 \cdot \left(NC_g\right)^{-0.0323}$

Contact Life Factor

$C_{Lg} := 2.466 \cdot \left(NC_g\right)^{-0.056}$

Reliability Factor (99% reliability)

$K_R := 1$

$C_R := K_R$

Elastic Coefficient:

$C_p := 191 \cdot \sqrt{MPa}$

Geometry Factor, I (maybe just use the table)

$I_g := I_p$

Bending and Contact Stress calculation for Gear

Subscript ending with g indicates gear

Bending Stress Calculations for gear

$$\sigma_{bg} := \frac{W_{tg}}{F_g \cdot J_g \cdot m} \cdot \frac{K_a \cdot K_{mg}}{K_{vg}} \cdot K_s \cdot K_B \cdot K_I = 2.4153 \cdot 10^{8}\ Pa$$

Contact Stress Calculation for Pinion

$$\sigma_{cg} := C_p \cdot \sqrt{\frac{W_{tg}}{F_g \cdot I_g \cdot d_g} \cdot \frac{C_a \cdot C_{mg}}{C_{vg}} \cdot C_s \cdot C_f} = 632.7531\ MPa$$

Bending and Contact Strength calculation for Input Pinion

Subscript ending with p indicates pinion

Uncorrected Bending Stress (depends on the material)

$$S_{fbg'} := S_{fbp'}$$

Uncorrected Contact Stress (depends on the material)

$$S_{fcg'} := S_{fcp'}$$

Corrected Bending Stress (depends on the material)

$$S_{fbg} := \frac{K_{Lg}}{K_T \cdot K_R} \cdot S_{fbg'} = 370.3578 \text{ MPa}$$

Corrected Contact Stress (depends on the material)

$$S_{fcg} := \frac{C_{Lg} \cdot C_H}{C_T \cdot C_R} \cdot S_{fcg'} = 1252.838 \text{ MPa}$$

Safety factor calculation for Gear

Subscript ending with g indicates gear

Bending Safety Factor:

$$n_{bg} := \frac{S_{fbg}}{\sigma_{bg}} = 1.5334$$

Contact Safety Factor:

$$n_{cg} := \left(\frac{S_{fcg}}{\sigma_{cg}} \right)^2 = 3.9203$$

FOLLOWED THE SAME PROCEDURE AS ITERATION 1

Input Pinion and Gear Pair

Fundamental specifications

Subscript of g is for gear and p is for pinion in this 1st gear pair

Torque Input

$T_{in} := 56\ N\,m$

Input Pinion and Gear Specification

$F_p := 16\ mm$ $\quad F_g := 16\ mm$ $\quad m := 2\ mm$ $\quad T_p := T_{in}$

$N_p := 30$ $\quad N_g := 100$ $\quad p_d := \frac{1}{m} = 500 \cdot \frac{1}{m}$

$a_p := 3\ mm$ $\quad a_g := 3\ mm$ $\quad \Psi := 21.5\ deg$

$d_p := N_p \cdot m$ $\quad d_g := N_g \cdot m$ $\quad \phi := 20\ deg$

$$\phi_t := \operatorname{atan}\left(\frac{\tan(\phi)}{\cos(\Psi)}\right) = 21.365\ deg$$

$$\omega_g := \omega_{input} \cdot \frac{N_p}{N_g} = 94.2478\ \frac{rad}{s}$$

$$T_g := \frac{T_p \cdot \omega_{input}}{\omega_g} = 186.6667\ N\,m$$

Input Shaft Speed

$$\omega_{input} := 3000\ rpm = 314.1593\ \frac{rad}{s}$$

Define m_n and Z (if table is not used to find I value

**value of addendum is used here

Number of cycles (lifespan) of pinion and gear until failure

$$NC_p := \left(16.632 \cdot 10^{7}\right)$$

$$NC_g := NC_p \cdot \frac{N_p}{N_g} = 4.9896 \cdot 10^{7}$$

Force Input:

$$r_p := \frac{d_p}{2} = 0.03\ m$$

$$r_g := \frac{d_g}{2} = 0.1\ m$$

$$W_{tp} := \left(\frac{T_p}{r_p}\right) = 1866.6667\ N$$

$$W_{tg} := \frac{T_g}{r_g} = 1866.6667\ N$$

Geometry Factor, J and I

Table 1: AGMA 908-B89 Geometry Factors for Spur and Helical Gears

I AND J FACTORS FOR:[1]

[illegible] DEG. PRESSURE ANGLE
[illegible] DEG. HELIX ANGLE
[illegible] TOOL EDGE RADIUS
EQUAL ADDENDUM ($x_1 = x_2 = 0$)

1.250 WHOLE DEPTH FACTOR
0.024 TOOTH THINNING FOR BACKLASH
LOADED AT TIP

GEAR TEETH		12 P	12 G	14 P	14 G	17 P	17 G	21 P	21 G	26 P	26 G	35 P	35 G	55 P	55 G	135 P	135 G
12	I																
	J	U	U														
14	I																
	J	U	U	U	U												
17	I					0.129											
	J	U	U	U	U	0.44	0.44										
21	I					0.140		0.129									
	J	U	U	U	U	0.42	0.46	0.47	0.47								
26	I					0.156		0.145		0.133							
	J	U	U	U	U	0.43	0.49	0.48	0.49	0.50	0.50						
35	I					0.177		0.167		0.155		0.138					
	J	U	U	U	U	0.46	0.51	0.49	0.52	0.51	0.53	0.54	0.54				
55	I					0.205		0.197		0.188		0.173		0.144			
	J	U	U	U	U	0.47	0.54	0.50	0.55	0.52	0.56	0.53	0.57	0.58	0.58		
135	I					0.245		0.242		0.236		0.229		0.209		0.151	
	J	U	U	U	U	0.48	0.58	0.51	0.59	0.54	0.60	0.57	0.61	0.60	0.62	0.64	0.64

FOLLOWED THE SAME PROCEDURE AS ITERATION 1

Geometric Factors Derived from python code

$I_p := 0.21076388888888886$

$J_p := 0.5445833333333333$

$J_g := 0.5869444444444445$

Bending and Contact Stress Factors for Input Pinion

Subscript ending with p indicates that the factor only applies to pinion stress calculations

Application Factor (multi-cylinder moderate shock)

$K_a := 1.25$

$C_a := K_a$

Rim Thickness Factor (Solid disk gears)

$K_B := 1$

Hardness Factor (all gears same hardness)

$C_H := 1$

Idler Factor

$K_I := 1$

Temperature Factor (less than 250F)

$K_T := 1$

$C_T := 1$

Surface Finish

$C_f := 1$

Size Factor

$K_s := 1$

$C_s := K_s$

Load Distribution Factor

$K_{mp} := 1.6$

$C_{mp} := K_{mp}$

Bending Life Factor

$K_{Lp} := 1.6831 \cdot \left(NC_p\right)^{-0.0323} = 0.9132$

Contact Life Factor

$C_{Lp} := 2.466 \cdot \left(NC_p\right)^{-0.056} = 0.8543$

Reliability Factor (99% reliability: AGMA standard)

$K_R := 1$

$C_R := K_R$

Reliability (%)	K_R
90	0.85
99	1.00
99.9	1.25
99.99	1.50

Dynamic Factor

$Q_{vp} := 11$

$V_{tp} := \omega_{input} \cdot r_p$

$$B_p := \frac{\left(12 - Q_{vp}\right)^{\frac{2}{3}}}{4}$$

$A_p := 50 + 56 \cdot \left(1 - B_p\right)$

$$K_{vp} := \left(\frac{A_p}{\left(A_p + \sqrt{200 \cdot V_{tp} \frac{s}{m}}\right)}\right)^{B_p} = 0.9079$$

$C_{vp} := K_{vp}$

Elastic Coefficient

$$C_p := 191 \cdot \sqrt{MPa} = 1.91 \cdot 10^5 \frac{kg^{\frac{1}{2}}}{m^{\frac{1}{2}} s}$$

FOLLOWED THE SAME PROCEDURE AS ITERATION 1

Bending and Contact Stress calculation for Input Pinion

Subscript ending with p indicates pinion

Bending Stress Calculations for input pinion

$$\sigma_{bp} := \frac{W_{tp}}{F_p \cdot J_p \cdot m} \cdot \frac{K_a \cdot K_{mp}}{K_{vp}} \cdot K_S \cdot K_B \cdot K_I = 2.3597 \cdot 10^{8}\ \text{Pa}$$

Contact Stress Calculation for Pinion

$$\sigma_{cp} := C_p \cdot \sqrt{\frac{W_{tp}}{F_p \cdot I_p \cdot d_p} \cdot \frac{C_a \cdot C_{mp}}{C_{vp}} \cdot C_s \cdot C_f} = 861.0598\ \text{MPa}$$

Bending and Contact Strength calculation for Input Pinion

Subscript ending with p indicates pinion

Uncorrected Bending Stress (depends on the material)

$$S_{fbp'} := 380\ \text{MPa}$$

Uncorrected Contact Stress (depends on the material)

$$S_{fcp'} := 1310\ \text{MPa}$$

Corrected Bending Strength (depends on the material)

$$S_{fbp} := \frac{K_{Lp}}{K_T \cdot K_R} \cdot S_{fbp'} = 347.022\ \text{MPa}$$

Corrected Contact Strength (depends on the material)

$$S_{fcp} := \frac{C_{Lp} \cdot C_H}{C_T \cdot C_R} \cdot S_{fcp'} = 1119.1582\ \text{MPa}$$

Safety factor calculation for Input Pinion

Subscript ending with p indicates pinion

Bending Safety Factor:

$$n_{bp} := \frac{S_{fbp}}{\sigma_{bp}} = 1.4706$$

Contact Safety Factor:

$$n_{cp} := \left(\frac{S_{fcp}}{\sigma_{cp}}\right)^2 = 1.6893$$

Input Pinion is done ---

FOLLOWED THE SAME PROCEDURE AS ITERATION 1

Bending and Contact Stress Factors for Gear

Subscript ending with g indicates that the factor only applies to gear stress calculations

Application Factor (multi-cylinder moderate shock)

$K_a := 1.25$

$C_a := K_a$

Rim Thickness Factor (Solid disk gears)

$K_B := 1$

Hardness Factor (all gears same hardness)

$C_H := 1$

Idler Factor

$K_I := 1$

Temperature Factor (less than 250F)

$K_T := 1$

$C_T := 1$

Surface Finish

$C_f := 1$

Dynamic Factor

$Q_{vg} := 11$

$V_{tg} := \omega_g \cdot r_g$

$$B_g := \frac{(12 - Q_{vg})^{\frac{2}{3}}}{4}$$

$A_g := 50 + 56 \cdot (1 - B_g)$

$$K_{vg} := \left(\frac{A_g}{\left(A_g + \sqrt{200 \cdot V_{tp} \frac{s}{m}} \right)} \right)^{B_g}$$

$C_{vg} := K_{vg}$

Size Factor

$K_s := 1$

$C_s := K_s$

Load Distribution Factor

$K_{mg} := 1.6$

$C_{mg} := K_{mg}$

Bending Life Factor

$K_{Lg} := 1.6831 \cdot (NC_g)^{-0.0323}$

Contact Life Factor

$C_{Lg} := 2.466 \cdot (NC_g)^{-0.056}$

Reliability Factor (99% reliability)

$K_R := 1.25$

$C_R := K_R$

Elastic Coefficient

$C_p := 191 \cdot \sqrt{MPa}$

Geometry Factor, I (maybe just use the table)

$I_g := I_p$

Bending and Contact Stress calculation for Gear

Subscript ending with g indicates gear

Bending Stress Calculations for gear

$$\sigma_{bg} := \frac{W_{tg}}{F_g \cdot J_g \cdot m} \cdot \frac{K_a \cdot K_{mg}}{K_{vg}} \cdot K_s \cdot K_B \cdot K_I = 2.1894 \cdot 10^8 \ Pa$$

Contact Stress Calculation for Pinion

$$\sigma_{cg} := C_p \cdot \sqrt{\frac{W_{tg}}{F_g \cdot I_g \cdot d_g} \cdot \frac{C_a \cdot C_{mg}}{C_{vg}} \cdot C_s \cdot C_f} = 471.6219 \ MPa$$

FOLLOWED THE SAME PROCEDURE AS ITERATION 1

Bending and Contact Strength calculation for Input Pinion

Subscript ending with p indicates pinion

Uncorrected Bending Stress (depends on the material)

$S_{fbg'} := S_{fbp'}$

Uncorrected Contact Stress (depends on the material)

$S_{fcg'} := S_{fcp'}$

Corrected Bending Stress (depends on the material)

$$S_{fbg} := \frac{K_{Lg}}{K_T \cdot K_R} \cdot S_{fbg'} = 288.6264\ \text{MPa}$$

Corrected Contact Stress (depends on the material)

$$S_{fcg} := \frac{C_{Lg} \cdot C_H}{C_T \cdot C_R} \cdot S_{fcg'} = 957.7732\ \text{MPa}$$

Safety factor calculation for Gear

Subscript ending with g indicates gear

Bending Safety Factor:

$$n_{bg} := \frac{S_{fbg}}{\sigma_{bg}} = 1.3183$$

Contact Safety Factor:

$$n_{cg} := \left(\frac{S_{fcg}}{\sigma_{cg}}\right)^2 = 4.1242$$

FOLLOWED THE SAME PROCEDURE AS ITERATION 1

Idler Pinion and Output Gear Pair

Fundamental specifications

Subscript of g is for gear and p is for pinion in this 1st gear pair

Input Pinion and Gear Specification

$N_p := 25$ $\quad N_g := 60$

$F_p := 25\ mm$ $\quad F_g := 25\ mm$ $\quad m := 3\ mm$

$d_p := N_p \cdot m$ $\quad d_g := N_g \cdot m$ $\quad p_d := \frac{1}{m} = 333.3333 \cdot \frac{1}{m}$

$a_p := 3\ mm$ $\quad a_g := 3\ mm$ $\quad \Psi := 21.5\ deg$

$\phi := 20\ deg$

$$\phi_t := atan\left(\frac{\tan(\phi)}{\cos(\Psi)}\right) = 21.365\ deg$$

$$\omega_g := \omega_{input} \cdot \frac{N_p}{N_g} = 39.2699\ \frac{rad}{s}$$

$$T_g := \frac{T_p \cdot r_g}{r_p} = 448\ N\ m$$

Input Shaft Speed

$\omega_{input} := \omega_g = 94.2478\ Hz$

Torque Input (idler shaft torque)

$T_{in} := T_g = 186.6667\ N\ m$

$T_p := T_{in} = 186.6667\ N\ m$

Define m_n and Z (if table is not used to find I value)

**value of addendum is used here

Number of cycles (lifespan) of pinion and gear until failure

$NC_p := NC_g$

$$NC_g := NC_p \cdot \frac{N_p}{N_g} = 2.079 \cdot 10^7$$

Force Input:

$$r_p := \frac{d_p}{2} = 0.0375\ m$$

$$r_g := \frac{d_g}{2} = 0.09\ m$$

$$W_{tp} := \left(\frac{T_p}{r_p}\right) = 4977.7778\ N$$

$$W_{tg} := \frac{T_g}{r_g} = 4977.7778\ N$$

Geometry Factor, J and I

Table 1: AGMA 908-B89 Geometry Factors for Spur and Helical Gears

I AND *J* FACTORS FOR:[1]

20.0 DEG. PRESSURE ANGLE
20.0 DEG. HELIX ANGLE
0.250 TOOL EDGE RADIUS
EQUAL ADDENDUM ($x_1 = x_2 = 0$)

2.250 WHOLE DEPTH FACTOR
0.024 TOOTH THINNING FOR BACKLASH
LOADED AT TIP

GEAR		PINION TEETH 12		14		17		21		26		35		55		135	
TEETH		P	G	P	G	P	G	P	G	P	G	P	G	P	G	P	G
12	I																
	J	U	U														
14	I																
	J	U	U	U	U												
17	I					0.123											
	J	U	U	U	U	0.44	0.44										
21	I					0.140		0.129									
	J	U	U	U	U	0.45	0.46	0.47	0.47								
26	I					0.156		0.145		0.133							
	J	U	U	U	U	0.45	0.49	0.48	0.49	0.50	0.50						
35	I					0.177		0.167		0.155		0.138					
	J	U	U	U	U	0.46	0.51	0.49	0.52	0.51	0.53	0.54	0.54				
55	I					0.205		0.197		0.188		0.172		0.144			
	J	U	U	U	U	0.47	0.54	0.50	0.55	0.52	0.56	0.53	0.57	0.58	0.58		
135	I					0.245		0.242		0.238		0.229		0.209		0.151	
	J	U	U	U	U	0.48	0.58	0.51	0.59	0.54	0.60	0.57	0.61	0.60	0.62	0.64	0.64

FOLLOWED THE SAME PROCEDURE AS ITERATION 1

Geometric Factors Derived from python code

$I_p := 0.1928625$

$J_p := 0.517125$

$J_g := 0.5605$

Bending and Contact Stress Factors for Input Pinion

Subscript ending with p indicates that the factor only applies to pinion stress calculations

Application Factor (multi-cylinder moderate shock)

$K_a := 1.25$

$C_a := K_a$

Rim Thickness Factor (Solid disk gears)

$K_B := 1$

Hardness Factor (all gears same hardness)

$C_H := 1$

Idler Factor

$K_I := 1$

Temperature Factor (less than 250F)

$K_T := 1$

$C_T := 1$

Surface Finish

$C_f := 1$

Size Factor

$K_s := 1$

$C_s := K_s$

Load Distribution Factor

$K_{mp} := 1.6$

$C_{mp} := K_{mp}$

Bending Life Factor

$K_{Lp} := 1.6831 \cdot \left(NC_p\right)^{-0.0323} = 0.9494$

Contact Life Factor

$C_{Lp} := 2.466 \cdot \left(NC_p\right)^{-0.056} = 0.9139$

Reliability Factor (99% reliability)

$K_R := 1$

$C_R := K_R$

Reliability (%)	K_R
90	0.85
99	1.00
99.9	1.25
99.99	1.50

Dynamic Factor

$Q_{vp} := 11$

$V_{tp} := \omega_{input} \cdot r_p$

$$B_p := \frac{\left(12 - Q_{vp}\right)^{\frac{2}{3}}}{4}$$

$A_p := 50 + 56 \cdot \left(1 - B_p\right)$

$$K_{vp} := \left(\frac{A_p}{\left(A_p + \sqrt{200 \cdot V_{tp} \frac{s}{m}}\right)}\right)^{B_p} = 0.9385$$

$C_{vp} := K_{vp}$

Elastic Coefficient

$$C_p := 191 \cdot \sqrt{MPa} = 1.91 \cdot 10^5 \frac{kg^{\frac{1}{2}}}{m^{\frac{1}{2}} s}$$

FOLLOWED THE SAME PROCEDURE AS ITERATION 1

Bending and Contact Stress calculation for Input Pinion

Subscript ending with p indicates pinion

Bending Stress Calculations for input pinion

$$\sigma_{bp} := \frac{W_{tp}}{F_p \cdot J_p \cdot m} \cdot \frac{K_a \cdot K_{mp}}{K_{vp}} \cdot K_s \cdot K_B \cdot K_I = 2.7351 \cdot 10^8 \text{ Pa}$$

Contact Stress Calculation for Pinion

$$\sigma_{cp} := C_p \cdot \sqrt{\frac{W_{tp}}{F_p \cdot I_p \cdot d_p} \cdot \frac{C_a \cdot C_{mp}}{C_{vp}} \cdot C_s \cdot C_f} = 1034.4813 \text{ MPa}$$

Bending and Contact Strength calculation for Input Pinion

Subscript ending with p indicates pinion

Uncorrected Bending Strength (depends on the material)

$$S_{fbp'} := 380 \text{ MPa}$$

Uncorrected Contact Strength (depends on the material)

$$S_{fcp'} := 1310 \text{ MPa}$$

Corrected Bending Strength (depends on the material)

$$S_{fbp} := \frac{K_{Lp}}{K_T \cdot K_R} \cdot S_{fbp'} = 360.7829 \text{ MPa}$$

Corrected Contact Strength (depends on the material)

$$S_{fcp} := \frac{C_{Lp} \cdot C_H}{C_T \cdot C_R} \cdot S_{fcp'} = 1197.2165 \text{ MPa}$$

Safety factor calculation for Input Pinion

Subscript ending with p indicates pinion

Bending Safety Factor:

$$n_{bp} := \frac{S_{fbp}}{\sigma_{bp}} = 1.3191$$

Contact Safety Factor:

$$n_{cp} := \left(\frac{S_{fcp}}{\sigma_{cp}}\right)^2 = 1.3394$$

Input Pinion is done

FOLLOWED THE SAME PROCEDURE AS ITERATION 1

Bending and Contact Stress Factors for Gear

Subscript ending with g indicates that the factor only applies to gear stress calculations

Application Factor (multi-cylinder moderate shock)

$K_a := 1.25$

$C_a := K_a$

Rim Thickness Factor (Solid disk gears)

$K_B := 1$

Hardness Factor (all gears same hardness)

$C_H := 1$

Idler Factor

$K_I := 1$

Temperature Factor (less than 250F)

$K_T := 1$

$C_T := 1$

Surface Finish

$C_f := 1$

Dynamic Factor

$Q_{vg} := 11$

$V_{tg} := \omega_g \cdot r_g$

$$B_g := \frac{\left(12 - Q_{vg}\right)^{\frac{2}{3}}}{4}$$

$A_g := 50 + 56 \cdot \left(1 - B_g\right)$

$$K_{vg} := \left(\frac{A_g}{\left(A_g + \sqrt{200 \cdot V_{tp} \frac{s}{m}}\right)}\right)^{B_g}$$

$C_{vg} := K_{vg}$

Size Factor

$K_s := 1$

$C_s := K_s$

Load Distribution Factor

$K_{mg} := 1.6$

$C_{mg} := K_{mg}$

Bending Life Factor

$K_{Lg} := 1.6831 \cdot \left(NC_g\right)^{-0.0323}$

Contact Life Factor

$C_{Lg} := 2.466 \cdot \left(NC_g\right)^{-0.056}$

Reliability Factor (99% reliability)

$K_R := 1$

$C_R := K_R$

Elastic Coefficient

$C_p := 191 \cdot \sqrt{MPa}$

Geometry Factor, I (maybe just use the table)

$I_g := I_p$

Bending and Contact Stress calculation for Gear

Subscript ending with g indicates gear

Bending Stress Calculations for gear

$$\sigma_{bg} := \frac{W_{tg}}{F_g \cdot J_g \cdot m} \cdot \frac{K_a \cdot K_{mg}}{K_{vg}} \cdot K_s \cdot K_B \cdot K_I = 2.5234 \cdot 10^{8}\ Pa$$

Contact Stress Calculation for Pinion

$$\sigma_{cg} := C_p \cdot \sqrt{\frac{W_{tg}}{F_g \cdot I_g \cdot d_g} \cdot \frac{C_a \cdot C_{mg}}{C_{vg}} \cdot C_s \cdot C_f} = 667.7548\ MPa$$

FOLLOWED THE SAME PROCEDURE AS ITERATION 1

Bending and Contact Strength calculation for Input Pinion

Subscript ending with p indicates pinion

Uncorrected Bending Stress (depends on the material)

$S_{fbg'} := S_{fbp'}$

Uncorrected Contact Stress (depends on the material)

$S_{fcg'} := S_{fcp'}$

Corrected Bending Stress (depends on the material)

$$S_{fbg} := \frac{K_{Lg}}{K_T \cdot K_R} \cdot S_{fbg'} = 371.1306 \text{ MPa}$$

Corrected Contact Stress (depends on the material)

$$S_{fcg} := \frac{C_{Lg} \cdot C_H}{C_T \cdot C_R} \cdot S_{fcg'} = 1257.3741 \text{ MPa}$$

Safety factor calculation for Gear

Subscript ending with g indicates gear

Bending Safety Factor:

$$n_{bg} := \frac{S_{fbg}}{\sigma_{bg}} = 1.4707$$

Contact Safety Factor:

$$n_{cg} := \left(\frac{S_{fcg}}{\sigma_{cg}}\right)^2 = 3.5456$$

FOLLOWED THE SAME PROCEDURE AS ITERATION 1 AND 2

Input Pinion and Gear Pair

Fundamental specifications

Subscript of g is for gear and p is for pinion in this 1st gear pair

Torque Input

$T_{in} := 56\ N\,m$

Input Pinion and Gear Specification

$F_p := 25\ mm$ $F_g := 25\ mm$ $m := 3\ mm$ $T_p := T_{in}$

$N_p := 20$ $N_g := 60$ $p_d := \frac{1}{m} = 333.3333 \cdot \frac{1}{m}$

$a_p := 3\ mm$ $a_g := 3\ mm$ $\Psi := 21.5\ deg$

$d_p := N_p \cdot m$ $d_g := N_g \cdot m$ $\phi := 20\ deg$

$$\phi_t := atan\left(\frac{\tan(\phi)}{\cos(\Psi)}\right) = 21.365\ deg$$

$$\omega_g := \omega_{input} \cdot \frac{N_p}{N_g} = 104.7198\ \frac{rad}{s}$$

$$T_g := \frac{T_p \cdot \omega_{input}}{\omega_g} = 168\ N\,m$$

Input Shaft Speed

$$\omega_{input} := 3000\ rpm = 314.1593\ \frac{rad}{s}$$

Define m_n and Z (if table is not used to find I value)

**value of addendum is used here

Number of cycles (lifespan) of pinion and gear until failure

$$NC_p := \left(16.632 \cdot 10^{7}\right)$$

$$NC_g := NC_p \cdot \frac{N_p}{N_g} = 5.544 \cdot 10^{7}$$

Force Input:

$$r_p := \frac{d_p}{2} = 0.03\ m$$

$$r_g := \frac{d_g}{2} = 0.09\ m$$

$$W_{tp} := \left(\frac{T_p}{r_p}\right) = 1866.6667\ N$$

$$W_{tg} := \frac{T_g}{r_g} = 1866.6667\ N$$

Geometry Factor, J and I

Table 1: AGMA 908-B89 Geometry Factors for Spur and Helical Gears

I AND J FACTORS FOR:[1]

20.0 DEG. PRESSURE ANGLE
20.0 DEG. HELIX ANGLE
0.250 TOOL EDGE RADIUS
EQUAL ADDENDUM $(x_1 = x_2 = 0)$

2.250 WHOLE DEPTH FACTOR
0.024 TOOTH THINNING FOR BACKLASH
LOADED AT TIP

		PINION TEETH															
GEAR		12		14		17		21		26		35		55		135	
TEETH		P	G	P	G	P	G	P	G	P	G	P	G	P	G	P	G
12	I																
	J	U	U														
14	I																
	J	U	U	U	U												
17	I					0.125											
	J	U	U	U	U	0.44	0.44										
21	I					0.140		0.129									
	J	U	U	U	U	0.45	0.46	0.47	0.47								
26	I					0.156		0.145		0.133							
	J	U	U	U	U	0.45	0.49	0.48	0.49	0.50	0.50						
35	I					0.177		0.167		0.155		0.138					
	J	U	U	U	U	0.46	0.51	0.49	0.52	0.51	0.53	0.54	0.54				
55	I					0.205		0.197		0.188		0.172		0.144			
	J	U	U	U	U	0.47	0.54	0.50	0.55	0.52	0.56	0.55	0.57	0.58	0.58		
135	I					0.245		0.242		0.238		0.229		0.209		0.151	
	J	U	U	U	U	0.48	0.58	0.51	0.59	0.54	0.60	0.57	0.61	0.60	0.62	0.64	0.64

FOLLOWED THE SAME PROCEDURE AS ITERATION 1 AND 2

Geometric Factors Derived from python code

$I_p := 0.201734375$

$J_p := 0.493125$

$J_g := 0.55$

Bending and Contact Stress Factors for Input Pinion

Subscript ending with p indicates that the factor only applies to pinion stress calculations

Application Factor (multi-cylinder moderate shock)

$K_a := 1.25$

$C_a := K_a$

Rim Thickness Factor (Solid disk gears)

$K_B := 1$

Hardness Factor (all gears same hardness)

$C_H := 1$

Idler Factor

$K_I := 1$

Temperature Factor (less than 250F)

$K_T := 1$

$C_T := 1$

Surface Finish

$C_f := 1$

Size Factor

$K_s := 1$

$C_s := K_s$

Load Distribution Factor

$K_{mp} := 1.6$

$C_{mp} := K_{mp}$

Bending Life Factor

$K_{Lp} := 1.6831 \cdot (NC_p)^{-0.0323} = 0.9132$

Contact Life Factor

$C_{Lp} := 2.466 \cdot (NC_p)^{-0.056} = 0.8543$

Reliability Factor (99% reliability: AGMA standard)

$K_R := 1$

$C_R := K_R$

Reliability (%)	K_R
90	0.85
99	1.00
99.9	1.25
99.99	1.50

Dynamic Factor

$Q_{vp} := 11$

$V_{tp} := \omega_{input} \cdot r_p$

$$B_p := \frac{(12 - Q_{vp})^{\frac{2}{3}}}{4}$$

$$A_p := 50 + 56 \cdot (1 - B_p)$$

$$K_{vp} := \left(\frac{A_p}{\left(A_p + \sqrt{200 \cdot V_{tp} \frac{s}{m}} \right)} \right)^{B_p} = 0.9079$$

$C_{vp} := K_{vp}$

Elastic Coefficient

$$C_p := 191 \cdot \sqrt{MPa} = 1.91 \cdot 10^5 \frac{kg^{\frac{1}{2}}}{m^{\frac{1}{2}} s}$$

FOLLOWED THE SAME PROCEDURE AS ITERATION 1 AND 2

Bending and Contact Stress calculation for Input Pinion

Subscript ending with p indicates pinion

Bending Stress Calculations for input pinion

$$\sigma_{bp} := \frac{W_{tp}}{F_p \cdot J_p \cdot m} \cdot \frac{K_a \cdot K_{mp}}{K_{vp}} \cdot K_S \cdot K_B \cdot K_I = 1.1119 \cdot 10^{8}\ \text{Pa}$$

Contact Stress Calculation for Pinion

$$\sigma_{cp} := C_p \cdot \sqrt{\frac{W_{tp}}{F_p \cdot I_p \cdot d_p} \cdot \frac{C_a \cdot C_{mp}}{C_{vp}} \cdot C_s \cdot C_f} = 704.0953\ \text{MPa}$$

Bending and Contact Strength calculation for Input Pinion

Subscript ending with p indicates pinion

Uncorrected Bending Stress (depends on the material)

$$S_{fbp'} := 380\ \text{MPa}$$

Uncorrected Contact Stress (depends on the material)

$$S_{fcp'} := 1310\ \text{MPa}$$

Corrected Bending Strength (depends on the material)

$$S_{fbp} := \frac{K_{Lp}}{K_T \cdot K_R} \cdot S_{fbp'} = 347.022\ \text{MPa}$$

Corrected Contact Strength (depends on the material)

$$S_{fcp} := \frac{C_{Lp} \cdot C_H}{C_T \cdot C_R} \cdot S_{fcp'} = 1119.1582\ \text{MPa}$$

Safety factor calculation for Input Pinion

Subscript ending with p indicates pinion

Bending Safety Factor:

$$n_{bp} := \frac{S_{fbp}}{\sigma_{bp}} = 3.1211$$

Contact Safety Factor:

$$n_{cp} := \left(\frac{S_{fcp}}{\sigma_{cp}}\right)^2 = 2.5265$$

Input Pinion is done ---

FOLLOWED THE SAME PROCEDURE AS ITERATION 1 AND 2

Bending and Contact Stress Factors for Gear

Subscript ending with g indicates that the factor only applies to gear stress calculations

Application Factor (multi-cylinder moderate shock)

$K_a := 1.25$

$C_a := K_a$

Rim Thickness Factor (Solid disk gears)

$K_B := 1$

Hardness Factor (all gears same hardness)

$C_H := 1$

Idler Factor

$K_I := 1$

Temperature Factor (less than 250F)

$K_T := 1$

$C_T := 1$

Surface Finish

$C_f := 1$

Dynamic Factor

$Q_{vg} := 11$

$V_{tg} := \omega_g \cdot r_g$

$$B_g := \frac{(12 - Q_{vg})^{\frac{2}{3}}}{4}$$

$A_g := 50 + 56 \cdot (1 - B_g)$

$$K_{vg} := \left(\frac{A_g}{\left(A_g + \sqrt{200 \cdot V_{tp} \frac{s}{m}} \right)} \right)^{B_g}$$

$C_{vg} := K_{vg}$

Size Factor

$K_s := 1$

$C_s := K_s$

Load Distribution Factor

$K_{mg} := 1.6$

$C_{mg} := K_{mg}$

Bending Life Factor

$K_{Lg} := 1.6831 \cdot (NC_g)^{-0.0323}$

Contact Life Factor

$C_{Lg} := 2.466 \cdot (NC_g)^{-0.056}$

Reliability Factor (99% reliability)

$K_R := 1.25$

$C_R := K_R$

Elastic Coefficient

$C_p := 191 \cdot \sqrt{MPa}$

Geometry Factor, I (maybe just use the table)

$I_g := I_p$

Bending and Contact Stress calculation for Gear

Subscript ending with g indicates gear

Bending Stress Calculations for gear

$$\sigma_{bg} := \frac{W_{tg}}{F_g \cdot J_g \cdot m} \cdot \frac{K_a \cdot K_{mg}}{K_{vg}} \cdot K_s \cdot K_B \cdot K_I = 9.9688 \cdot 10^7 \ Pa$$

Contact Stress Calculation for Pinion

$$\sigma_{cg} := C_p \cdot \sqrt{\frac{W_{tg}}{F_g \cdot I_g \cdot d_g} \cdot \frac{C_a \cdot C_{mg}}{C_{vg}} \cdot C_s \cdot C_f} = 406.5096 \ MPa$$

FOLLOWED THE SAME PROCEDURE AS ITERATION 1 AND 2

Bending and Contact Strength calculation for Input Pinion

Subscript ending with p indicates pinion

Uncorrected Bending Stress (depends on the material)

$S_{fbg'} := S_{fbp'}$

Uncorrected Contact Stress (depends on the material)

$S_{fcg'} := S_{fcp'}$

Corrected Bending Stress (depends on the material)

$$S_{fbg} := \frac{K_{Lg}}{K_T \cdot K_R} \cdot S_{fbg'} = 287.6458 \text{ MPa}$$

Corrected Contact Stress (depends on the material)

$$S_{fcg} := \frac{C_{Lg} \cdot C_H}{C_T \cdot C_R} \cdot S_{fcg'} = 952.1388 \text{ MPa}$$

Safety factor calculation for Gear

Subscript ending with g indicates gear

Bending Safety Factor:

$$n_{bg} := \frac{S_{fbg}}{\sigma_{bg}} = 2.8855$$

Contact Safety Factor:

$$n_{cg} := \left(\frac{S_{fcg}}{\sigma_{cg}}\right)^2 = 5.486$$

FOLLOWED THE SAME PROCEDURE AS ITERATION 1 AND 2

Idler Pinion and Output Gear Pair

Fundamental specifications

Subscript of g is for gear and p is for pinion in this 1st gear pair

Input Pinion and Gear Specification

$N_p := 24$ $N_g := 60$

$F_p := 25\ mm$ $F_g := 25\ mm$ $m := 3\ mm$

$d_p := N_p \cdot m$ $d_g := N_g \cdot m$

$a_p := 3\ mm$ $a_g := 3\ mm$

$P_d := \frac{1}{m} = 333.3333 \cdot \frac{1}{m}$

$\Psi := 21.5\ deg$

$\phi := 20\ deg$

$$\phi_t := atan\left(\frac{\tan(\phi)}{\cos(\Psi)}\right) = 21.365\ deg$$

$$\omega_g := \omega_{input} \cdot \frac{N_p}{N_g} = 41.8879\ \frac{rad}{s}$$

$$T_g := \frac{T_p \cdot r_g}{r_p} = 420\ N\ m$$

Input Shaft Speed

$\omega_{input} := \omega_g = 104.7198\ Hz$

Torque Input (idler shaft torque)

$T_{in} := T_g = 168\ N\ m$

$T_p := T_{in} = 168\ N\ m$

Define m_n and Z (if table is not used to find I value)

**value of addendum is used here

Number of cycles (lifespan) of pinion and gear until failure

$NC_p := NC_g$

$$NC_g := NC_p \cdot \frac{N_p}{N_g} = 2.2176 \cdot 10^7$$

Force Input:

$$r_p := \frac{d_p}{2} = 0.036\ m$$

$$r_g := \frac{d_g}{2} = 0.09\ m$$

$$W_{tp} := \left(\frac{T_p}{r_p}\right) = 4666.6667\ N$$

$$W_{tg} := \frac{T_g}{r_g} = 4666.6667\ N$$

Geometry Factor, J and I

Table 1: AGMA 908-B89 Geometry Factors for Spur and Helical Gears

I AND *J* FACTORS FOR:[1]

20.0 DEG. PRESSURE ANGLE
20.0 DEG. HELIX ANGLE
0.250 TOOL EDGE RADIUS
EQUAL ADDENDUM ($x_1 = x_2 = 0$)

2.250 WHOLE DEPTH FACTOR
0.024 TOOTH THINNING FOR BACKLASH
LOADED AT TIP

GEAR TEETH		PINION TEETH 12 P	12 G	14 P	14 G	17 P	17 G	21 P	21 G	26 P	26 G	35 P	35 G	55 P	55 G	135 P	135 G
12	I																
	J	U	U														
14	I																
	J	U	U	U	U												
17	I					0.125											
	J	U	U	U	U	0.44	0.44										
21	I					0.140		0.129									
	J	U	U	U	U	0.45	0.46	0.47	0.47								
26	I					0.156		0.145		0.133							
	J	U	U	U	U	0.45	0.49	0.48	0.49	0.50	0.50						
35	I					0.177		0.167		0.155		0.138					
	J	U	U	U	U	0.46	0.51	0.49	0.52	0.51	0.53	0.54	0.54				
55	I					0.205		0.197		0.188		0.172		0.144			
	J	U	U	U	U	0.47	0.54	0.50	0.55	0.52	0.56	0.55	0.57	0.58	0.58		
135	I					0.245		0.242		0.238		0.229		0.209		0.151	
	J	U	U	U	U	0.48	0.58	0.51	0.59	0.54	0.60	0.57	0.61	0.60	0.62	0.64	0.64

Geometric Factors Derived from python code

$I_p := 0.1946$

$J_p := 0.513$

$J_g := 0.5585$

Bending and Contact Stress Factors for Input Pinion

Subscript ending with p indicates that the factor only applies to pinion stress calculations

Application Factor (multi-cylinder moderate shock)

$K_a := 1.25$

$C_a := K_a$

Rim Thickness Factor (Solid disk gears)

$K_B := 1$

Hardness Factor (all gears same hardness)

$C_H := 1$

Idler Factor

$K_I := 1$

Temperature Factor (less than 250F)

$K_T := 1$

$C_T := 1$

Surface Finish

$C_f := 1$

Size Factor

$K_s := 1$

$C_s := K_s$

Load Distribution Factor

$K_{mp} := 1.6$

$C_{mp} := K_{mp}$

Bending Life Factor

$K_{Lp} := 1.6831 \cdot \left(NC_p\right)^{-0.0323} = 0.9462$

Contact Life Factor

$C_{Lp} := 2.466 \cdot \left(NC_p\right)^{-0.056} = 0.9085$

Reliability Factor (99% reliability)

$K_R := 1$

$C_R := K_R$

Reliability (%)	K_R
90	0.85
99	1.00
99.9	1.25
99.99	1.50

Dynamic Factor

$Q_{vp} := 11$

$V_{tp} := \omega_{input} \cdot r_p$

$$B_p := \frac{\left(12 - Q_{vp}\right)^{\frac{2}{3}}}{4}$$

$A_p := 50 + 56 \cdot \left(1 - B_p\right)$

$$K_{vp} := \left(\frac{A_p}{\left(A_p + \sqrt{200 \cdot V_{tp} \frac{s}{m}}\right)}\right)^{B_p} = 0.9368$$

$C_{vp} := K_{vp}$

Elastic Coefficient

$$C_p := 191 \cdot \sqrt{MPa} = 1.91 \cdot 10^5 \frac{kg^{\frac{1}{2}}}{m^{\frac{1}{2}} s}$$

FOLLOWED THE SAME PROCEDURE AS ITERATION 1 AND 2

Bending and Contact Stress calculation for Input Pinion

Subscript ending with p indicates pinion

Bending Stress Calculations for input pinion

$$\sigma_{bp} := \frac{W_{tp}}{F_p \cdot J_p \cdot m} \cdot \frac{K_a \cdot K_{mp}}{K_{vp}} \cdot K_s \cdot K_B \cdot K_I = 2.5895 \cdot 10^{8}\ \text{Pa}$$

Contact Stress Calculation for Pinion

$$\sigma_{cp} := C_p \cdot \sqrt{\frac{W_{tp}}{F_p \cdot I_p \cdot d_p} \cdot \frac{C_a \cdot C_{mp}}{C_{vp}} \cdot C_s \cdot C_f} = 1018.645\ \text{MPa}$$

Bending and Contact Strength calculation for Input Pinion

Subscript ending with p indicates pinion

Uncorrected Bending Strength (depends on the material)

$$S_{fbp'} := 380\ \text{MPa}$$

Uncorrected Contact Strength (depends on the material)

$$S_{fcp'} := 1310\ \text{MPa}$$

Corrected Bending Strength (depends on the material)

$$S_{fbp} := \frac{K_{Lp}}{K_T \cdot K_R} \cdot S_{fbp'} = 359.5572\ \text{MPa}$$

Corrected Contact Strength (depends on the material)

$$S_{fcp} := \frac{C_{Lp} \cdot C_H}{C_T \cdot C_R} \cdot S_{fcp'} = 1190.1735\ \text{MPa}$$

Safety factor calculation for Input Pinion

Subscript ending with p indicates pinion

Bending Safety Factor:

$$n_{bp} := \frac{S_{fbp}}{\sigma_{bp}} = 1.3885$$

Contact Safety Factor:

$$n_{cp} := \left(\frac{S_{fcp}}{\sigma_{cp}}\right)^2 = 1.3651$$

Input Pinion is done

FOLLOWED THE SAME PROCEDURE AS ITERATION 1 AND 2

Bending and Contact Stress Factors for Gear

Subscript ending with g indicates that the factor only applies to gear stress calculations

Application Factor (multi-cylinder moderate shock)

$K_a := 1.25$

$C_a := K_a$

Rim Thickness Factor (Solid disk gears)

$K_B := 1$

Hardness Factor (all gears same hardness)

$C_H := 1$

Idler Factor

$K_I := 1$

Temperature Factor (less than 250F)

$K_T := 1$

$C_T := 1$

Surface Finish

$C_f := 1$

Dynamic Factor

$Q_{vg} := 11$

$V_{tg} := \omega_g \cdot r_g$

$$B_g := \frac{(12 - Q_{vg})^{\frac{2}{3}}}{4}$$

$A_g := 50 + 56 \cdot (1 - B_g)$

$$K_{vg} := \left(\frac{A_g}{\left(A_g + \sqrt{200 \cdot V_{tp} \cdot \frac{s}{m}} \right)} \right)^{B_g}$$

$C_{vg} := K_{vg}$

Size Factor

$K_s := 1$

$C_s := K_s$

Load Distribution Factor

$K_{mg} := 1.6$

$C_{mg} := K_{mg}$

Bending Life Factor

$K_{Lg} := 1.6831 \cdot (NC_g)^{-0.0323}$

Contact Life Factor

$C_{Lg} := 2.466 \cdot (NC_g)^{-0.056}$

Reliability Factor (99% reliability)

$K_R := 1$

$C_R := K_R$

Elastic Coefficient

$C_p := 191 \cdot \sqrt{MPa}$

Geometry Factor, I (maybe just use the table)

$I_g := I_p$

Bending and Contact Stress calculation for Gear

Subscript ending with g indicates gear

Bending Stress Calculations for gear

$$\sigma_{bg} := \frac{W_{tg}}{F_g \cdot J_g \cdot m} \cdot \frac{K_a \cdot K_{mg}}{K_{vg}} \cdot K_s \cdot K_B \cdot K_I = 2.3785 \cdot 10^8 \ Pa$$

Contact Stress Calculation for Pinion

$$\sigma_{cg} := C_p \cdot \sqrt{\frac{W_{tg}}{F_g \cdot I_g \cdot d_g} \cdot \frac{C_a \cdot C_{mg}}{C_{vg}} \cdot C_s \cdot C_f} = 644.2476 \ MPa$$

FOLLOWED THE SAME PROCEDURE AS ITERATION 1 AND 2

Bending and Contact Strength calculation for Input Pinion

Subscript ending with p indicates pinion

Uncorrected Bending Stress (depends on the material)

$S_{fbg'} := S_{fbp'}$

Uncorrected Contact Stress (depends on the material)

$S_{fcg'} := S_{fcp'}$

Corrected Bending Stress (depends on the material)

$$S_{fbg} := \frac{K_{Lg}}{K_T \cdot K_R} \cdot S_{fbg'} = 370.3578 \text{ MPa}$$

Corrected Contact Stress (depends on the material)

$$S_{fcg} := \frac{C_{Lg} \cdot C_H}{C_T \cdot C_R} \cdot S_{fcg'} = 1252.838 \text{ MPa}$$

Safety factor calculation for Gear

Subscript ending with g indicates gear

Bending Safety Factor:

$$n_{bg} := \frac{S_{fbg}}{\sigma_{bg}} = 1.5571$$

Contact Safety Factor:

$$n_{cg} := \left(\frac{S_{fcg}}{\sigma_{cg}}\right)^2 = 3.7817$$

***C5 -** The following python code is used to calculate the geometry factors by interpolating the AGMA standard tables. Then the values were plugged into **C4**.*

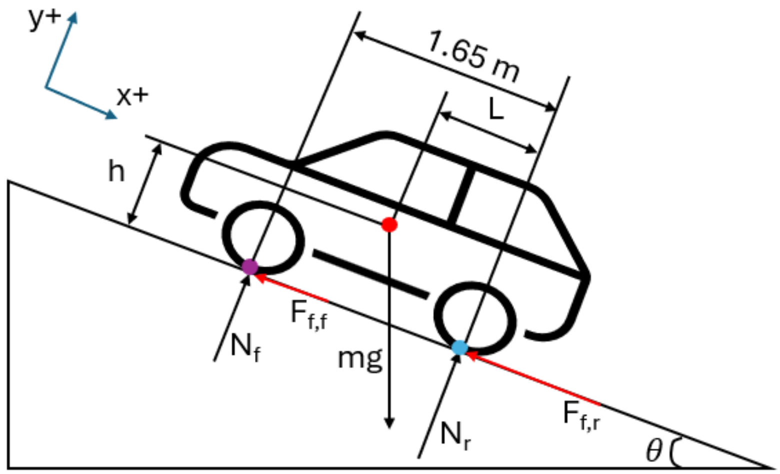
```python
data_Pinion = {
    26: {17: 0.45, 21: 0.48, 26: 0.5, 35: 0, 55: 0},
    35: {17: 0.46, 21: 0.49, 26: 0.51, 35: 0.54, 55: 0},
    55: {17: 0.47, 21: 0.5, 26: 0.52, 35: 0.55, 55: 0.58},
    135: {17: 0.48, 21: 0.51, 26: 0.54, 35: 0.57, 55: 0.6}
}

data_Gear = {
    26: {17: 0.49, 21: 0.49, 26: 0.5, 35: 0, 55: 0},
    35: {17: 0.51, 21: 0.52, 26: 0.53, 35: 0.54, 55: 0},
    55: {17: 0.54, 21: 0.55, 26: 0.56, 35: 0.57, 55: 0.58},
    135: {17: 0.58, 21: 0.59, 26: 0.6, 35: 0.61, 55: 0.62}
}

data_I_gear_pinion = {
    26: {17: 0.156, 21: 0.145, 26: 0.133, 35: 0, 55: 0},
    35: {17: 0.117, 21: 0.167, 26: 0.155, 35: 0.138, 55: 0},
    55: {17: 0.205, 21: 0.197, 26: 0.188, 35: 0.172, 55: 0.144},
    135: {17: 0.245, 21: 0.242, 26: 0.238, 35: 0.225, 55: 0.209}
}

# Helper function for linear interpolation
def linear_interpolate(x0, y0, x1, y1, x):
    return y0 + (y1 - y0) * (x - x0) / (x1 - x0)

# Interpolation function for a data table
def double_interpolate(data, gear_teeth, pinion_teeth):
    # Find the nearest neighbors for `Gear Teeth` and `Pinion Teeth`
    gear_keys = sorted(data.keys())
    pinion_keys = sorted(data[gear_keys[0]].keys())

    gear_low = max([g for g in gear_keys if g <= gear_teeth], default=None)
    gear_high = min([g for g in gear_keys if g >= gear_teeth], default=None)

    pinion_low = max([p for p in pinion_keys if p <= pinion_teeth], default=None)
    pinion_high = min([p for p in pinion_keys if p >= pinion_teeth], default=None)

    # Check if interpolation is possible
    if gear_low is None or gear_high is None or pinion_low is None or pinion_high is None:
        return "Interpolation not possible with the given data range."

    # Interpolate along the `Gear Teeth` direction for both `pinion_low` and `pinion_high`
    f_pinion_low = linear_interpolate(gear_low, data[gear_low][pinion_low], gear_high, data[gear_high][pinion_low], gear_teeth)
    f_pinion_high = linear_interpolate(gear_low, data[gear_low][pinion_high], gear_high, data[gear_high][pinion_high], gear_teeth)

    # Interpolate along the `Pinion Teeth` direction using the results from above
    interpolated_value = linear_interpolate(pinion_low, f_pinion_low, pinion_high, f_pinion_high, pinion_teeth)

    return interpolated_value

while True:
    # Input values
    gear_teeth = int(input("Enter Gear Teeth:\n"))
    pinion_teeth = int(input("Enter Pinion Teeth:\n"))


    # Perform interpolation for both tables
    J_p = double_interpolate(data_Pinion, gear_teeth, pinion_teeth)
    J_g = double_interpolate(data_Gear, gear_teeth, pinion_teeth)
    I = double_interpolate(data_I_gear_pinion, gear_teeth, pinion_teeth)

    # Print geometry factor
    print(f"Geometry Factor for Pinion J_p = {J_p}")
    print(f"Geometry Factor for Gear J_g = {J_g}")
    print(f"Geometry Factor I = {I}")
```

C6 - Incline Friction and Torque Analysis

Incline Friction and Torque Analysis

Free Body Diagram of a vehicle on an incline for maximum torque requirement:

Figure 1 - Free body diagram (FBD) of the cart on incline



Center of mass	● (red)
Point A	● (blue)
Point B	● (purple)
Static Friction, Front Axle	$F_{f,f}$
Static Friction, Rear Axle	$F_{f,r}$
Weight of the Vehicle	mg
Distance between point A and Center of Mass	L
Distance between point B and Center of Mass	H
Incline Angle	θ
Normal Force, Front Axle	N_f
Normal Force, Rear Axle	N_r

Figure x – Free Body Diagram of a vehicle on incline

Since the location of the center of mass is not certain as it depends on the location of the passengers within the vehicle as well as the number of passengers, there were ranges given for variable L and h:

$$0 \leq L \leq 1.65m$$

$$0 \leq h \leq 1.75m$$

Note that the maximum values of L and h are the wheelbase length and height of the golf cart respectively, based on our reference golf cart model[1].

The first condition the vehicle needs to satisfy is the no tip condition. The center of mass should be located adequately that the vehicle does not tip about Point A. To find the no tip condition:

$$\sum \quad M_B = mgLcos\theta - 1.65N_f - mghsin\theta = 0 \qquad (1)$$

[1] "Tempo lithium-ion fleet golf car," Tempo Lithium Ion Golf Cart | Fleet Golf Vehicles | Club Car, https://www.clubcar.com/en/golf-operations/fleet-golf/tempo-lithium-ion (accessed Sep. 21, 2024).

$$N_f \geq 0N \tag{2}$$

$$N_f = mg\left(\frac{Lcos\theta - hsin\theta}{1.65}\right) \geq 0 \tag{3}$$

Then, solving the inequality:

$$1.65 \geq L \geq htan\theta \tag{4}$$

Based on the value of normal force on front axle, normal force on the rear axle can be calculated:

$$\sum \quad F_y = N_f + N_r - mgcos\theta = 0 \tag{5}$$

$$\therefore N_r = mgcos\theta - N_f = mg\left[cos\theta - \left(\frac{Lcos\theta - hsin\theta}{1.65}\right)\right] \tag{6}$$

Now that all the necessary equations were obtained to solve static friction (i.e. non-slip friction) at the contact between the incline surface and the tires:

$$F_{f,r} = \mu_s N_r = \mu_s mg\left[cos\theta - \left(\frac{Lcos\theta - hsin\theta}{1.65}\right)\right] \tag{7}$$

$$F_{f,f} = \mu_s N_f = \mu_s mg\left(\frac{Lcos\theta - hsin\theta}{1.65}\right) \tag{8}$$

Given rear wheel drive system of our vehicle and hence the torque of the rear axle converting to the surface friction:

$$2T_{max} = r_{wheel} \times F_{f,r} = r_{wheel}\mu_s mg\left[cos\theta - \left(\frac{Lcos\theta - hsin\theta}{1.65}\right)\right] \tag{9}$$

Note there’s 2 multiplied to torque because the torque represents the rear axle torque, and it acts on two rear tires in contact with the surface. Also, this is represented as maximum torque, because torque exceeding this value will cause the force from torque on the tire surface to be greater than the static friction, causing the tire to slip.

Second condition the vehicle should satisfy is that it should be able to drive up the incline. For the cart to drive upwards:

$$\sum \quad F_x = mgsin\theta - F_{f,f} - F_{f,r} \leq 0 \tag{10}$$

$$\therefore F_{f,f} + F_{f,r} \geq mgsin\theta \tag{11}$$

Substituting the equation (7) and (8):

$$\mu_s mg\left[cos\theta - \left(\frac{Lcos\theta - hsin\theta}{1.65}\right) + \left(\frac{Lcos\theta - hsin\theta}{1.65}\right)\right] = \mu_s mgcos\theta \;\geq\; mgsin\theta \tag{12}$$

$$\therefore \mu_s \geq tan\theta \quad \text{(for 0 to 90 degrees)} \tag{13}$$

Surprisingly, for the vehicle to drive up or at least stay parked on the incline without the tire slipping, it only depends on the angle of the incline and the coefficient of friction between the surface and tire.

With all the set of conditions to find the torque requirement for the rear axle, two scenarios were investigated:

1. At maximum incline of $15°$ and $h = 1.75m$:
    a. Condition 1: $1.65m \geq L \geq 0.4689m$
    b. Condition 2: $\mu_s \geq tan15° \approx 0.268$
    c. $r_{wheel} = 9\,in = 0.2286\,m$
    d. $m = 600kg$ for conservative estimate
    e. $g = 9.81\frac{m}{s^2}$

    For the first scenario, choose the lowest possible coefficient of friction, and recalling equation 9:

$$T_{max} = \frac{1}{2} \times r_{wheel} \times 0.268 \times mg\left[cos15° - \left(\frac{Lcos15° - 1.75sin15°}{1.65}\right)\right] \quad (14)$$

    Plotting equation, treating L as an independent variable and Torque as the dependent variable, maximum torque was assessed as 124.4 Nm, where L = 1.65m.

2. At maximum incline of $15°$ and $h = 1.75\,m$:
    a. Condition 1: $1.65m \geq L \geq 0m$
    b. Condition 2: $\mu_s \geq tan15° \approx 0.268$
    c. $r_{wheel} = 9\,in = 0.2286\,m$
    d. $m = 600kg$ for conservative estimate
    e. $g = 9.81\frac{m}{s^2}$

    For the first scenario, choose the maximum possible coefficient of friction, and recalling equation 9:

$$T_{max} = \frac{1}{2} \times r_{wheel} \times 1.0 \times mg\left[cos15° - \left(\frac{Lcos15° - 1.75sin15°}{1.65}\right)\right] \quad (14)$$

    Plotting equation, treating L as an independent variable and Torque as the dependent variable, maximum torque was assessed as 464.01 Nm, where L = 1.65m.

After investigating these two scenarios, the output of our gearbox should provide the conservative torque of maximum 464 Nm. Torque beyond this value will exceed the maximum friction, causing the tire to screech. 464 Nm is a conservative estimate, and having a torque higher than 124.4Nm will provide sufficient static friction to overcome the weight force component parallel to the incline that pushes the cart down the hill.

On a flat ground, the cart will move forward for any torque value greater than 0 Nm, to calculate by substituting 0 degrees to the angle, θ, in equation (9). Also, maximum torque (before tire screech) required to drive up the incline considering different terrains ranges from 124.4 - 464 Nm. Therefore, the output torque should be in the range of this range for the cart to drive up the hill, but ideally good to have it closer to 464 Nm.

Therefore, knowing our input torque is 56 Nm. Ideal gear ratio to achieve this should be:

$$\frac{124.4\ Nm}{56\ Nm}:1 = 2.22:1 \text{ to } \frac{464\ Nm}{56\ Nm}:1 = 8.286:1.$$

However, this doesn’t mean that our gear ratio has to be restricted to this range, as the cart screeching would not be a problem (cars screeching on the road it's pretty often too). Having a gear ratio greater than that would just be an excessive specification. However, it is definitely essential to achieve the above 2.22:1 ratio from above because that is the minimum output torque requirement for the extreme slippery terrain.

C7 *- Acceptable gear ratio calculation*

- Input RPM $\omega_{in} = 3000\, RPM$
- Output speed 1 $(v_1) = 25 km/h = 6.94444 m/s$
- Output speed 2 $(v_2) = 35 km/h = 9.72222 m/s$
- Tire Radius $r_t = 0.2286 m$

1. $v_1 \frac{m}{s} \times \frac{1}{r_t\, m} \times \frac{60s}{2\pi\, rad} = 290.090 RPM$
2. $v_2 \frac{m}{s} \times \frac{1}{r_t\, m} \times \frac{60s}{2\pi\, rad} = 406.126 RPM$

Therefore, Gear ratio 1:

$$3000 : 290.090 \approx 10.34 : 1$$

Gear ratio 2:

$$3000 : 406.126 \approx 7.39 : 1$$

C8 - *Output speed and output torque calculation*

Output Speed v_o:

$$v_o = \omega_{in} \times \frac{2\pi}{60} \times gear\ ratio \times r_{tire}$$

- ω_{in}=Input angular velocity in RPM
- r_{tire}=Radius of tire (m)

Output Torque T_o:

$$T_{in} \times gear\ ratio = T_o$$

- T_{in}=Input Torque from the motor (Nm)

Therefore, Gear ratio 1:

$$T_o = 579.04\, Nm$$

Therefore, Gear ratio 2:

$$T_o = 413.84\ Nm$$

Both gear ratios exceed the minimum required output torque of 124.4 Nm, ensuring sufficient static friction to counteract the weight force component parallel to the incline that pulls the cart downhill. Although the output torque for Gear Ratio 1 surpasses the maximum threshold of 464 Nm, which could cause tire screeching, this is not considered a limiting factor and is deemed acceptable for the design.

Design Safety Factor

The purpose of calculating a design safety factor is to provide an initial margin of safety that can be used when conducting shaft analysis.

Determine Safety Factor

Materials

AISI 4140 Steel properties from matweb.com

$n_m := 1$

n_m: materials	
1.0	Well known or experimentally determined from identical material and loading conditions
1.1	Handbook of manufacturer values
1.4	Material not well known or of variable properties

Figure 1: Material factor [1]

Stress

$n_s := 1.3$

n_s: stress	
1.1	Loads are well known with no anticipated overloads or shock. Stress analysis is accurate
1.3	Average load known (some overload); stress analysis may contain errors
1.7	Loads not well defined or known; stress analysis is of doubtful accuracy

Figure 2: Stress factor [1]

Geometry

$n_g := 1$

n_g: geometry	
1.0	Manufacturing tolerances tight and held
1.2	Dimensions not closely held

Figure 3: Geometry factor [1]

Failure Analysis

$n_f := 1.2$

n_f: failure analysis	
1.1	Failure analysis is based on simple uniaxial or multiaxial static loads, or fully reversible uniaxial fatigue
1.2	Failure analysis must be extended to handle complex loading cases
1.5	Failure analysis and theory not well developed

Figure 4: Failure analysis factor [1]

Reliability

$n_r := 1.2$

n_r: reliability	
1.1	Reliability not required to be high: less than 90%
1.2	Reliability between 90% to 98%
1.7	Reliability above 99%

Figure 5: Reliability factor [1]

Calculate Safety Factor

$$n := n_m \cdot n_s \cdot n_g \cdot n_f \cdot n_r = 1.872$$

Design safety factor is $n = 1.872$. Use this as initial safety factor in shaft analysis.

References

[1] D. Romanyk, Class Lecture, Topic: "Mechanical Loading and Analysis: Static Failure Analysis." Faculty of Engineering, University of Alberta, Edmonton, Alberta, 2024

C10 *- Shaft Analysis - MATLAB function minShaftDiameter.m for iteratively calculating minimum diameter (DE Elliptic failure criteria) to be used in shaft analysis Smath*

```
function dmin = minShaftDiameter(di, n, Sy, SeUncor, k, Ma, Mm, Ta, Tm, Faa, Fam, Kf, Kfs)
% Function to iteratively determine the minimum required diameter based
% off Distortion Energy Elliptic (DE Elliptic) failure for a shaft under
% bending, axial, and torsional stress.
%
% Inputs:   "di" is the initial guess for diameter (in)
%           "n" is the inital safety factor
%           "Sy" is the yield strength (psi)
%           "SeUncor" is the uncorrected endurance strength (psi)
%           "k" is the result of all the correction factors w/o kb
%           "Ma" is the alternating bending moment (lbf ft)
%           "Mm" is the mean bending moment (lbf ft)
%           "Ta" is the alternating torque (lbf ft)
%           "Tm" is the mean bending torque (lbf ft)
%           "Faa" is the alternating axial force (lbf)
%           "Fam" is the mean axial force (lbf)
%           "Kf" is the fatigue alternating SCF
%           "Kfs" is the fatigue alternating SCF for shear
%
% Output:   "dmin" is the minimum required diameter (in)
%
% Key assumptions:
% - diameter is between 0.11 and 2 (in)
% - Kfm = Kf & Kfsm = Kfs
```

```
% convert bending moment & torque into (lbf in)
Ma = Ma*12;
Mm = Mm*12;
Ta = Ta*12;
Tm = Tm*12;
dmin = di; % set initial guess for dmin
dmax = 2; % max allowable diameter (in)
Kfm = Kf; % fatigue mean SCF
Kfsm = Kfs; % fatigue mean SCF for shear
converged = false;
% iteratively solve for minimum diameter
while ~converged
  % Calculate alternating stresses
  sigma_b_a = (32*Kf*Ma)/(pi*dmin^3); % alternating bending stress
  sigma_a_a = (4*Kf*Faa)/(pi*dmin^2); % alternating axial stress
  tau_t_a = (16*Kfs*Ta)/(pi*dmin^3); % alternating torsional shear stress
  % Calculate mean stresses
  sigma_b_m = (32*Kfm*Mm)/(pi*dmin^3); % mean bending stress
  sigma_a_m = (4*Kfm*Fam)/(pi*dmin^2); % mean axial stress
  tau_t_m = (16*Kfsm*Tm)/(pi*dmin^3); % mean torsional shear stress

  % Calculate corrected endurance strength
  kb = 0.897*dmin^(-0.107); % size correction factor, kb
  Se = k*kb*SeUncor; % adjusted endurance strength
  % Von Mises stresses
  sigma_a_v = ((sigma_b_a+sigma_a_a)^2+3*tau_t_a^2)^(1/2); % alternating
  sigma_m_v = ((sigma_b_m+sigma_a_m)^2+3*tau_t_m^2)^(1/2); % mean
```

```
  % Check if DE Elliptic fatigue failure criteria is met
  lhs = ((n*sigma_a_v)/Se)^2+((n*sigma_m_v)/Sy)^2;
  if lhs <= 1
     converged = true; % found the minimum diameter
  else
     dmin = dmin + 0.0001; % increase diameter and try again
     if dmin > dmax
        error('No feasible diameter found within assumed range.');
     end
  end
end
end
```

C11 *- Shaft Analysis - MATLAB script minShaftMain.m that inputs values from Smath shaft analysis and calls function minShaftDiameter.m*

```
%% Input Shaft

% input known values

di = 0.11; % (in)

n = 1.872;

Sy = 143000; % (psi)

SeUncor = 78573.6; % (psi)

k = 0.613; % ka*kc*kd*ke*kf

Ma = 48.3524; % (lbf*ft)

Mm = 0; % (lbf*ft)

Ta = 20.65; % (lbf*ft)

Tm = 20.65; % (lbf*ft)

Faa = 165.288; % (lbf)

Fam = 165.288; % (lbf)

Kf = 1.9; % Kf=Kfm

Kfs = 2.64; % Kfs=Kfsm

dmin1 = minShaftDiameter(di, n, Sy, SeUncor, k, Ma, Mm, Ta, Tm, Faa, Fam, Kf, Kfs);

% Output the result

fprintf('Minimum diameter (dmin) for gear 1 on input shaft = %.4f in \n', dmin1);

%% Idler Shaft

% input known values

di = 0.11; % (in)

n = 1.872;

Sy = 143000; % (psi)

SeUncor = 78573.6; % (psi)
```

```
k = 0.613; % ka*kc*kd*ke*kf

Ta = 61.95; % (lbf*ft)

Tm = 61.95; % (lbf*ft)

Faa = 123.966; % (lbf)

Fam = 123.966; % (lbf)

Kf = 1.9; % Kf=Kfm

Kfs = 2.64; % Kfs=Kfsm

% Gear 2

Ma = 56.8137; % (lbf*ft)

Mm = 0; % (lbf*ft)

dmin2 = minShaftDiameter(di, n, Sy, SeUncor, k, Ma, Mm, Ta, Tm, Faa, Fam, Kf, Kfs);

% Output the result

fprintf('Minimum diameter (dmin) for gear 2 on idler shaft = %.4f in \n', dmin2);

% Gear 3

Ma = 106.383; % (lbf*ft)

Mm = 0; % (lbf*ft)

dmin3 = minShaftDiameter(di, n, Sy, SeUncor, k, Ma, Mm, Ta, Tm, Faa, Fam, Kf, Kfs);

% Output the result

fprintf('Minimum diameter (dmin) for gear 3 on idler shaft = %.4f in \n', dmin3);

%% Output Shaft

% input known values

di = 1.2598; % (in)

n = 1.872;

Sy = 143000; % (psi)

SeUncor = 78573.6; % (psi)

k = 0.613; % ka*kc*kd*ke*kf

Ma = 172.3345; % (lbf*ft)
```

```
Mm = 0; % (lbf*ft)

Ta = 154.875; % (lbf*ft)

Tm = 154.875; % (lbf*ft)

Faa = 206.61; % (lbf)

Fam = 206.61; % (lbf)

Kf = 1.9; % Kf=Kfm

Kfs = 2.64; % Kfs=Kfsm

dmin4 = minShaftDiameter(di, n, Sy, SeUncor, k, Ma, Mm, Ta, Tm, Faa, Fam, Kf, Kfs);

% Output the result

fprintf('Minimum diameter (dmin) for gear 4 on output shaft = %.4f in \n', dmin4);
```

C12 - Shaft Analysis - Input Shaft

MECE 360: Shaft Analysis, Input Shaft

Annotation Legend
Blue: Notes
Yellow: Final Answers

Problem Statement

The intention of the following calculation is to aid the design of a golf cart transmission. The simply supported shaft made of AISI 4140 Steel below represents the first shaft in a three shaft transmission, the input shaft. For this application, the gear on the shaft is helical and is driving a gear on the idler shaft. The shaft will be connected to the moter via spline gear and rotates at the motor input speed. The gear is straddle mounted and the shaft is supported on either side by bearings (axial load to be carried on the left).

The objective of this design calculation sheet is to solve for the minimum required diameter for the shaft shown in Figure 1 below. Additional calculations are presented to ensure that the evaluation criterial (including shaft twist angle, linear deflection at the gear, and angular deflection at gear) is satisfied. As well, a final calculation for fatigue and yeild safety factor safety factors are included.

Problem Diagram

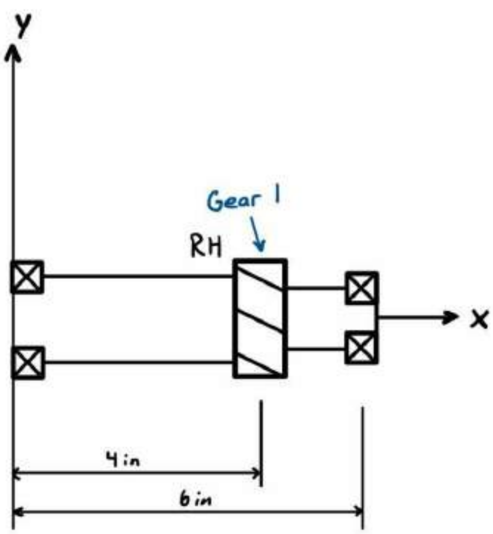
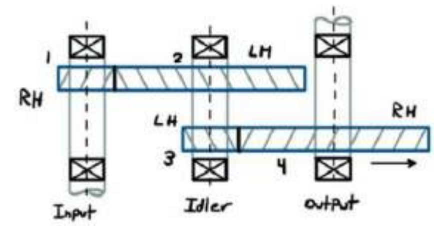


Figure 1. Schematic of design problem: Simply supported shaft with one straddle mounted gear and bearings on each end.



Note the shafts are not in the same plane

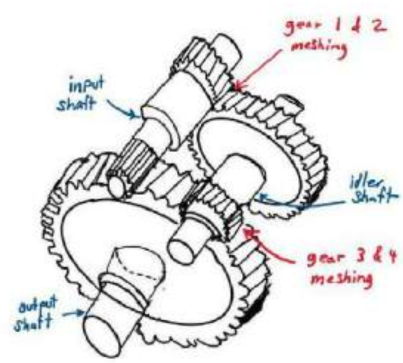


Figure 2. Full layout for all three shafts and gears

Evaluation Criteria

Based on the problem objective, the following criteria must be satisfied:

1. Strength Failure (DE Elliptic Criteria)

- First Cycle Yield: $\sigma'_{max} < \frac{S_y}{n}$
- Fatigue Failure: $\left(\frac{n\sigma'_a}{S_e}\right)^2 + \left(\frac{n\sigma'_m}{S_y}\right)^2 < 1$

2. Shaft twist ≤ 3 deg/m
3. Linaer deflection at gears ≤ 0.127 mm
4. Angular deflection at gears ≤ 0.03 deg (0.0005 rad)
5. Angular deflection at bearings ≤ 0.004 rad

Assumptions

The following assumptions were make for the given problem:

1. Assume machined shaft
2. Assume combined loading, but bending is dominant
3. Assume the operating temperature is 40 °C (104 °F)
4. Assume there are no miscellaneous correction factors
5. Assume lifespan ≥ 10^6 cycles therefore, infinite life
6. Assume 95% reliaitliy
7. Assume constant torque, but will fluctuate between 0 (when not in use) and operating torque
8. Assume forces on shaft are fully reversed, due to shaft rotation causing a critical element to switch between compression and tension of equal magnitudes
9. Assume that bearings and gears act on shaft as point loads
10. Assume that axial load from helical gears goes to bearing with highest radial load
11. Assume that for the stress concentration factors, Kfm = Kf and Kfms = Kfs
12. Assume gears are mounted using parallel keyways

Sketches

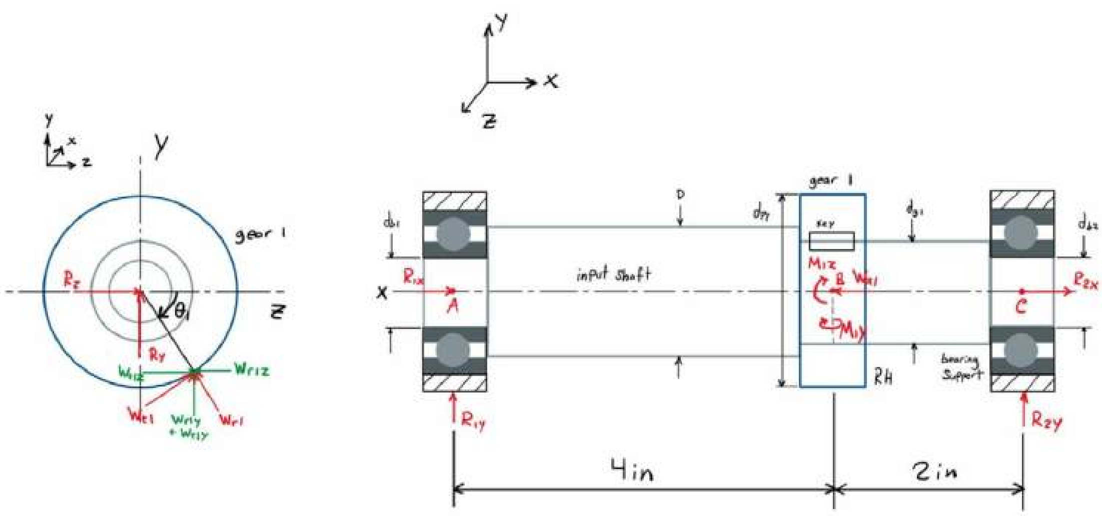


Figure 3. Free Body Diagram (FBD) of design problem with labelled reaction forces on shaft

FBD Variables

At Left Bearing (A)
R1x - Axial Reaction Force
R1y - Y-Direction Reaction Force
R1z - Z-Direction Reaction Force

At Right Bearing (C)
R2x - Axial Reaction Force
R2y - Y-Direction Reaction Force
R2z - Z-Direction Reaction Force

At Gear 1 (B)
Wa1 - Axial Force
Wr1 - Radial Force
Wt1 - Tangential Force
M1z - Z-Direction Moment
M1y - Y-Direction Moment

Misc.
θ1 - Angle between idler shaft and the input shaft

Given for Motor:

$RPM_{in} := 3000\ rpm$ $T_{run} := 41.3\ lbf\ ft$ $T_{max} := 44.69\ lbf\ ft$ (max torque on startup)

Gear Analysis Summary: Pressure Angle $\phi := 20\ °$ Helix Angle $\psi := 21.5\ °$

Number of Teeth	Module	Pitch Diameter
$N_1 := 20\ teeth$	$m_1 := 3\ \frac{mm}{teeth}$	$d_{p1} := m_1 \cdot N_1 = 60\ mm$
$N_2 := 60\ teeth$	$m_2 := 3\ \frac{mm}{teeth}$	$d_{p2} := m_2 \cdot N_2 = 180\ mm$
$N_3 := 24\ teeth$	$m_3 := 3\ \frac{mm}{teeth}$	$d_{p3} := m_3 \cdot N_3 = 72\ mm$
$N_4 := 60\ teeth$	$m_4 := 3\ \frac{mm}{teeth}$	$d_{p4} := m_4 \cdot N_4 = 180\ mm$

Input Shaft Inputs

Shaft Velocity	$\omega := RPM_{in} = 314.1593\ \frac{rad}{s}$	Design Factor of Safety	$n := 1.872$
Shaft Torque	$T := T_{run} = 41.3\ lbf\ ft$	Operation Temperature	$T_{oper} := 104\ °F$
Ultimate Strength	$S_{ut} := 155.9\ kpsi$	Shaft Length	$l := 6\ in$
Yield Strength	$S_y := 143\ kpsi$	Length to Gear 1	$l_{g1} := 4\ in$
Young's Modulus	$E := 29700\ kpsi$	Angle Gear 2 to Gear 1	$\theta_1 := 45\ °$
Shear Modulus	$G := 11600\ kpsi$	Angle Gear 3 to Gear 4	$\theta_4 := 45\ °$

Material properties taken from matweb.com [1]

Find the forces exerted on the input shaft (gears 1):

Transverse Presure Angle $\phi_t := \operatorname{atan}\left(\frac{\tan(\phi)}{\cos(\psi)}\right) = 21.365\ °$

Loads on Gear 1

$$W_{t1} := \frac{T}{0.5 \cdot d_{p1}} = 419.608\ lbf$$

$$W_{r1} := W_{t1} \cdot \tan(\phi_t) = 164.1465\ lbf$$

$$W_{a1} := W_{t1} \cdot \tan(\psi) = 165.288\ lbf$$

$$W_1 := \frac{W_{t1}}{\cos(\psi) \cdot \cos(\phi)} = 479.9324\ lbf$$

$$M_{1z} := W_{a1} \cdot \frac{d_{p1}}{2} \cdot \sin\left(\theta_1\right) = 138.0428 \text{ lbf in}$$

$$M_{1y} := W_{a1} \cdot \frac{d_{p1}}{2} \cdot \cos\left(\theta_1\right) = 138.0428 \text{ lbf in}$$

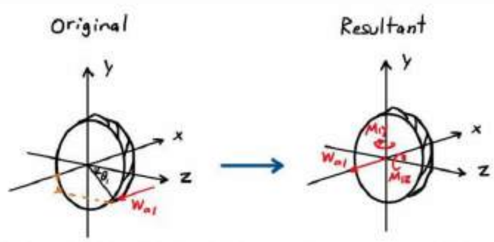


Figure 4. Visualization of two resulting moments, M1z and M1y, due to axial load.

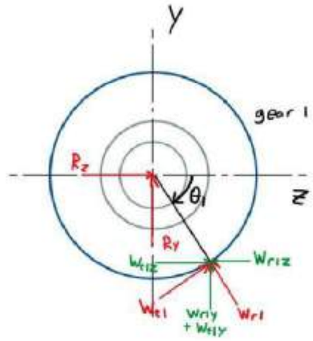


Figure 5. Snapshot of Figure 3.

$$W_{r1z} := W_{r1} \cdot \cos\left(\theta_1\right) = 116.0691 \text{ lbf}$$

$$W_{r1y} := W_{r1} \cdot \sin\left(\theta_1\right) = 116.0691 \text{ lbf}$$

$$W_{t1z} := W_{t1} \cdot \cos\left(90\,° - \theta_1\right) = 296.7077 \text{ lbf}$$

$$W_{t1y} := W_{t1} \cdot \sin\left(90\,° - \theta_1\right) = 296.7077 \text{ lbf}$$

Reaction Forces

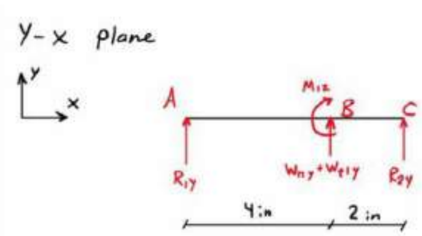


Figure 6. Reaction forces FBD in the y-x plane

$$\circlearrowleft + \sum M_A = 0 \implies 4(W_{r1y} + W_{t1y}) - M_{1z} + 6R_{2y} = 0$$

$$R_{2y} := \frac{M_{1z} - l_{g1} \cdot \left(W_{r1y} + W_{t1y}\right)}{l}$$

$$R_{2y} = -252.1774 \text{ lbf}$$

$$+\uparrow \sum F_y = 0 \implies R_{1y} + W_{r1y} + W_{t1y} + R_{2y} = 0$$

$$R_{1y} := -W_{r1y} - W_{t1y} - R_{2y}$$

$$R_{1y} = -160.5994 \text{ lbf}$$

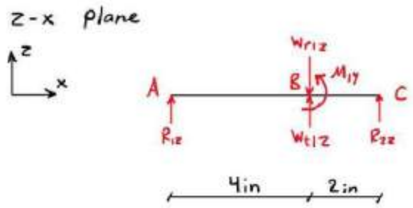


Figure 7. Reaction forces FBD in the z-x plane

$$(+\sum M_A = 0 \Rightarrow 4(W_{t1z} - W_{r1z}) + M_{1y} + 6R_{2z} = 0$$

$$R_{2z} := \frac{-M_{1y} - l_{g1} \cdot (W_{t1z} - W_{r1z})}{l}$$

$$R_{2z} = -143.4328 \text{ lbf}$$

$$+\uparrow \sum F_y = 0 \Rightarrow R_{1z} + W_{t1z} - W_{r1z} + R_{2z} = 0$$

$$R_{1z} := W_{r1z} - W_{t1z} - R_{2z}$$

$$R_{1z} = -37.2057 \text{ lbf}$$

Assume all the axial load is on bearing 1:

$R_{2x} := 0 \text{ lbf}$ $\quad R_{1x} := W_{a1} = 165.288 \text{ lbf}$

Reaction Force Summary

$R_{1y} = -160.5994 \text{ lbf}$ $\quad R_{1z} = -37.2057 \text{ lbf}$ $\quad R_{1x} = 165.288 \text{ lbf}$

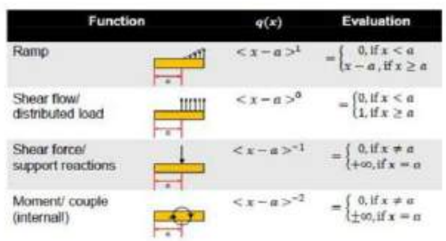
$R_{2y} = -252.1774 \text{ lbf}$ $\quad R_{2z} = -143.4328 \text{ lbf}$ $\quad R_{2x} = 0 \text{ lbf}$

Axial load:

$F_a := R_{1x} = 165.288 \text{ lbf}$

Singularity Function

```
s(x, a, n) := if (((x - a) > 0) ∧ (n ≥ 0))
                (x - a)^n
              else
                if (((x - a) = 0) ∧ (n = 0))
                  1
                else
                  0
```



Function	q(x)	Evaluation
Ramp	$<x-a>^1$	$=\begin{cases}0, \text{if } x<a \\ x-a, \text{if } x \ge a\end{cases}$
Shear flow/ distributed load	$<x-a>^0$	$=\begin{cases}0, \text{if } x<a \\ 1, \text{if } x \ge a\end{cases}$
Shear force/ support reactions	$<x-a>^{-1}$	$=\begin{cases}0, \text{if } x \ne a \\ +\infty, \text{if } x = a\end{cases}$
Moment/ couple (internall)	$<x-a>^{-2}$	$=\begin{cases}0, \text{if } x \ne a \\ \pm\infty, \text{if } x = a\end{cases}$

Figure 8. Singularity Function (to solve for Vy(x) and Mz(x) [2]

Singularity Function Equations for shear force and bending moment

Shear Force $\quad V_y(x) = \int q_y(x)dx$

Bending Moment $\quad M_z(x) = \int V_y(x)dx$

On X-Y plane:

$$q_y(x) := R_{1y} \cdot s(x, 0, -1) + \left(\left(W_{r1y} + W_{t1y}\right) \cdot s\left(x, l_{g1}, -1\right) + M_{1z} \cdot s\left(x, l_{g1}, -2\right)\right) + R_{2y} \cdot s(x, l, -1)$$

$$v_y(x) := R_{1y} \cdot s(x, 0, 0) + \left(\left(W_{r1y} + W_{t1y}\right) \cdot s\left(x, l_{g1}, 0\right) + M_{1z} \cdot s\left(x, l_{g1}, -1\right)\right) + R_{2y} \cdot s(x, l, 0)$$

$$M_z(x) := R_{1y} \cdot s(x, 0, 1) + \left(\left(W_{r1y} + W_{t1y}\right) \cdot s\left(x, l_{g1}, 1\right) + M_{1z} \cdot s\left(x, l_{g1}, 0\right)\right) + R_{2y} \cdot s(x, l, 1)$$

On X-Z plane:

$$q_z(x) := R_{1z} \cdot s(x, 0, -1) + \left(W_{t1z} - W_{r1z}\right) \cdot s\left(x, l_{g1}, -1\right) - M_{1y} \cdot s\left(x, l_{g1}, -2\right) + R_{2z} \cdot s(x, l, -1)$$

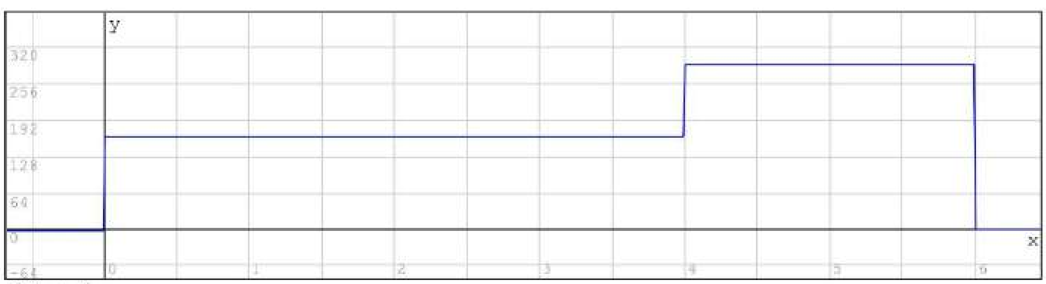
$$v_z(x) := R_{1z} \cdot s(x, 0, 0) + \left(W_{t1z} - W_{r1z}\right) \cdot s\left(x, l_{g1}, 0\right) - M_{1y} \cdot s\left(x, l_{g1}, -1\right) + R_{2z} \cdot s(x, l, 0)$$

$$M_y(x) := R_{1z} \cdot s(x, 0, 1) + \left(W_{t1z} - W_{r1z}\right) \cdot s\left(x, l_{g1}, 1\right) - M_{1y} \cdot s\left(x, l_{g1}, 0\right) + R_{2z} \cdot s(x, l, 1)$$

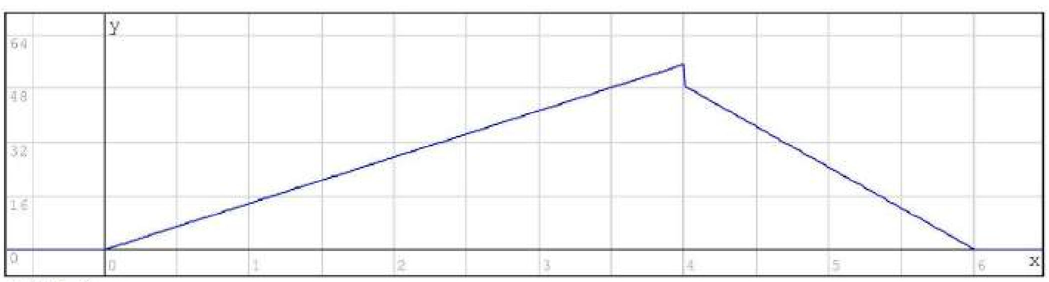
Total V and M:

$$V(x) := \sqrt{v_y(x)^2 + v_z(x)^2}$$

$$M(x) := \sqrt{M_y(x)^2 + M_z(x)^2}$$



$\frac{V(x\ in)}{lbf}$



$\frac{M(x\ in)}{lbf\ ft}$

The resultant moment at gear 1

$$M_{g1} := M\left(l_{g1}\right) = 48.3524\ lbf\ ft$$

Consider Loads on Startup (Max Torque)

Loads on Gear 1

$$W_{t1} := \frac{T_{max}}{0.5 \cdot d_{p1}} = 454.0504 \text{ lbf}$$

$$W_{r1} := W_{t1} \cdot \tan\left(\phi_t\right) = 177.6201 \text{ lbf}$$

$$W_{a1} := W_{t1} \cdot \tan\left(\psi\right) = 178.8552 \text{ lbf}$$

$$M_{1z} := W_{a1} \cdot \frac{d_{p1}}{2} \cdot \sin\left(\theta_1\right) = 149.3737 \text{ lbf in}$$

$$M_{1y} := W_{a1} \cdot \frac{d_{p1}}{2} \cdot \cos\left(\theta_1\right) = 149.3737 \text{ lbf in}$$

$$W_{r1z} := W_{r1} \cdot \cos\left(\theta_1\right) = 125.5964 \text{ lbf}$$

$$W_{r1y} := W_{r1} \cdot \sin\left(\theta_1\right) = 125.5964 \text{ lbf}$$

$$W_{t1z} := W_{t1} \cdot \cos\left(90\,° - \theta_1\right) = 321.0621 \text{ lbf}$$

$$W_{t1y} := W_{t1} \cdot \sin\left(90\,° - \theta_1\right) = 321.0621 \text{ lbf}$$

Reaction Forces

y-x plane:

$$R_{2y} := \frac{M_{1z} - l_{g1} \cdot \left(W_{r1y} + W_{t1y}\right)}{l} = -272.8767 \text{ lbf}$$

$$R_{1y} := -W_{r1y} - W_{t1y} - R_{2y} = -173.7818 \text{ lbf}$$

z-x plane:

$$R_{2z} := \frac{-M_{1y} - l_{g1} \cdot \left(W_{t1z} - W_{r1z}\right)}{l} = -155.2061 \text{ lbf}$$

$$R_{1z} := W_{r1z} - W_{t1z} - R_{2z} = -40.2596 \text{ lbf}$$

axial load on bearing 1:

$$R_{2x} := 0 \text{ lbf} \qquad R_{1x} := W_{a1} = 178.8552 \text{ lbf}$$

max axial force:

$$F_{a'max} := R_{1x} = 178.8552 \text{ lbf}$$

Singularity Function

On X-Y plane:

$$q_y(x) := R_{1y} \cdot s(x, 0, -1) + \left(\left(W_{r1y} + W_{t1y}\right) \cdot s\left(x, l_{g1}, -1\right) + M_{1z} \cdot s\left(x, l_{g1}, -2\right)\right) + R_{2y} \cdot s(x, l, -1)$$

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$$v_y(x) := R_{1y} \cdot s(x, 0, 0) + \left(\left(W_{r1y} + W_{t1y} \right) \cdot s\left(x, l_{g1}, 0\right) + M_{1z} \cdot s\left(x, l_{g1}, -1\right) \right) + R_{2y} \cdot s(x, l, 0)$$

$$M_z(x) := R_{1y} \cdot s(x, 0, 1) + \left(\left(W_{r1y} + W_{t1y} \right) \cdot s\left(x, l_{g1}, 1\right) + M_{1z} \cdot s\left(x, l_{g1}, 0\right) \right) + R_{2y} \cdot s(x, l, 1)$$

On X-Z plane:

$$q_z(x) := R_{1z} \cdot s(x, 0, -1) + \left(W_{t1z} - W_{r1z} \right) \cdot s\left(x, l_{g1}, -1\right) - M_{1y} \cdot s\left(x, l_{g1}, -2\right) + R_{2z} \cdot s(x, l, -1)$$

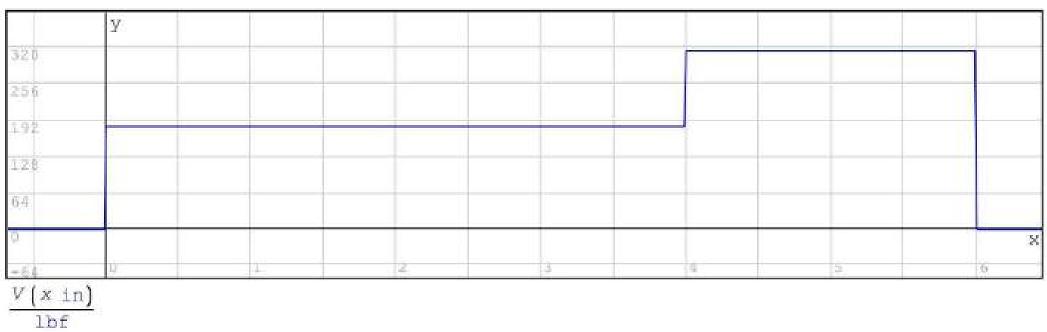
$$v_z(x) := R_{1z} \cdot s(x, 0, 0) + \left(W_{t1z} - W_{r1z} \right) \cdot s\left(x, l_{g1}, 0\right) - M_{1y} \cdot s\left(x, l_{g1}, -1\right) + R_{2z} \cdot s(x, l, 0)$$

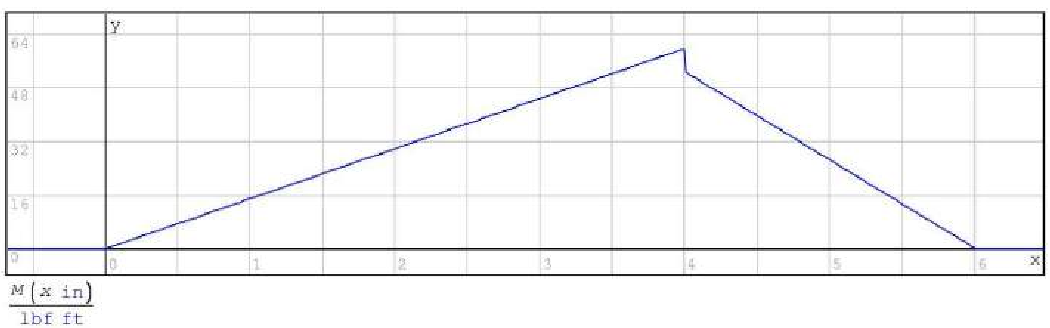
$$M_y(x) := R_{1z} \cdot s(x, 0, 1) + \left(W_{t1z} - W_{r1z} \right) \cdot s\left(x, l_{g1}, 1\right) - M_{1y} \cdot s\left(x, l_{g1}, 0\right) + R_{2z} \cdot s(x, l, 1)$$

Total V and M:

$$V(x) := \sqrt{v_y(x)^2 + v_z(x)^2}$$

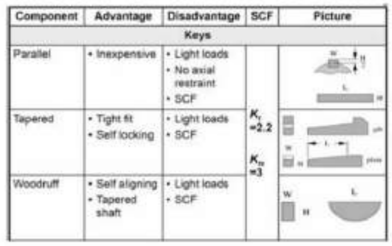
$$M(x) := \sqrt{M_y(x)^2 + M_z(x)^2}$$





The resultant moment at gear 1: $M_{g1'max} := M\left(l_{g1}\right) = 52.3213\ \text{lbf ft}$ Max moment for startup torque

Sizing the Shaft Based on Fatigue Strength



Component	Advantage	Disadvantage	SCF	Picture
Keys				
Parallel	• Inexpensive	• Light loads • No axial restraint • SCF	$K_t = 2.2$ $K_{ts} = 3$	
Tapered	• Tight fit • Self locking	• Light loads • SCF		
Woodruff	• Self aligning • Tapered shaft	• Light loads • SCF		

Figure 9. Keys SCF [3]

Gear 1

Determine minimum diameter at gear 1 assuming keyway has greatest SCF

$K_t := 2.2$

SCF for key

$K_{ts} := 3$

$r_{key} := 0.01\ \text{in}$

$r_{shoulder} := 0.1\ \text{in}$

$q := 0.75 \qquad q_s := 0.82$

$K_f := 1 + q \cdot (K_t - 1) = 1.9$

$K_{fs} := 1 + q_s \cdot (K_{ts} - 1) = 2.64$

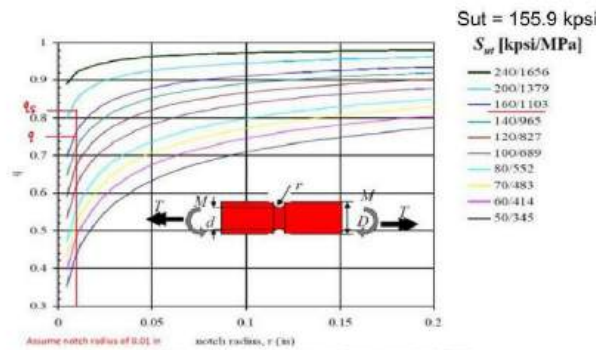


Figure 10. Notch Sensitivity (to solve for q and qs) [4]

Torque on idler shaft

The torque will be constant throughout the input shaft, but fluctuate between 0 (when not in use) and operating shaft torque.

$$T_{mean} := \frac{T + 0}{2} = 20.65\ \text{lbf ft} \qquad T_{alt} := \frac{T - 0}{2} = 20.65\ \text{lbf ft}$$

Bending moment will be fully reversed loading

$$M_{mean} := \frac{M_{g1} + (-M_{g1})}{2} = 0\ \text{lbf ft} \qquad M_{alt} := \frac{M_{g1} - (-M_{g1})}{2} = 48.3524\ \text{lbf ft}$$

Axial force will be constant throughout the input shaft, but fluctuate between 0 (when not in use) and operating shaft torque.

$$F_{a'mean} := \frac{F_a + 0}{2} = 82.644\ \text{lbf} \qquad F_{a'alt} := \frac{F_a - 0}{2} = 82.644\ \text{lbf}$$

Endurance Strength

Uncorrected endurance strength

$$S'_e := \text{if } S_{ut} \le 1463\ \text{MPa} \quad 0.504 \cdot S_{ut} \quad \text{else} \quad 737\ \text{MPa}$$

$S'_e = 78573.6\ \text{psi}$

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Surface Condition Factor, ka:

Assuming the shaft is machined

$a := 2.7 \qquad b := -0.265$

$k_a := a \cdot \left(S_{ut} \cdot \frac{1}{kpsi} \right)^b = 0.7084$

	MPa		kpsi	
Surface finish	a	b	a	B
Ground (standard unless otherwise indicated)	*1.58*	*-0.085*	*1.34*	*-0.085*
Machined or Cold drawn	4.51	-0.265	2.7	-0.265
Hot-rolled	57.7	-0.718	14.4	-0.718
As-forged	272	-0.995	39.9	-0.995

Size Correction Factor, kb:

Assuming bending and torsion with rotation

Assume dg1 is between 0.11 and 2 in:

kb = 0.897*d^(-0.107)

$$k_b = \begin{cases} 0.879d^{-0.107} & 0.11 \le d \le 2\,\text{in} \\ 0.91d^{-0.157} & 2 < d \le 10\,\text{in} \\ 1.24d^{-0.107} & 2.79 \le d \le 51\,\text{mm} \\ 1.51d^{-0.157} & 51 < d \le 254\,\text{mm} \end{cases}$$

We must come back to check this

Load Correction Factor, kc:

Combined loading, bending is considered dominant

$k_c := 1$

$$k_c = \begin{cases} 1 & \text{Bending} \\ 0.85 & \text{Axial} \\ 0.59 & \text{Pure torsion} \end{cases}$$

Temperature Correction Factor, kd:

Temperature Correction for operating temperature of 104 F

$$k_d := 0.975 + 0.00032 \cdot \left(\frac{T_{oper}}{°F} \right) - 0.115 \cdot 10^{-5} \cdot \left(\frac{T_{oper}}{°F} \right)^2 + 0.104 \cdot 10^{-8} \cdot \left(\frac{T_{oper}}{°F} \right)^3 - 0.595 \cdot 10^{-12} \cdot \left(\frac{T_{oper}}{°F} \right)^4 = 0.9969$$

Reliability Factor, ke:

Assume reliability to be 95%

$k_e := 0.868$

Reliability	Ke
50	1
90	0.897
95	0.868
99	0.814
99.9	0.753
99.99	0.702
99.999	0.659
99.9999	0.62

Miscellaneous Correction Factor, kf:

Assume no miscellaneous correction

$k_f := 1$

Result of all correction factors without kb:

$k := k_a \cdot k_c \cdot k_d \cdot k_e \cdot k_f = 0.613$

Minimum Diameter Iteration Calculation

Iterate using MATLAB code to find minimum shaft diameter, d. The MATLAB function takes an initial diameter and checks if it passes DE Elliptic Failure Criteria, if not increase diameter and try again, if yes then minimum diameter has been found.

DE Elliptic Failure Criteria: $\left(\frac{n\sigma'_a}{S_e}\right)^2 + \left(\frac{n\sigma'_m}{S_y}\right)^2 < 1$

Von Mises Stresses accounting for bending stress, axial stress, and torsional stress:

$$\sigma'_{alt} = \left(\left(\frac{32 \cdot K_f \cdot M_{alt}}{\pi \cdot d^3} + \frac{4 \cdot K_f \cdot F_{a'alt}}{\pi \cdot d^2}\right)^2 + 3 \cdot \left(\frac{16 \cdot K_{fs} \cdot T_{alt}}{\pi \cdot d^3}\right)^2\right)^{\frac{1}{2}}$$

$$\sigma'_{mean} = \left(\left(\frac{32 \cdot K_{fm} \cdot M_{mean}}{\pi \cdot d^3} + \frac{4 \cdot K_{fm} \cdot F_{a'mean}}{\pi \cdot d^2}\right)^2 + 3 \cdot \left(\frac{16 \cdot K_{fsm} \cdot T_{mean}}{\pi \cdot d^3}\right)^2\right)^{\frac{1}{2}}$$

Corrected Endurance Strength:

$S_e = k_a \cdot k_b \cdot k_c \cdot k_d \cdot k_e \cdot k_f \cdot S'_e$ kb is a function of diameter

Assume $K_f \cdot \sigma'_{max} < S_y$ so $K_{fm} := K_f$ and $K_{fsm} := K_{fs}$

Plug in an intial diameter guess of d = 0.11 in (minimum assumed diameter range) into the MATLAB function minShaftDiameter.m, as well as other required input values.
The MATLAB function iteratively tests for a minimum shaft diameter that meets DE Elliptic Failure Criteria using the above formulas.

Computed minimum shaft diameter at gear 1: $d_{g1} := 0.8210 \text{ in}$

Calculate final endurance strength using minimum diameter:

$k_b := 0.897 \cdot \left(\frac{d_{g1}}{\text{in}}\right)^{-0.107} = 0.9161$ $S_e := k_a \cdot k_b \cdot k_c \cdot k_d \cdot k_e \cdot k_f \cdot S'_e = 44.1249 \text{ kpsi}$

Check Yield Failure Criteria

DE Failure criteria states: $\sigma' < \frac{S_y}{n}$

$$\sigma' := \left(\left(\frac{32 \cdot K_f \cdot M_{g1'max}}{\pi \cdot d_{g1}^3} + \frac{4 \cdot K_f \cdot F_{a'max}}{\pi \cdot d_{g1}^2}\right)^2 + 3 \cdot \left(\frac{16 \cdot K_{fs} \cdot T_{max}}{\pi \cdot d_{g1}^3}\right)^2\right)^{\frac{1}{2}} = 31.9384 \text{ kpsi}$$

$\sigma' = 31.9384 \text{ kpsi}$ ≤ $\frac{S_y}{n} = 76.3889 \text{ kpsi}$ Therefore, first cycle yield passes.

Check the assumptions

$$k_b = \begin{cases} 0.879d^{-0.107} & 0.11 \le d \le 2\,\text{in} \\ 0.91d^{-0.157} & 2 < d \le 10\,\text{in} \\ 1.24d^{-0.107} & 2.79 \le d \le 51\,\text{mm} \\ 1.51d^{-0.157} & 51 < d \le 254\,\text{mm} \end{cases}$$

0.11 ≤ dg1 = 0.8210 in ≤ 2 in

Assumption is valid

Moment of Inertia:

$$I := \pi \cdot \frac{{d_{g1}}^4}{64} = 0.0223\ \text{in}^4$$

Polar Moment of Inertia:

$$J := 2 \cdot I = 0.0446\ \text{in}^4$$

Cross-Sectional Area:

$$A := \frac{1}{4} \cdot \pi \cdot {d_{g1}}^2 = 0.5294\ \text{in}^2$$

$$\sigma'_{max} := \sqrt{\left(\frac{M_{g1} \cdot \frac{d_{g1}}{2}}{I} + \frac{W_{a1}}{A}\right)^2 + 3 \cdot \left(\frac{T \cdot \frac{d_{g1}}{2}}{J}\right)^2} = 13.5574\ \text{kpsi}$$

$K_f \cdot \sigma'_{max} = 25.7591$ kpsi ≤ $S_y = 143$ kpsi

Therefore, Kfm = Kf was a valid assumption

Check the SCF of the keyway is greater than the SCF of the step

Assume Diameter of the center part is

$D := 1.25$ in $\quad \frac{D}{d_{g1}} = 1.5225 \quad \frac{r_{shoulder}}{d_{g1}} = 0.1218$

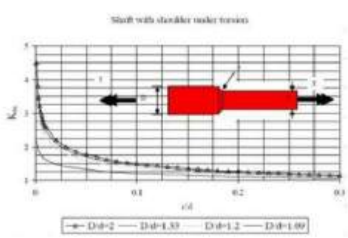

Figure 11. Step Shaft = Shaft with shoulder under torsion [4]

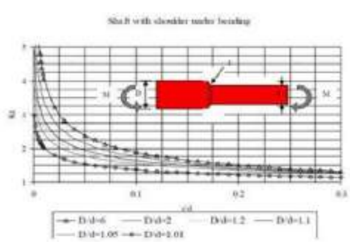

Figure 12. Step Shaft = Shaft with shoulder under bending [4]

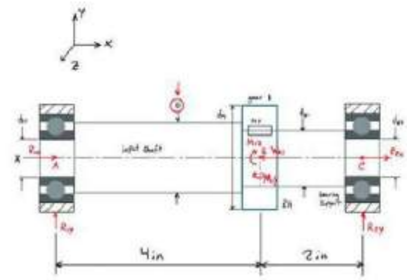

Figure 13. FBD to show "D"

$K_{ts} := 1.25 \quad K_t := 1.6$

$q := 0.9 \quad q_s := 0.925$

Step SCF:

$K_{f2} := 1 + q \cdot (K_t - 1) = 1.54$ ≤

$K_{fs2} := 1 + q_s \cdot (K_{ts} - 1) = 1.2312$ ≤

Keyway SCF:

$K_f = 1.9$

$K_{fs} = 2.64$

Sut = 155.9 kpsi

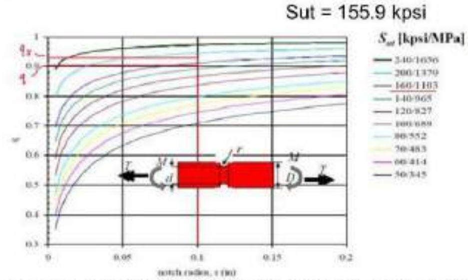

Figure 14. Notch Sensitivity (to solve for q and qs) [4]

Therefore assuming the SCF of the keyway is greater than the SCF of the shoulder is valid

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Minimum diameter of shaft where gear is located:

$d_{g1} = 0.821\ in$

$d_{g1} := 30\ mm = 1.1811\ in$ size up to chosen gear bore diameter

Assume the diameters at the bearings

$d_{b1} := 25\ mm = 0.9843\ in$ $d_{b2} := 25\ mm = 0.9843\ in$ chosen bearing bore diameters

Slope and Deflection Calculations

$I_1 := \pi \cdot \frac{d_{b1}^4}{64} = 0.0461\ in^4$ $I_2 := \pi \cdot \frac{d_{g1}^4}{64} = 0.0955\ in^4$ $I_3 := \pi \cdot \frac{d_{b2}^4}{64} = 0.0461\ in^4$

$J_1 := 2 \cdot I_1 = 0.0921\ in^4$ $J_2 := 2 \cdot I_2 = 0.1911\ in^4$ $J_3 := 2 \cdot I_3 = 0.0921\ in^4$

Singularity Function Equations for Angular Deflection and Deflection of Shaft

$$\theta_z(x) = \int \frac{M_z(x)}{EI} dx + C_1 \quad \text{Slope}$$

$$y(x) = \int \left[\int \frac{M_z(x)}{EI} dx + C_1 \right] dx + C_2 \quad \text{Deflection}$$

$$\theta_z(x) := \frac{1}{E \cdot I_1} \cdot \frac{R_{1y}}{2} \cdot s(x, 0, 2) + \frac{1}{E \cdot I_2} \cdot \left(\frac{\left(W_{r1y} + W_{t1y}\right)}{2} \cdot s\left(x, l_{g1}, 2\right) + M_{1z} \cdot s\left(x, l_{g1}, 1\right) \right) + \\ + \frac{1}{E \cdot I_3} \cdot \frac{R_{2y}}{2} \cdot s(x, l, 2) + c_{1y}$$

$$y(x) := \frac{1}{E \cdot I_1} \cdot \frac{R_{1y}}{6} \cdot s(x, 0, 3) + \frac{1}{E \cdot I_2} \cdot \left(\frac{\left(W_{r1y} + W_{t1y}\right)}{6} \cdot s\left(x, l_{g1}, 3\right) + \frac{M_{1z}}{2} \cdot s\left(x, l_{g1}, 2\right) \right) + \\ + \frac{1}{E \cdot I_3} \cdot \frac{R_{2y}}{6} \cdot s(x, l, 3) + c_{1y} \cdot x + c_{2y}$$

$$\theta_y(x) := \frac{1}{E \cdot I_1} \cdot \frac{R_{1z}}{2} \cdot s(x, 0, 2) + \frac{1}{E \cdot I_2} \cdot \left(\frac{\left(W_{t1z} - W_{r1z}\right)}{2} \cdot s\left(x, l_{g1}, 2\right) - M_{1y} \cdot s\left(x, l_{g1}, 1\right) \right) + \\ + \frac{1}{E \cdot I_3} \cdot \frac{R_{2z}}{2} \cdot s(x, l, 2) + c_{1z}$$

$$z(x) := \frac{1}{E \cdot I_1} \cdot \frac{R_{1z}}{6} \cdot s(x, 0, 3) + \frac{1}{E \cdot I_2} \cdot \left(\frac{\left(W_{t1z} - W_{r1z}\right)}{6} \cdot s\left(x, l_{g1}, 3\right) - \frac{M_{1y}}{2} \cdot s\left(x, l_{g1}, 2\right) \right) + \\ + \frac{1}{E \cdot I_3} \cdot \frac{R_{2z}}{6} \cdot s(x, l, 3) + c_{1z} \cdot x + c_{2z}$$

Solve for constants using boundary conditions: (No linear deflection at bearings)

Y-Direction

$y(0) = 0\ \text{in} \qquad y(l) = 0\ \text{in}$

$c_{2y} := 0$

$$c_{1y} := \frac{\frac{1}{E \cdot I_1} \cdot \frac{R_{1y}}{6} \cdot s(l, 0, 3) + \frac{1}{E \cdot I_2} \cdot \left(\frac{\left(W_{r1y} + W_{t1y}\right)}{6} \cdot s\left(l, l_{g1}, 3\right) + \frac{M_{1z}}{2} \cdot s\left(l, l_{g1}, 2\right) \right)}{-l}$$

$c_{1y} = 0.0007$

Z-Direction

$z(0) = 0\ \text{in} \qquad z(l) = 0\ \text{in}$

$c_{2z} := 0$

$$c_{1z} := \frac{\frac{1}{E \cdot I_1} \cdot \frac{R_{1z}}{6} \cdot s(l, 0, 3) + \frac{1}{E \cdot I_2} \cdot \left(\frac{\left(W_{t1z} - W_{r1z}\right)}{6} \cdot s\left(l, l_{g1}, 3\right) - \frac{M_{1y}}{2} \cdot s\left(l, l_{g1}, 2\right) \right)}{-l}$$

$c_{1z} = 0.0002$

$$\theta_z(x) := \frac{1}{E \cdot I_1} \cdot \frac{R_{1y}}{2} \cdot s(x, 0, 2) + \frac{1}{E \cdot I_2} \cdot \left(\frac{\left(W_{r1y} + W_{t1y}\right)}{2} \cdot s\left(x, l_{g1}, 2\right) + M_{1z} \cdot s\left(x, l_{g1}, 1\right) \right) + \\ + \frac{1}{E \cdot I_3} \cdot \frac{R_{2y}}{2} \cdot s(x, l, 2) + c_{1y}$$

$$y(x) := \frac{1}{E \cdot I_1} \cdot \frac{R_{1y}}{6} \cdot s(x, 0, 3) + \frac{1}{E \cdot I_2} \cdot \left(\frac{\left(W_{r1y} + W_{t1y}\right)}{6} \cdot s\left(x, l_{g1}, 3\right) + \frac{M_{1z}}{2} \cdot s\left(x, l_{g1}, 2\right) \right) + \\ + \frac{1}{E \cdot I_3} \cdot \frac{R_{2y}}{6} \cdot s(x, l, 3) + c_{1y} \cdot x + c_{2y}$$

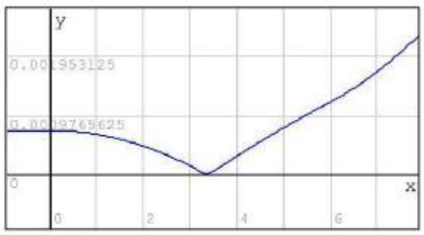
$$\theta_y(x) := \frac{1}{E \cdot I_1} \cdot \frac{R_{1z}}{2} \cdot s(x, 0, 2) + \frac{1}{E \cdot I_2} \cdot \left(\frac{\left(W_{t1z} - W_{r1z}\right)}{2} \cdot s\left(x, l_{g1}, 2\right) - M_{1y} \cdot s\left(x, l_{g1}, 1\right) \right) + \\ + \frac{1}{E \cdot I_3} \cdot \frac{R_{2z}}{2} \cdot s(x, l, 2) + c_{1z}$$

$$z(x) := \frac{1}{E \cdot I_1} \cdot \frac{R_{1z}}{6} \cdot s(x, 0, 3) + \frac{1}{E \cdot I_2} \cdot \left(\frac{\left(W_{t1z} - W_{r1z}\right)}{6} \cdot s\left(x, l_{g1}, 3\right) - \frac{M_{1y}}{2} \cdot s\left(x, l_{g1}, 2\right) \right) + \\ + \frac{1}{E \cdot I_3} \cdot \frac{R_{2z}}{6} \cdot s(x, l, 3) + c_{1z} \cdot x + c_{2z}$$

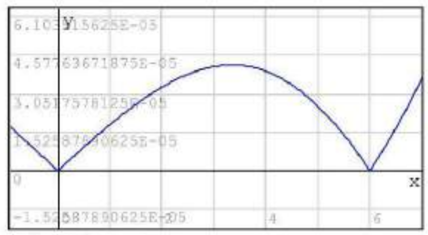
Total Angular Deflection and Deflection

$$\theta(x) := \sqrt{\theta_z(x)^2 + \theta_y(x)^2}$$

$$\delta(x) := \sqrt{y(x)^2 + z(x)^2}$$



$\theta(x \text{ in})$



$\delta(x \text{ in})$

Angular Deflection at Bearings

Criteria ≤ 0.004 rad

$\theta_{R1} := \theta(0) = 0.0007 \text{ rad}$ Angular deflection at beginning of shaft

$\theta_{R2} := \theta(l) = 0.0012 \text{ rad}$ Angular deflection at end of shaft

Both supports angular deflection does not exceed the allowable value. Therefore, criteria is met!

PASS

Angular Deflection at Gear

Criteria ≤ 0.0005 rad

$\theta_{G1} := \theta(l_{g1}) = 0.0003 \text{ rad}$ Angular deflection at gear 1

Gear angular deflection does not exceed the allowable value. Therefore, criteria is met!

PASS

Linear Deflection at Gear

Criteria ≤ 0.127 mm

$\delta_{G1} := \delta(l_{g1}) = 0.039 \text{ mm}$ Deflection at gear 1

Both gears deflection does not exceed the allowable value. Therefore, criteria is met!

PASS

Angle of Twist

Only need to check where J is the lowest and there are the most SCFs

Criteria ≤ 3 deg/m

$$\phi_{twist} := \frac{T}{G \cdot J_1} = 1.046 \, \frac{\text{deg}}{\text{m}}$$

Angle of twist does not exceed the allowable value. Therefore, criteria does pass.

PASS

Summary of Results

As a reminder, the evaluation criteria we need to satisfy are:

2. Shaft twist	≤ 3 deg/m
3. Linaer deflection at gears	≤ 0.127 mm
4. Angular deflection at gears	≤ 0.03 deg (0.0005 rad)
5. Angular deflection at bearings	≤ 0.004 rad

Minimum Diameters Summary

At Gear 1: $d_{g1} = 1.1811\ in$ (= 30mm)

At Bearing 1: $d_{b1} = 0.9843\ in$

At Bearing 2: $d_{b2} = 0.9843\ in$ (=25mm)

Evaluation Criteria

$\theta_{R1} = 0.0007$

$\theta_{R2} = 0.0012$

$\theta_{G1} = 0.0003$

$\delta_{G1} = 0.039\ mm$

$\phi_{twist} = 1.046\ \frac{deg}{m}$

Initial Conclusions

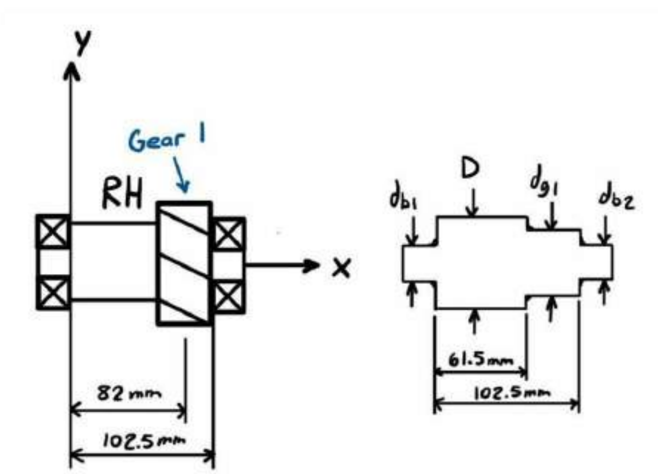
Calculations have been carried out, following a re-itteration method, a minimum diameter at gear 1 of 1.1811 in (30 mm) and a minimum diameter around both bearings of 1 in has been specified which meets all required conditions and defined evaluation criteria.

Optimized Shaft Length

Problem Statement

The objective of the following calculation is to confirm a new design (new shaft length) meets evaluation criteria. Due to the intended use of the shafts to rotate in both directions (drive forward and reverse), need to consider a shaft design that supports the helical gears axially in both directions. Consider a design where the gears are "sandwiched" along the shaft to prevent any axial thrusts (shaft step on one side and support bearing on other).

Problem Diagram



From Gear Analysis, we are given that the gear face width is 41 mm, use this to size the shaft appropriately.

Figure 15. Schematic of design problem: Simply supported shaft with two straddle mounted gears and bearings on each end.

Evaluation Criteria

Same evaluation criteria as orignial shaft analysis

Assumptions

Same assumptions as orignial shaft analysis

New Shaft Lengths

Shaft Length $l := 102.5\ mm = 4.0354\ in$

Length to Gear 1 $l_{g1} := 82\ mm = 3.2283\ in$

Find the forces exerted on the input shaft (gears 1):

Transverse Presure Angle $\phi_t := atan\left(\frac{\tan(\phi)}{\cos(\psi)}\right) = 21.365\ °$

Loads on Gear 1

$$W_{t1} := \frac{T}{0.5 \cdot d_{p1}} = 419.608\ lbf$$

$$W_{r1} := W_{t1} \cdot \tan(\phi_t) = 164.1465\ lbf$$

$$W_{a1} := W_{t1} \cdot \tan(\psi) = 165.288\ lbf$$

$$W_1 := \frac{W_{t1}}{\cos(\psi) \cdot \cos(\phi)} = 479.9324\ lbf$$

$$M_{1z} := W_{a1} \cdot \frac{d_{p1}}{2} \cdot \sin(\theta_1) = 138.0428\ lbf\ in$$

$$M_{1y} := W_{a1} \cdot \frac{d_{p1}}{2} \cdot \cos(\theta_1) = 138.0428\ lbf\ in$$

$$W_{r1z} := W_{r1} \cdot \cos(\theta_1) = 116.0691\ lbf$$

$$W_{r1y} := W_{r1} \cdot \sin(\theta_1) = 116.0691\ lbf$$

$$W_{t1z} := W_{t1} \cdot \cos(90\ ° - \theta_1) = 296.7077\ lbf$$

$$W_{t1y} := W_{t1} \cdot \sin(90\ ° - \theta_1) = 296.7077\ lbf$$

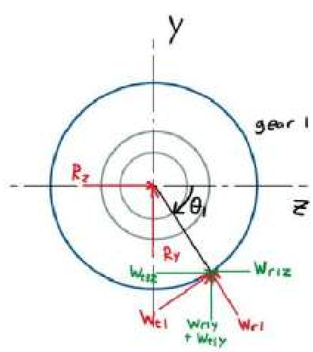


Figure 16. Snapshot of Figure 3.

Reaction Forces

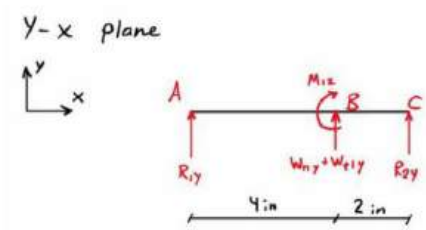


Figure 17. Reaction forces FBD in the y-x plane

$$\circlearrowleft + \sum M_A = 0 \Rightarrow 4(W_{r1y} + W_{t1y}) - M_{1z} + 6R_{2y} = 0$$

$$R_{2y} := \frac{M_{1z} - l_{g1} \cdot \left(W_{r1y} + W_{t1y}\right)}{l}$$

$$R_{2y} = -296.0137 \text{ lbf}$$

$$+\uparrow \sum F_y = 0 \Rightarrow R_{1y} + W_{r1y} + W_{t1y} + R_{2y} = 0$$

$$R_{1y} := -W_{r1y} - W_{t1y} - R_{2y}$$

$$R_{1y} = -116.763 \text{ lbf}$$

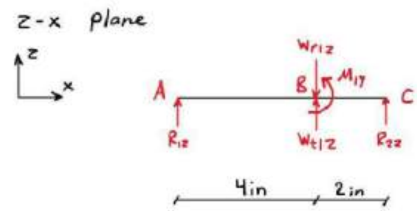


Figure 18. Reaction forces FBD in the z-x plane

$$\circlearrowleft + \sum M_A = 0 \Rightarrow 4(W_{t1z} - W_{r1z}) + M_{1y} + 6R_{2z} = 0$$

$$R_{2z} := \frac{-M_{1y} - l_{g1} \cdot \left(W_{t1z} - W_{r1z}\right)}{l}$$

$$R_{2z} = -178.7185 \text{ lbf}$$

$$+\uparrow \sum F_y = 0 \Rightarrow R_{1z} + W_{t1z} - W_{r1z} + R_{2z} = 0$$

$$R_{1z} := W_{r1z} - W_{t1z} - R_{2z}$$

$$R_{1z} = -1.92 \text{ lbf}$$

Assume all the axial load is on bearing 1:

$R_{2x} := 0 \text{ lbf}$ $\qquad$ $R_{1x} := W_{a1} = 165.288 \text{ lbf}$

Reaction Force Summary

$R_{1y} = -116.763 \text{ lbf}$	$R_{1z} = -1.92 \text{ lbf}$	$R_{1x} = 165.288 \text{ lbf}$	Axial load:
$R_{2y} = -296.0137 \text{ lbf}$	$R_{2z} = -178.7185 \text{ lbf}$	$R_{2x} = 0 \text{ lbf}$	$F_a := R_{1x} = 165.288 \text{ lbf}$

Singularity Function

On X-Y plane:

$$q_y(x) := R_{1y} \cdot s(x, 0, -1) + \left(\left(W_{r1y} + W_{t1y}\right) \cdot s\left(x, l_{g1}, -1\right) + M_{1z} \cdot s\left(x, l_{g1}, -2\right)\right) + R_{2y} \cdot s(x, l, -1)$$

$$v_y(x) := R_{1y} \cdot s(x, 0, 0) + \left(\left(W_{r1y} + W_{t1y}\right) \cdot s\left(x, l_{g1}, 0\right) + M_{1z} \cdot s\left(x, l_{g1}, -1\right)\right) + R_{2y} \cdot s(x, l, 0)$$

$$M_z(x) := R_{1y} \cdot s(x, 0, 1) + \left(\left(W_{r1y} + W_{t1y}\right) \cdot s\left(x, l_{g1}, 1\right) + M_{1z} \cdot s\left(x, l_{g1}, 0\right)\right) + R_{2y} \cdot s(x, l, 1)$$

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On X-Z plane:

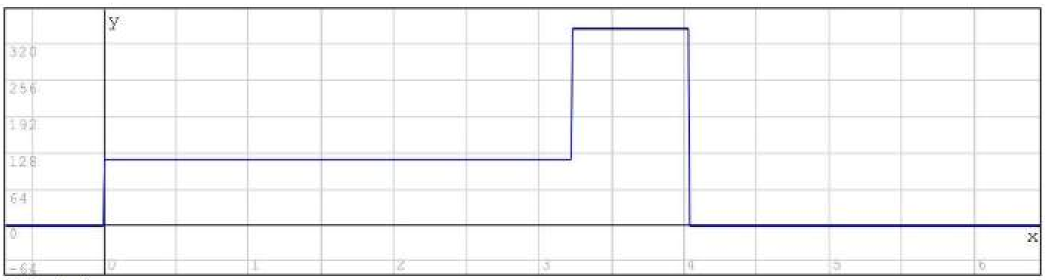
$$q_z(x) := R_{1z} \cdot s(x, 0, -1) + \left(W_{t1z} - W_{r1z}\right) \cdot s\left(x, l_{g1}, -1\right) - M_{1y} \cdot s\left(x, l_{g1}, -2\right) + R_{2z} \cdot s(x, l, -1)$$

$$v_z(x) := R_{1z} \cdot s(x, 0, 0) + \left(W_{t1z} - W_{r1z}\right) \cdot s\left(x, l_{g1}, 0\right) - M_{1y} \cdot s\left(x, l_{g1}, -1\right) + R_{2z} \cdot s(x, l, 0)$$

$$M_y(x) := R_{1z} \cdot s(x, 0, 1) + \left(W_{t1z} - W_{r1z}\right) \cdot s\left(x, l_{g1}, 1\right) - M_{1y} \cdot s\left(x, l_{g1}, 0\right) + R_{2z} \cdot s(x, l, 1)$$

Total V and M:

$$V(x) := \sqrt{v_y(x)^2 + v_z(x)^2} \qquad M(x) := \sqrt{M_y(x)^2 + M_z(x)^2}$$



$\frac{V(x\ in)}{lbf}$

$\frac{M(x\ in)}{lbf\ ft}$

The resultant moment at gear 1: $M\left(l_{g1}\right) = 23.2563\ lbf\ ft$

Torque on input shaft

Mean and alternating torque from before:

$T_{mean} = 20.65\ lbf\ ft$ $\qquad$ $T_{alt} = 20.65\ lbf\ ft$

Bending moment will be fully reversed loading

$M_{mean} := \frac{M\left(l_{g1}\right)+\left(-M\left(l_{g1}\right)\right)}{2} = 0 \text{ lbf ft}$ $\qquad M_{alt} := \frac{M\left(l_{g1}\right)-\left(-M\left(l_{g1}\right)\right)}{2} = 23.2563 \text{ lbf ft}$

Axial force will be constant throughout the input shaft, but fluctuate between 0 (when not in use) and operating shaft torque.

$F_{a'mean} := \frac{F_a + 0}{2} = 82.644 \text{ lbf}$ $\qquad F_{a'alt} := \frac{F_a - 0}{2} = 82.644 \text{ lbf}$

Minimum diameter of shaft where gear is located:

$d_{g1} = 1.1811 \text{ in}$ Previously determined minimum diameter

Assumed the diameters at the bearings:

$d_{b1} = 0.9843 \text{ in}$ $\qquad d_{b2} = 0.9843 \text{ in}$ Previously mentioned bearing diameters

Slope and Deflection Calculations

$I_1 := \pi \cdot \frac{{d_{b1}}^4}{64} = 0.0461 \text{ in}^4$ $\qquad I_2 := \pi \cdot \frac{{d_{g1}}^4}{64} = 0.0955 \text{ in}^4$ $\qquad I_3 := \pi \cdot \frac{{d_{b2}}^4}{64} = 0.0461 \text{ in}^4$

$J_1 := 2 \cdot I_1 = 0.0921 \text{ in}^4$ $\qquad J_2 := 2 \cdot I_2 = 0.1911 \text{ in}^4$ $\qquad J_3 := 2 \cdot I_3 = 0.0921 \text{ in}^4$

Solve for constants using boundary conditions: (No linear deflection at bearings)

Y-Direction

$y(0) = 0 \text{ in}$ $\qquad y(l) = 0 \text{ in}$

$c_{2y} := 0$

$$c_{1y} := \frac{\frac{1}{E \cdot I_1} \cdot \frac{R_{1y}}{6} \cdot s(l, 0, 3) + \frac{1}{E \cdot I_2} \cdot \left(\frac{\left(W_{r1y} + W_{t1y}\right)}{6} \cdot s\left(l, l_{g1}, 3\right) + \frac{M_{1z}}{2} \cdot s\left(l, l_{g1}, 2\right)\right)}{-l}$$

$c_{1y} = 0.0002$

Z-Direction

$z(0) = 0 \text{ in}$ $\qquad z(l) = 0 \text{ in}$

$c_{2z} := 0$

$$c_{1z} := \frac{\frac{1}{E \cdot I_1} \cdot \frac{R_{1z}}{6} \cdot s(l, 0, 3) + \frac{1}{E \cdot I_2} \cdot \left(\frac{\left(W_{t1z} - W_{r1z}\right)}{6} \cdot s\left(l, l_{g1}, 3\right) - \frac{M_{1y}}{2} \cdot s\left(l, l_{g1}, 2\right)\right)}{-l}$$

$$c_{1z} = 6.3533 \cdot 10^{-6}$$

$$\theta_z(x) := \frac{1}{E \cdot I_1} \cdot \frac{R_{1y}}{2} \cdot s(x, 0, 2) + \frac{1}{E \cdot I_2} \cdot \left(\frac{\left(W_{r1y} + W_{t1y} \right)}{2} \cdot s\left(x, l_{g1}, 2\right) + M_{1z} \cdot s\left(x, l_{g1}, 1\right) \right) + \\ + \frac{1}{E \cdot I_3} \cdot \frac{R_{2y}}{2} \cdot s(x, l, 2) + c_{1y}$$

$$y(x) := \frac{1}{E \cdot I_1} \cdot \frac{R_{1y}}{6} \cdot s(x, 0, 3) + \frac{1}{E \cdot I_2} \cdot \left(\frac{\left(W_{r1y} + W_{t1y} \right)}{6} \cdot s\left(x, l_{g1}, 3\right) + \frac{M_{1z}}{2} \cdot s\left(x, l_{g1}, 2\right) \right) + \\ + \frac{1}{E \cdot I_3} \cdot \frac{R_{2y}}{6} \cdot s(x, l, 3) + c_{1y} \cdot x + c_{2y}$$

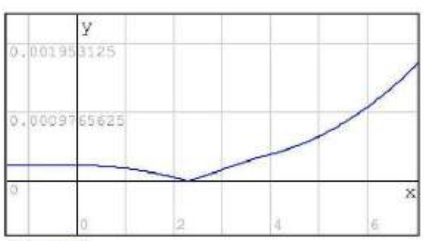
$$\theta_y(x) := \frac{1}{E \cdot I_1} \cdot \frac{R_{1z}}{2} \cdot s(x, 0, 2) + \frac{1}{E \cdot I_2} \cdot \left(\frac{\left(W_{t1z} - W_{r1z} \right)}{2} \cdot s\left(x, l_{g1}, 2\right) - M_{1y} \cdot s\left(x, l_{g1}, 1\right) \right) + \\ + \frac{1}{E \cdot I_3} \cdot \frac{R_{2z}}{2} \cdot s(x, l, 2) + c_{1z}$$

$$z(x) := \frac{1}{E \cdot I_1} \cdot \frac{R_{1z}}{6} \cdot s(x, 0, 3) + \frac{1}{E \cdot I_2} \cdot \left(\frac{\left(W_{t1z} - W_{r1z} \right)}{6} \cdot s\left(x, l_{g1}, 3\right) - \frac{M_{1y}}{2} \cdot s\left(x, l_{g1}, 2\right) \right) + \\ + \frac{1}{E \cdot I_3} \cdot \frac{R_{2z}}{6} \cdot s(x, l, 3) + c_{1z} \cdot x + c_{2z}$$

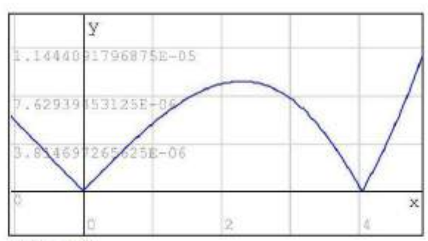
Total Angular Deflection and Deflection

$$\theta(x) := \sqrt{\theta_z(x)^2 + \theta_y(x)^2}$$

$$\delta(x) := \sqrt{y(x)^2 + z(x)^2}$$



$\theta(x \text{ in})$



$\delta(x \text{ in})$

Angular Deflection at Bearings — **Criteria** ≤ 0.004 rad

$\theta_{R1} := \theta(0) = 0.0002\ rad$ Angular deflection at beginning of shaft

$\theta_{R2} := \theta(l) = 0.0004\ rad$ Angular deflection at end of shaft

Both supports angular deflection does not exceed the allowable value. Therefore, criteria is met!

PASS

Angular Deflection at Gear — **Criteria** ≤ 0.0005 rad

$\theta_{G1} := \theta(l_{g1}) = 0.0002\ rad$ Angular deflection at gear 1

Gear angular deflection does not exceed the allowable value. Therefore, criteria is met!

PASS

Linear Deflection at Gear — **Criteria** ≤ 0.127 mm

$\delta_{G1} := \delta(l_{g1}) = 0.0063\ mm$ Deflection at gear 1

Both gears deflection does not exceed the allowable value. Therefore, criteria is met!

PASS

Angle of Twist — **Criteria** ≤ 3 deg/m

Only need to check where J is the lowest and there are the most SCFs

$$\phi_{twist} := \frac{T}{G \cdot J_1} = 1.046\ \frac{deg}{m}$$

Angle of twist does not exceed the allowable value. Therefore, criteria does pass.

PASS

Summary of Results

As a reminder, the evaluation criteria we need to satisfy are:

1. Shaft twist	≤ 3 deg/m
2. Linaer deflection at gears	≤ 0.127 mm
3. Angular deflection at gears	≤ 0.03 deg (0.0005 rad)
4. Angular deflection at bearings	≤ 0.004 rad

Minimum Diameters Summary

At Gear 1: $d_{g1} = 1.1811\ in$ (= 30mm)

At Bearing 1: $d_{b1} = 0.9843\ in$

At Bearing 2: $d_{b2} = 0.9843\ in$ (=25mm)

Evaluation Criteria

$\theta_{R1} = 0.0002$

$\theta_{R2} = 0.0004$

$\theta_{G1} = 0.0002$

$\delta_{G1} = 0.0063\ mm$

$\phi_{twist} = 1.046\ \frac{deg}{m}$

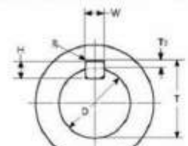
Given that all criteria are now satisfied and optimized, safety factors can be recalculated for new diameter. We need to find new notch radius and large diameter.

Key SCF

$r_{key} = 0.01\ \text{in}$ — Our initial assumption still satisfies new diameter

$K_f = 1.9 \qquad K_{fs} = 2.64$

Metric Key Keyway Dimensions Per ISO/R773 - Js9 Width Tolerance



Key & Keyway Dimensions - Millimeters										
Shaft Diameter "D"		Key Size Nominal		Keyway Width Hub "W"			Keyway Depth Hub "T2"		Keyway Radius "R"	
Over	Thru	Width "W"	Height "H"	Nominal	Min	Max	Min	Max	Min	Max
22	30	8	7	8	-.0180	+0180	3.3	3.5	0.16	0.25
30	38	10	8	10	-.0180	+0180	3.3	3.5	0.25	0.40
38	44	12	8	12	-0215	+.0215	3.3	3.5	0.25	0.40
44	50	14	9	14	-0215	+.0215	3.8	4.0	0.25	0.40

Figure 19. Standard Key and Keyway dimension (used to find rkey) [5]

Check the SCF of the keyway is greater than the SCF of the step

$D := 35\ \text{mm} = 1.378\ \text{in}$ — new chosen large diameter

$r_{shoulder} := 0.1 \cdot d_{g1} = 0.1181\ \text{in}$ — assume ratio of r/d = 0.1

$\frac{D}{d_{g1}} = 1.1667$

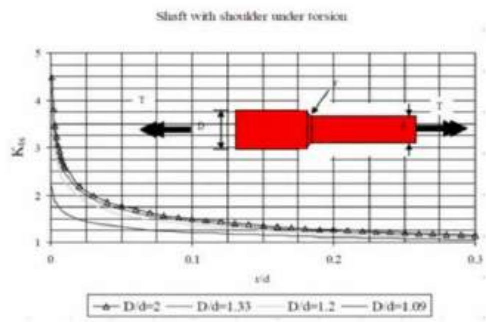


Figure 20. Step Shaft = Shaft with shoulder under torsion [4]

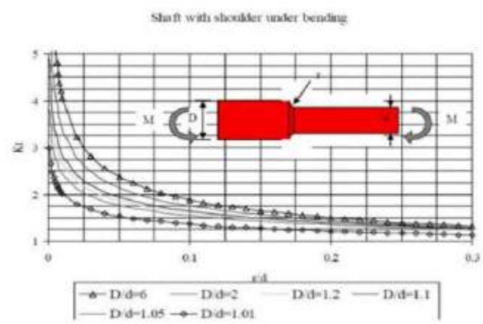


Figure 21. Step Shaft = Shaft with shoulder under bending [4]

$K_{ts} := 1.5 \qquad K_t := 1.6$

Step SCF:		Keyway SCF:
$K_{f2} := 1 + q \cdot (K_t - 1) = 1.54$	≤	$K_f = 1.9$
$K_{fs2} := 1 + q_s \cdot (K_{ts} - 1) = 1.4625$	≤	$K_{fs} = 2.64$

Therefore, keyway SCF is still the main failure point

As previously stated in the initial length analysis, we can assume Kf = Kfm and Kfs = Kfsm

$K_{fm} := K_f = 1.9 \qquad K_{fsm} := K_{fs} = 2.64$

Fatigue and Yield Safety Factor Calculations

Although there are many failure theroies available for design criteria considerations, in this case the yielding safety factor will be calculated along with a fatigue safety factor. The fatigue safety factor will be found according to AGMA, guidelines suggest use of DE Elliptic criteria for evaluation of shaft failure:

DE Elliptic Fatigue Safety Factor

$$n_f \equiv \frac{1}{\sqrt{\left(\frac{\sigma'_{alt}}{S_e}\right)^2 + \left(\frac{\sigma'_{mean}}{S_y}\right)^2}}$$

Yield Safety Factor

$$n_y \equiv \frac{1}{\frac{\sigma'_{alt}}{S_y} + \frac{\sigma'_{mean}}{S_y}}$$

$$d_{min} := d_{g1} = 1.1811 \text{ in}$$

Alternating and Mean Von Mises Stresses

$$\sigma'_{alt} := \left(\left(\frac{32 \cdot K_f \cdot M_{alt}}{\pi \cdot d_{min}^3} + \frac{4 \cdot K_f \cdot F_{a'alt}}{\pi \cdot d_{min}^2}\right)^2 + 3 \cdot \left(\frac{16 \cdot K_{fs} \cdot T_{alt}}{\pi \cdot d_{min}^3}\right)^2\right)^{\frac{1}{2}}$$

$$\sigma'_{mean} := \left(\left(\frac{32 \cdot K_{fm} \cdot M_{mean}}{\pi \cdot d_{min}^3} + \frac{4 \cdot K_{fm} \cdot F_{a'mean}}{\pi \cdot d_{min}^2}\right)^2 + 3 \cdot \left(\frac{16 \cdot K_{fsm} \cdot T_{mean}}{\pi \cdot d_{min}^3}\right)^2\right)^{\frac{1}{2}}$$

$$n_f := \frac{1}{\sqrt{\left(\frac{\sigma'_{alt}}{S_e}\right)^2 + \left(\frac{\sigma'_{mean}}{S_y}\right)^2}}$$

$$n_y := \frac{1}{\frac{\sigma'_{alt}}{S_y} + \frac{\sigma'_{mean}}{S_y}}$$

$$n_f = 8.7999$$

$$n_y = 17.0206$$

Conclusion

Calculations have been carried out, following a re-itteration method, a minimum diameter at gear 1 of 1.1811" (30 mm) and a minimum diameter around both bearings of 0.9843" (25 mm) has been specified which meets all required conditions and defined evaluation criteria. Additionally, the fatigue and yield safety factors were recalculated to be 8.80 and 17.02, respectively for this shaft diameter. It can be noted that the obtatined safety factors are both greater than the design safety factor of 1.872, indicating the shaft is safe for use.

References

[1] "AISI 4140 Steel, oil quenched, 25 mm round [845C quench, 540C tempered]," MatWeb. Accessed on: Nov 1, 2024. [Online]. Available: https://www.matweb.com/search/DataSheet.aspx?MatGUID=07d1795c3f034c97b52cccda78ae1409

[2] D. Romanyk, Class Lecture, Topic: "Singularity Functions." Faculty of Engineering, University of Alberta, Edmonton, Alberta, 2024

[3] D. Romanyk, Class Lecture, Topic: "Shaft Analysis." Faculty of Engineering, University of Alberta, Edmonton, Alberta, 2024

[4] D. Romanyk, Class Lecture, Topic: "Stress Concentration Factors." Faculty of Engineering, University of Alberta, Edmonton, Alberta, 2024

[5] "Keyway Chart," Hallite. Accessed on Nov 7, 2024. [Online]. Available: https://hallite.com/au/hallite-transeals/transmission-products/keyway-chart/

C13 *- Shaft Analysis - Idler Shaft*

MECE 360: Birdie Boys Design Calculations

MECE 360: Shaft Analysis, Idler Shaft

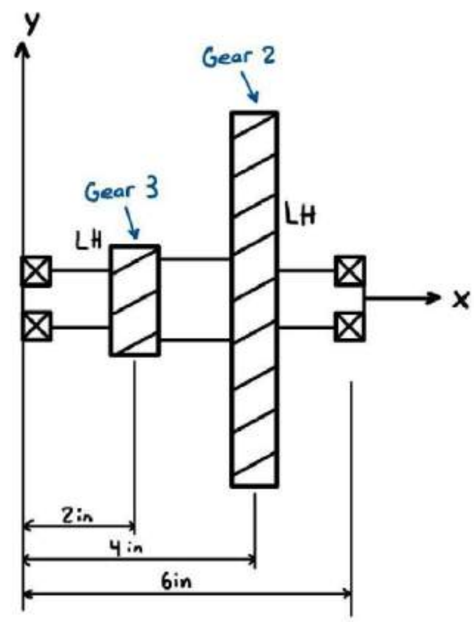
Annotation Legend
Blue: Notes
Yellow: Final Answers

Problem Statement

The intention of the following calculation is to aid the design of a golf cart transmission. The simply supported shaft made of AISI 4140 Steel below represents the second shaft in a three shaft transmission, the idler shaft. For this application, both the gears on the shaft are helical and the the bigger one on the right is being driving by the input shaft and the smaller one on the left is driving the output shaft. Both gears are straddle mounted and the shaft is supported on either side by bearings (axial load to be carried on the left). It can also be noted that the two mating gears are not located in the same plane, Figure 2.

The objective of this design calculation code is to solve for the minimum required diameters for the step shaft shown in Figure 1 below. Additional calculations are presened to ensure that the evaluation criterial (including shaft twist angle, linear deflection at the gears, and angular deflection between gears) is satisfied. As well, a final calculation for fatigue and yeild safety factors are included.

Problem Diagram



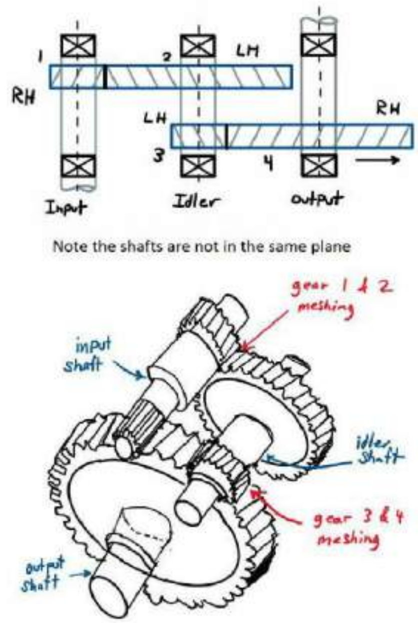


Figure 2. Full layout for all three shafts and gears.

Figure 1. Schematic of design problem: Simply supported shaft with two straddle mounted gears and bearings on each end (Note that both gears are helical and their mating gears are not in the same plane).

Evaluation Criteria

Based on the problem objective, the following criteria must be satisfied:

1. Strength Failure (DE Elliptic Criteria)

- First Cycle Yield: $\sigma'_{max} < \frac{S_y}{n}$ - Fatigue Failure: $\left(\frac{n\sigma'_a}{S_e}\right)^2 + \left(\frac{n\sigma'_m}{S_y}\right)^2 < 1$

2. Shaft twist ≤ 3 deg/m
3. Linaer deflection at gears ≤ 0.127 mm
4. Angular deflection at gears ≤ 0.03 deg (0.0005 rad)
5. Angular deflection at bearings ≤ 0.004 rad

Page 1

Assumptions

The following assumptions were make for the given problem:

1. Assume machined shaft
2. Assume combined loading, but bending is dominant
3. Assume the operating temperature is 40 °C (104 °F)
4. Assume there are no miscellaneous correction factors
5. Assume lifespan ≥ 10^6 cycles therefore, infinite life
6. Assume 95% reliaitliy
7. Assume constant torque, but will fluctuate between 0 (when not in use) and operating torque
8. Assume forces on shaft are fully reversed, due to shaft rotation causing a critical element to switch between compression and tension of equal magnitudes
9. Assume that bearings and gears act on shaft as point loads
10. Assume that axial load from helical gears goes to bearing with highest radial load
11. Assume that for the stress concentration factors, Kfm = Kf and Kfms = Kfs
12. Assume gears are mounted using parallel keyways

Sketches

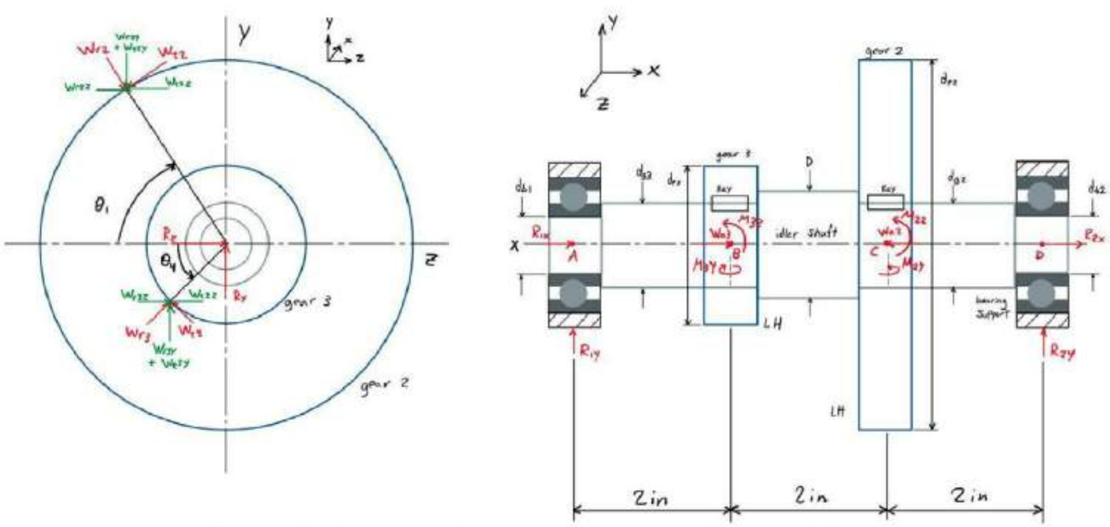


Figure 3. Free Body Diagram (FBD) of design problem with labelled reaction forces on shaft

FBD Variables

At Left Bearing (A)
R1x - Axial Reaction Force
R1y - Y-Direction Reaction Force
R1z - Z-Direction Reaction Force

At Right Bearing (D)
R2x - Axial Reaction Force
R2y - Y-Direction Reaction Force
R2z - Z-Direction Reaction Force

At Gear 3 (B)
Wa3 - Axial Force
Wr3 - Radial Force
Wt3 - Tangential Force
M3z - Z-Direction Moment
M3y - Y-Direction Moment

At Gear 2 (C)
Wa2 - Axial Force
Wr2 - Radial Force
Wt2 - Tangential Force
M2z - Z-Direction Moment
M2y - Y-Direction Moment

Misc. **θ1** and **θ4** - angle between idler shaft and the input and output shafts

Given for Motor:

$RPM_{in} := 3000\ rpm$ $T_{in} := 41.3\ lbf\ ft$ $T_{max} := 44.69\ lbf\ ft$ (max torque on startup)

Gear Analysis Summary: Pressure Angle $\phi := 20\ °$ Helix Angle $\psi := 21.5\ °$

Number of Teeth	Module	Pitch Diameter
$N_1 := 20\ teeth$	$m_1 := 3\ \frac{mm}{teeth}$	$d_{p1} := m_1 \cdot N_1 = 60\ mm$
$N_2 := 60\ teeth$	$m_2 := 3\ \frac{mm}{teeth}$	$d_{p2} := m_2 \cdot N_2 = 180\ mm$
$N_3 := 24\ teeth$	$m_3 := 3\ \frac{mm}{teeth}$	$d_{p3} := m_3 \cdot N_3 = 72\ mm$
$N_4 := 60\ teeth$	$m_4 := 3\ \frac{mm}{teeth}$	$d_{p4} := m_4 \cdot N_4 = 180\ mm$

Idler Shaft Inputs

Shaft Velocity $\omega := RPM_{in} \cdot \frac{N_1}{N_2} = 104.7198\ \frac{rad}{s}$

Shaft Torque $T := T_{in} \cdot \frac{N_2}{N_1} = 123.9\ lbf\ ft$

Max Torque $T_{max} := T_{max} \cdot \frac{N_2}{N_1} = 134.07\ lbf\ ft$

Ultimate Strength $S_{ut} := 155.9\ kpsi$

Yield Strength $S_y := 143\ kpsi$

Young's Modulus $E := 29700\ kpsi$

Shear Modulus $G := 11600\ kpsi$

Material properties taken from matweb.com [1]

Design Factor of Safety $n := 1.872$

Operation Temperature $T_{oper} := 104\ °F$

Shaft Length $l := 6\ in$

Length to Gear 3 $l_{g3} := 2\ in$

Length to Gear 2 $l_{g2} := 4\ in$

Angle Gear 2 to Gear 1 $\theta_1 := 45\ °$

Angle Gear 3 to Gear 4 $\theta_4 := 45\ °$

Find the forces exerted on the idler shaft (gears 2 and 3):

Transverse Presure Angle $\phi_t := atan\left(\frac{tan(\phi)}{cos(\psi)}\right) = 21.365\ °$

Loads on Gear 2

$$W_{t2} := \frac{T}{0.5 \cdot d_{p2}} = 419.608\ lbf$$

$$W_{r2} := W_{t2} \cdot tan(\phi_t) = 164.1465\ lbf$$

$$W_{a2} := W_{t2} \cdot tan(\psi) = 165.288\ lbf$$

$$W_2 := \frac{W_{t2}}{cos(\psi) \cdot cos(\phi)} = 479.9324\ lbf$$

Loads on Gear 3

$$W_{t3} := \frac{T}{0.5 \cdot d_{p3}} = 1049.02\ lbf$$

$$W_{r3} := W_{t3} \cdot tan(\phi_t) = 410.3663\ lbf$$

$$W_{a3} := W_{t3} \cdot tan(\psi) = 413.22\ lbf$$

$$W_3 := \frac{W_{t3}}{cos(\psi) \cdot cos(\phi)} = 1199.8309\ lbf$$

$M_{2z} := W_{a2} \cdot \frac{d_{p2}}{2} \cdot \sin\left(\theta_1\right) = 34.5107 \text{ lbf ft}$

$M_{3z} := W_{a3} \cdot \frac{d_{p3}}{2} \cdot \sin\left(\theta_4\right) = 34.5107 \text{ lbf ft}$

$M_{2y} := W_{a2} \cdot \frac{d_{p2}}{2} \cdot \cos\left(\theta_1\right) = 34.5107 \text{ lbf ft}$

$M_{3y} := W_{a3} \cdot \frac{d_{p3}}{2} \cdot \cos\left(\theta_4\right) = 34.5107 \text{ lbf ft}$

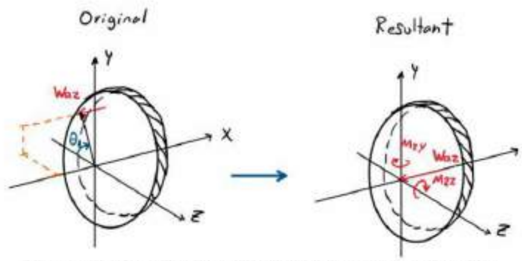


Figure 4. Visualization of two resulting moments, M2z and M2y, due to axial load.

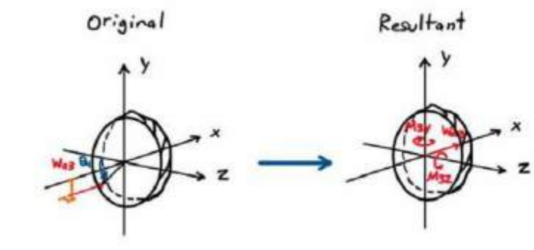


Figure 5. Visualization of two resulting moments, M3z and M3y, due to axial load.

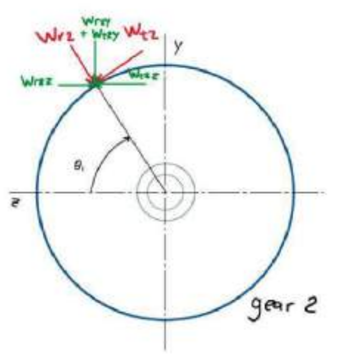


Figure 6. Snapshot of Figure 3.

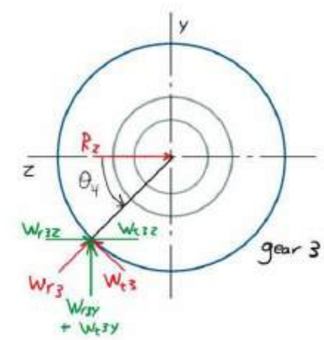


Figure 7. Snapshot of Figure 3.

$W_{r2z} := W_{r2} \cdot \cos\left(\theta_1\right) = 116.0691 \text{ lbf}$

$W_{r3z} := W_{r3} \cdot \cos\left(\theta_4\right) = 290.1728 \text{ lbf}$

$W_{r2y} := W_{r2} \cdot \sin\left(\theta_1\right) = 116.0691 \text{ lbf}$

$W_{r3y} := W_{r3} \cdot \sin\left(\theta_4\right) = 290.1728 \text{ lbf}$

$W_{t2z} := W_{t2} \cdot \cos\left(90\ ° - \theta_1\right) = 296.7077 \text{ lbf}$

$W_{t3z} := W_{t3} \cdot \cos\left(90\ ° - \theta_4\right) = 741.7692 \text{ lbf}$

$W_{t2y} := W_{t2} \cdot \sin\left(90\ ° - \theta_1\right) = 296.7077 \text{ lbf}$

$W_{t3y} := W_{t3} \cdot \sin\left(90\ ° - \theta_4\right) = 741.7692 \text{ lbf}$

Reaction Forces

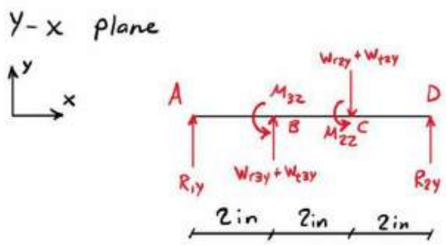


Figure 8. Reaction forces FBD in the y-x plane

$$\circlearrowleft + \sum M_A = 0 \Rightarrow 2(W_{r3y}+W_{t3y}) + M_{3z} - 4(W_{r2y}+W_{t2y}) + M_{2z} + 6R_{2y} = 0$$

$$R_{2y} := \frac{-M_{2z} - M_{3z} - l_{g3}\cdot\left(W_{r3y}+W_{t3y}\right) + l_{g2}\cdot\left(W_{r2y}+W_{t2y}\right)}{l}$$

$$R_{2y} = -206.839 \text{ lbf}$$

$$+\uparrow \sum F_y = 0 \Rightarrow R_{1y} + W_{r3y} + W_{t3y} - W_{r2y} - W_{t2y} + R_{2y} = 0$$

$$R_{1y} := -W_{r3y} - W_{t3y} + W_{r2y} + W_{t2y} - R_{2y}$$

$$R_{1y} = -412.3262 \text{ lbf}$$

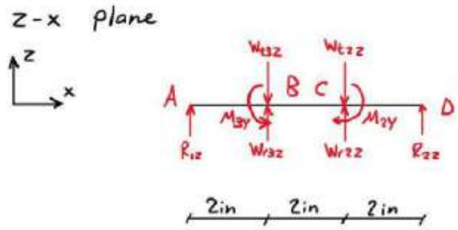


Figure 9. Reaction forces FBD in the z-x plane

$$\circlearrowleft + \sum M_A = 0 \Rightarrow 2(W_{r3z}-W_{t3z}) + M_{3y} + 4(W_{r2z}-W_{t2z}) - M_{2y} + 6R_{2z} = 0$$

$$R_{2z} := \frac{M_{2y} - M_{3y} - l_{g3}\cdot\left(W_{r3z}-W_{t3z}\right) - l_{g2}\cdot\left(W_{r2z}-W_{t2z}\right)}{l}$$

$$R_{2z} = 270.9578 \text{ lbf}$$

$$+\uparrow \sum F_y = 0 \Rightarrow R_{1z} + W_{r3z} - W_{t3z} + W_{r2z} - W_{t2z} + R_{2z} = 0$$

$$R_{1z} := W_{t3z} - W_{r3z} + W_{t2z} - W_{r2z} - R_{2z}$$

$$R_{1z} = 361.2771 \text{ lbf}$$

Assume all the axial load is on bearing 1:

$R_{2x} := 0 \text{ lbf}$ $\quad R_{1x} := W_{a3} - W_{a2} = 247.932 \text{ lbf}$

Reaction Force Summary

$R_{1y} = -412.3262 \text{ lbf}$ $\quad R_{1z} = 361.2771 \text{ lbf}$ $\quad R_{1x} = 247.932 \text{ lbf}$

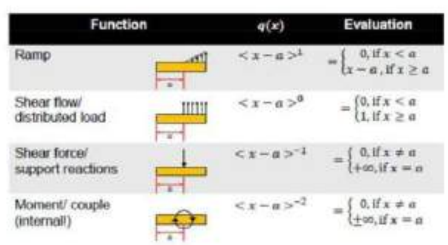
$R_{2y} = -206.839 \text{ lbf}$ $\quad R_{2z} = 270.9578 \text{ lbf}$ $\quad R_{2x} = 0 \text{ lbf}$

Axial load:

$F_a := R_{1x} = 247.932 \text{ lbf}$

Singularity Function

```
s(x, a, n) := if (((x - a) > 0) ∧ (n ≥ 0))
                  (x - a)^n
              else
                  if (((x - a) = 0) ∧ (n = 0))
                      1
                  else
                      0
```



Function		q(x)	Evaluation
Ramp		$<x-a>^1$	$=\begin{cases}0, \text{if } x<a \\ x-a, \text{if } x \geq a\end{cases}$
Shear flow/ distributed load		$<x-a>^0$	$=\begin{cases}0, \text{if } x<a \\ 1, \text{if } x \geq a\end{cases}$
Shear force/ support reactions		$<x-a>^{-1}$	$=\begin{cases}0, \text{if } x \neq a \\ +\infty, \text{if } x = a\end{cases}$
Moment/ couple (internal!)		$<x-a>^{-2}$	$=\begin{cases}0, \text{if } x \neq a \\ \pm\infty, \text{if } x = a\end{cases}$

Figure 10. Singularity Function (to solve for Vy(x) and Mz(x) [2]

Page 5

Singularity Function Equations for shear force and bending moment

Shear Force $$V_y(x) = \int q_y(x)dx$$

Bending Moment $$M_z(x) = \int V_y(x)dx$$

On X-Y plane:

$$q_y(x) := R_{1y} \cdot s(x, 0, -1) + \left(W_{r3y} + W_{t3y}\right) \cdot s\left(x, l_{g3}, -1\right) - M_{3z} \cdot s\left(x, l_{g3}, -2\right) + \\ + \left(-\left(W_{r2y} + W_{t2y}\right)\right) \cdot s\left(x, l_{g2}, -1\right) - M_{2z} \cdot s\left(x, l_{g2}, -2\right) + R_{2y} \cdot s(x, l, -1)$$

$$v_y(x) := R_{1y} \cdot s(x, 0, 0) + \left(W_{r3y} + W_{t3y}\right) \cdot s\left(x, l_{g3}, 0\right) - M_{3z} \cdot s\left(x, l_{g3}, -1\right) + \\ + \left(-\left(W_{r2y} + W_{t2y}\right)\right) \cdot s\left(x, l_{g2}, 0\right) - M_{2z} \cdot s\left(x, l_{g2}, -1\right) + R_{2y} \cdot s(x, l, 0)$$

$$M_z(x) := R_{1y} \cdot s(x, 0, 1) + \left(W_{r3y} + W_{t3y}\right) \cdot s\left(x, l_{g3}, 1\right) - M_{3z} \cdot s\left(x, l_{g3}, 0\right) + \\ + \left(-\left(W_{r2y} + W_{t2y}\right)\right) \cdot s\left(x, l_{g2}, 1\right) - M_{2z} \cdot s\left(x, l_{g2}, 0\right) + R_{2y} \cdot s(x, l, 1)$$

On X-Z plane:

$$q_z(x) := R_{1z} \cdot s(x, 0, -1) + \left(W_{r3z} - W_{t3z}\right) \cdot s\left(x, l_{g3}, -1\right) - M_{3y} \cdot s\left(x, l_{g3}, -2\right) + \left(W_{r2z} - W_{t2z}\right) \cdot s\left(x, l_{g2}, -1\right) + \\ + M_{2y} \cdot s\left(x, l_{g2}, -2\right) + R_{2z} \cdot s(x, l, -1)$$

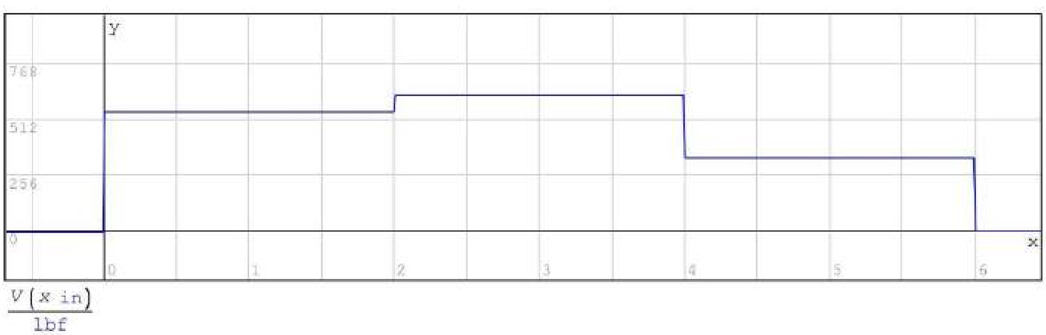
$$v_z(x) := R_{1z} \cdot s(x, 0, 0) + \left(W_{r3z} - W_{t3z}\right) \cdot s\left(x, l_{g3}, 0\right) - M_{3y} \cdot s\left(x, l_{g3}, -1\right) + \left(W_{r2z} - W_{t2z}\right) \cdot s\left(x, l_{g2}, 0\right) + \\ + M_{2y} \cdot s\left(x, l_{g2}, -1\right) + R_{2z} \cdot s(x, l, 0)$$

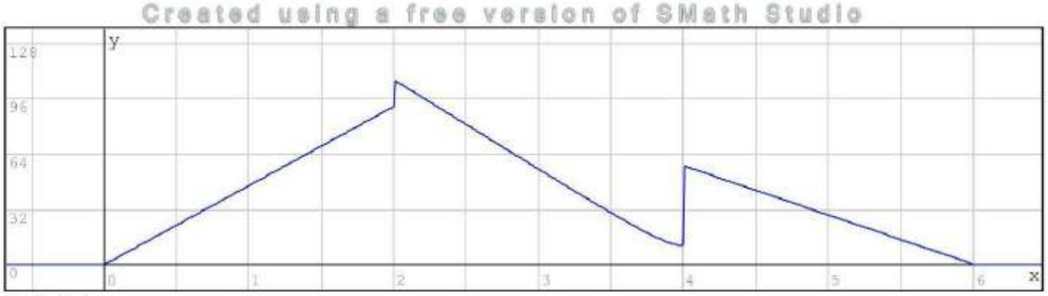
$$M_y(x) := R_{1z} \cdot s(x, 0, 1) + \left(W_{r3z} - W_{t3z}\right) \cdot s\left(x, l_{g3}, 1\right) - M_{3y} \cdot s\left(x, l_{g3}, 0\right) + \left(W_{r2z} - W_{t2z}\right) \cdot s\left(x, l_{g2}, 1\right) + \\ + M_{2y} \cdot s\left(x, l_{g2}, 0\right) + R_{2z} \cdot s(x, l, 1)$$

Total V and M:

$$V(x) := \sqrt{v_y(x)^2 + v_z(x)^2}$$

$$M(x) := \sqrt{M_y(x)^2 + M_z(x)^2}$$





$\frac{M(x\ in)}{lbf\ ft}$

The resultant moment at both gears

$M_{g3} := M(l_{g3}) = 106.3832\ lbf\ ft$ $\quad M_{g2} := M(l_{g2}) = 56.8137\ lbf\ ft$

Consider Loads on Startup (Max Torque)

Loads on Gear 2

$W_{t2} := \frac{T_{max}}{0.5 \cdot d_{p2}} = 454.0504\ lbf$

$W_{r2} := W_{t2} \cdot \tan(\phi_t) = 177.6201\ lbf$

$W_{a2} := W_{t2} \cdot \tan(\psi) = 178.8552\ lbf$

$M_{2z} := W_{a2} \cdot \frac{d_{p2}}{2} \cdot \sin(\theta_1) = 448.1211\ lbf\ in$

$M_{2y} := W_{a2} \cdot \frac{d_{p2}}{2} \cdot \cos(\theta_1) = 448.1211\ lbf\ in$

$W_{r2z} := W_{r2} \cdot \cos(\theta_1) = 125.5964\ lbf$

$W_{r2y} := W_{r2} \cdot \sin(\theta_1) = 125.5964\ lbf$

$W_{t2z} := W_{t2} \cdot \cos(90° - \theta_1) = 321.0621\ lbf$

$W_{t2y} := W_{t2} \cdot \sin(90° - \theta_1) = 321.0621\ lbf$

Loads on Gear 3

$W_{t3} := \frac{T_{max}}{0.5 \cdot d_{p3}} = 1135.126\ lbf$

$W_{r3} := W_{t3} \cdot \tan(\phi_t) = 444.0502\ lbf$

$W_{a3} := W_{t3} \cdot \tan(\psi) = 447.138\ lbf$

$M_{3z} := W_{a3} \cdot \frac{d_{p3}}{2} \cdot \sin(\theta_4) = 448.1211\ lbf\ in$

$M_{3y} := W_{a3} \cdot \frac{d_{p3}}{2} \cdot \cos(\theta_4) = 448.1211\ lbf\ in$

$W_{r3z} := W_{r3} \cdot \cos(\theta_4) = 313.9909\ lbf$

$W_{r3y} := W_{r3} \cdot \sin(\theta_4) = 313.9909\ lbf$

$W_{t3z} := W_{t3} \cdot \cos(90° - \theta_4) = 802.6553\ lbf$

$W_{t3y} := W_{t3} \cdot \sin(90° - \theta_4) = 802.6553\ lbf$

Reaction Forces

y-x plane:

$$R_{2y} := \frac{-M_{2z} - M_{3z} - l_{g3} \cdot (W_{r3y} + W_{t3y}) + l_{g2} \cdot (W_{r2y} + W_{t2y})}{l} = -223.8168\ lbf$$

$R_{1y} := -W_{r3y} - W_{t3y} + W_{r2y} + W_{t2y} - R_{2y} = -446.1709\ \text{lbf}$

z-x plane:

$$R_{2z} := \frac{M_{2y} - M_{3y} - l_{g3} \cdot \left(W_{r3z} - W_{t3z}\right) - l_{g2} \cdot \left(W_{r2z} - W_{t2z}\right)}{l} = 293.1986\ \text{lbf}$$

$R_{1z} := W_{t3z} - W_{r3z} + W_{t2z} - W_{r2z} - R_{2z} = 390.9315\ \text{lbf}$

axial load on bearing 1:

$R_{2x} := 0\ \text{lbf}$ $\quad$ $R_{1x} := W_{a3} - W_{a2} = 268.2828\ \text{lbf}$

max axial force: $F_{a'max} := R_{1x} = 268.2828\ \text{lbf}$

Singularity Function

On X-Y plane:

$$q_y(x) := R_{1y} \cdot s(x, 0, -1) + \left(W_{r3y} + W_{t3y}\right) \cdot s\left(x, l_{g3}, -1\right) - M_{3z} \cdot s\left(x, l_{g3}, -2\right) + \\ + \left(-\left(W_{r2y} + W_{t2y}\right)\right) \cdot s\left(x, l_{g2}, -1\right) - M_{2z} \cdot s\left(x, l_{g2}, -2\right) + R_{2y} \cdot s(x, l, -1)$$

$$v_y(x) := R_{1y} \cdot s(x, 0, 0) + \left(W_{r3y} + W_{t3y}\right) \cdot s\left(x, l_{g3}, 0\right) - M_{3z} \cdot s\left(x, l_{g3}, -1\right) + \\ + \left(-\left(W_{r2y} + W_{t2y}\right)\right) \cdot s\left(x, l_{g2}, 0\right) - M_{2z} \cdot s\left(x, l_{g2}, -1\right) + R_{2y} \cdot s(x, l, 0)$$

$$M_z(x) := R_{1y} \cdot s(x, 0, 1) + \left(W_{r3y} + W_{t3y}\right) \cdot s\left(x, l_{g3}, 1\right) - M_{3z} \cdot s\left(x, l_{g3}, 0\right) + \\ + \left(-\left(W_{r2y} + W_{t2y}\right)\right) \cdot s\left(x, l_{g2}, 1\right) - M_{2z} \cdot s\left(x, l_{g2}, 0\right) + R_{2y} \cdot s(x, l, 1)$$

On X-Z plane:

$$q_z(x) := R_{1z} \cdot s(x, 0, -1) + \left(W_{r3z} - W_{t3z}\right) \cdot s\left(x, l_{g3}, -1\right) - M_{3y} \cdot s\left(x, l_{g3}, -2\right) + \left(W_{r2z} - W_{t2z}\right) \cdot s\left(x, l_{g2}, -1\right) + \\ + M_{2y} \cdot s\left(x, l_{g2}, -2\right) + R_{2z} \cdot s(x, l, -1)$$

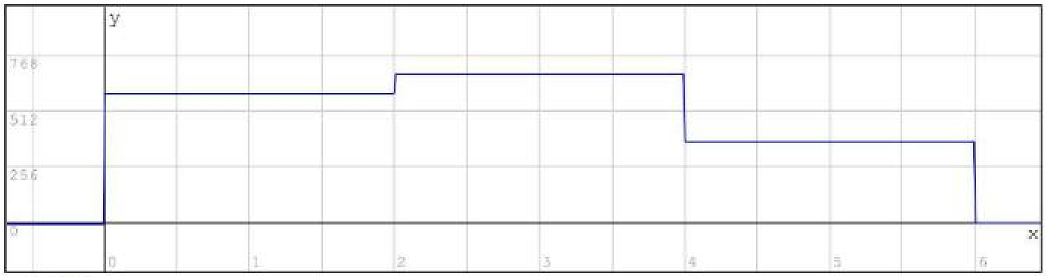
$$v_z(x) := R_{1z} \cdot s(x, 0, 0) + \left(W_{r3z} - W_{t3z}\right) \cdot s\left(x, l_{g3}, 0\right) - M_{3y} \cdot s\left(x, l_{g3}, -1\right) + \left(W_{r2z} - W_{t2z}\right) \cdot s\left(x, l_{g2}, 0\right) + \\ + M_{2y} \cdot s\left(x, l_{g2}, -1\right) + R_{2z} \cdot s(x, l, 0)$$

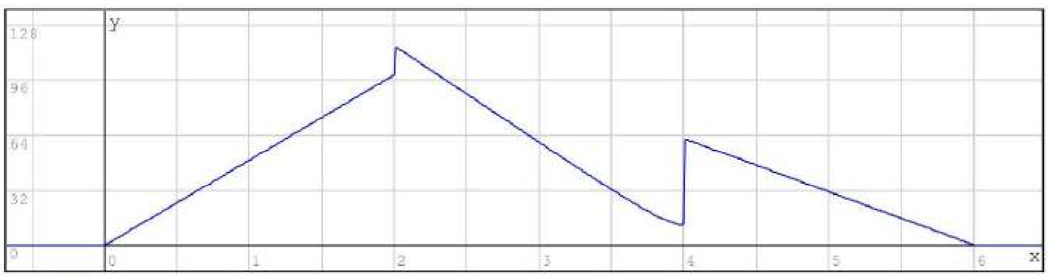
$$M_y(x) := R_{1z} \cdot s(x, 0, 1) + \left(W_{r3z} - W_{t3z}\right) \cdot s\left(x, l_{g3}, 1\right) - M_{3y} \cdot s\left(x, l_{g3}, 0\right) + \left(W_{r2z} - W_{t2z}\right) \cdot s\left(x, l_{g2}, 1\right) + \\ + M_{2y} \cdot s\left(x, l_{g2}, 0\right) + R_{2z} \cdot s(x, l, 1)$$

Total V and M:

$$V(x) := \sqrt{v_y(x)^2 + v_z(x)^2}$$

$$M(x) := \sqrt{M_y(x)^2 + M_z(x)^2}$$





The resultant moment at both gears

$M_{g3'max} := M\left(l_{g3}\right) = 115.1154\ \text{lbf ft}$ $\quad M_{g2'max} := M\left(l_{g2}\right) = 61.477\ \text{lbf ft}$ Max moments for startup torque

Sizing the Shaft Based on Fatigue Strength

Gear 3

Determine minimum diameter at gear 3 assuming keyway has greatest SCF.

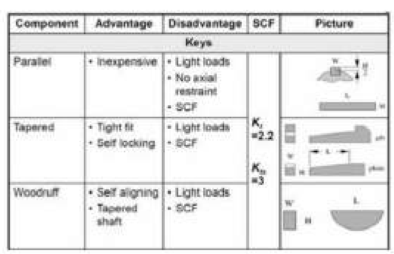
$K_t := 2.2$

$K_{ts} := 3$

SCF for key

$r_{key} := 0.01\ \text{in}$

$r_{shoulder} := 0.1\ \text{in}$



Component	Advantage	Disadvantage	SCF	Picture
Keys				
Parallel	• Inexpensive	• Light loads • No axial restraint • SCF	K_t =2.2 K_{ts} =3	
Tapered	• Tight fit • Self locking	• Light loads • SCF		
Woodruff	• Self aligning • Tapered shaft	• Light loads • SCF		

Figure 11. Keys SCF [3]

Sut = 155.9 kpsi

$q := 0.75 \qquad q_s := 0.82$

$K_f := 1 + q \cdot (K_t - 1) = 1.9$

$K_{fs} := 1 + q_s \cdot (K_{ts} - 1) = 2.64$

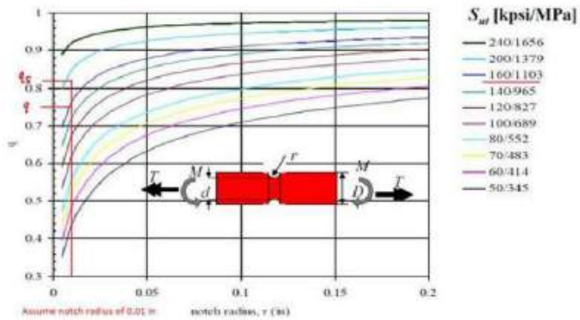


Figure 12. Notch Sensitivity (to solve for q and qs) [4]

Torque on idler shaft

The torque will be constant throughout the idler shaft, but fluctuate between 0 (when not in use) and operating shaft torque.

$T_{mean} := \frac{T + 0}{2} = 61.95 \text{ lbf ft} \qquad T_{alt} := \frac{T - 0}{2} = 61.95 \text{ lbf ft}$

Bending moment will be fully reversed loading

$M_{mean} := \frac{M_{g3} + (-M_{g3})}{2} = 0 \text{ lbf ft} \qquad M_{alt} := \frac{M_{g3} - (-M_{g3})}{2} = 106.3832 \text{ lbf ft}$

Axial force will be constant throughout the input shaft, but fluctuate between 0 (when not in use) and operating shaft torque.

$F_{a'mean} := \frac{F_a + 0}{2} = 123.966 \text{ lbf} \qquad F_{a'alt} := \frac{F_a - 0}{2} = 123.966 \text{ lbf}$

Endurance Strength

Uncorrected endurance strength

```
S'e := if Sut ≤ 1463 MPa
         0.504 · Sut
       else
         737 MPa
```

$S'_e = 78573.6 \text{ psi}$

Surface Condition Factor, ka:

Assuming the shaft is machined

$a := 2.7 \qquad b := -0.265$

$k_a := a \cdot \left(S_{ut} \cdot \frac{1}{\text{kpsi}}\right)^b = 0.7084$

	MPa		kpsi	
Surface finish	**a**	**b**	**a**	**B**
Ground (standard unless otherwise indicated)	*1.58*	*-0.085*	*1.34*	*-0.085*
Machined or Cold drawn	4.51	-0.265	2.7	-0.265
Hot-rolled	57.7	-0.718	14.4	-0.718
As-forged	272	-0.995	39.9	-0.995

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Size Correction Factor, kb:

Assuming bending and torsion with rotation

Assume dg3 is between 0.11 and 2 in:

kb = 0.879d^(-0.107)

$$k_b = \begin{cases} 0.879d^{-0.107} & 0.11 \le d \le 2\,\text{in} \\ 0.91d^{-0.157} & 2 < d \le 10\,\text{in} \\ 1.24d^{-0.107} & 2.79 \le d \le 51\,\text{mm} \\ 1.51d^{-0.157} & 51 < d \le 254\,\text{mm} \end{cases}$$

We must come back to check this

Load Correction Factor, kc:

Combined loading, bending is considered dominant

$k_c := 1$

$$k_c = \begin{cases} 1 & \text{Bending} \\ 0.85 & \text{Axial} \\ 0.59 & \text{Pure torsion} \end{cases}$$

Temperature Correction Factor, kd:

Temperature Correction for operating temperature of 104 F

$$k_d := 0.975 + 0.00032 \cdot \left(\frac{T_{oper}}{°F}\right) - 0.115 \cdot 10^{-5} \cdot \left(\frac{T_{oper}}{°F}\right)^2 + 0.104 \cdot 10^{-8} \cdot \left(\frac{T_{oper}}{°F}\right)^3 - 0.595 \cdot 10^{-12} \cdot \left(\frac{T_{oper}}{°F}\right)^4 = 0.9969$$

Reliability Factor, ke:

Assume reliability to be 95%

$k_e := 0.868$

Reliability	Ke
50	1
90	0.897
95	0.868
99	0.814
99.9	0.753
99.99	0.702
99.999	0.659
99.9999	0.62

Miscellaneous Correction Factor, kf:

Assume no miscellaneous correction

$k_f := 1$

Result of all Correction Factors without kb:

$k := k_a \cdot k_c \cdot k_d \cdot k_e \cdot k_f = 0.613$

Minimum Diameter Iteration Calculation

Iterate using MATLAB code to find minimum shaft diameter, d. The MATLAB function takes an initial diameter and checks if it passes DE Elliptic Failure Criteria, if not increase diameter and try again, if yes then minimum diameter has been found.

DE Elliptic Failure Criteria: $\left(\frac{n\sigma'_a}{S_e}\right)^2 + \left(\frac{n\sigma'_m}{S_y}\right)^2 < 1$

Von Mises Stresses accounting for bending stress, axial stress, and torsional stress:

$$\sigma'_{alt} = \left(\left(\frac{32 \cdot K_f \cdot M_{alt}}{\pi \cdot d^3} + \frac{4 \cdot K_f \cdot F_{a'alt}}{\pi \cdot d^2}\right)^2 + 3 \cdot \left(\frac{16 \cdot K_{fs} \cdot T_{alt}}{\pi \cdot d^3}\right)^2\right)^{\frac{1}{2}}$$

$$\sigma'_{mean} = \left(\left(\frac{32 \cdot K_{fm} \cdot M_{mean}}{\pi \cdot d^3} + \frac{4 \cdot K_{fm} \cdot F_{a'mean}}{\pi \cdot d^2}\right)^2 + 3 \cdot \left(\frac{16 \cdot K_{fsm} \cdot T_{mean}}{\pi \cdot d^3}\right)^2\right)^{\frac{1}{2}}$$

Corrected Endurance Strength:

$S_e \equiv k_a \cdot k_b \cdot k_c \cdot k_d \cdot k_e \cdot k_f \cdot S'_e$ kb is a function of diameter

Assume $K_f \cdot \sigma'_{max} < S_y$ so $K_{fm} := K_f$ and $K_{fsm} := K_{fs}$

Plug in an intial diameter guess of d = 0.8210 in (minimum diameter for input shaft which experiences less loads) into the MATLAB function minShaftDiameter.m, as well as other required input values.
The MATLAB function iteratively tests for a minimum shaft diameter that meets DE Elliptic Failure Criteria using the above formulas.

Computed minimum shaft diameter at gear 3: $d_{g3} := 1.1059\ \text{in}$

Calculate final endurance strength using minimum diameter:

$$k_b := 0.897 \cdot \left(\frac{d_{g3}}{\text{in}}\right)^{-0.107} = 0.8874 \qquad S_e := k_a \cdot k_b \cdot k_c \cdot k_d \cdot k_e \cdot k_f \cdot S'_e = 42.7406\ \text{kpsi}$$

Check Yield Failure Criteria

DE Failure criteria states: $\sigma' < \frac{S_y}{n}$

$$\sigma' := \left(\left(\frac{32 \cdot K_f \cdot M_{g3'max}}{\pi \cdot d_{g3}^{3}} + \frac{4 \cdot K_f \cdot F_{a'max}}{\pi \cdot d_{g3}^{2}}\right)^2 + 3 \cdot \left(\frac{16 \cdot K_{fs} \cdot T_{max}}{\pi \cdot d_{g3}^{3}}\right)^2\right)^{\frac{1}{2}} = 34.3412\ \text{kpsi}$$

$\sigma' = 34.3412\ \text{kpsi}$ ≤ $\frac{S_y}{n} = 76.3889\ \text{kpsi}$ Therefore, first cycle yield passes.

Check the assumptions

$$k_b = \begin{cases} 0.879d^{-0.107} & 0.11 \le d \le 2\ \text{in} \\ 0.91d^{-0.157} & 2 < d \le 10\ \text{in} \\ 1.24d^{-0.107} & 2.79 \le d \le 51\ \text{mm} \\ 1.51d^{-0.157} & 51 < d \le 254\ \text{mm} \end{cases}$$

0.11 ≤ dg3 = 1.1059 ≤ 2 in

Assumption is valid

Moment of Inertia: $I := \pi \cdot \frac{d_{g3}^{4}}{64} = 0.0734\ \text{in}^4$

Polar Moment of Inertia: $J := 2 \cdot I = 0.1468\ \text{in}^4$

Cross-Sectional Area: $A := \frac{1}{4} \cdot \pi \cdot d_{g3}^{2} = 0.9606\ \text{in}^2$

$$\sigma'_{max} := \sqrt{\left(\frac{M_{g3} \cdot \frac{d_{g3}}{2}}{I} + \frac{W_{a3}}{A}\right)^2 + 3 \cdot \left(\frac{T \cdot \frac{d_{g3}}{2}}{J}\right)^2} = 13.9867 \text{ kpsi}$$

$K_f \cdot \sigma'_{max} = 26.5748 \text{ kpsi} \quad \leq \quad S_y = 143 \text{ kpsi}$ Therefore, Kfm = Kf was a valid assumption

Check the SCF of the keyway is greater than the SCF of the step

Assume large diameter of the center part is

$D := 1.5 \text{ in}$ $\quad \frac{D}{d_{g3}} = 1.3564$ $\quad \frac{r_{shoulder}}{d_{g3}} = 0.0904$

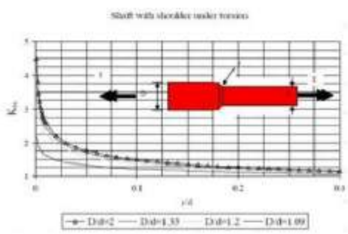


Figure 13. Step Shaft = Shaft with shoulder under torsion [4]

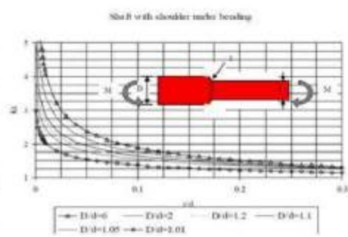


Figure 14. Step Shaft = Shaft with shoulder under bending [4]

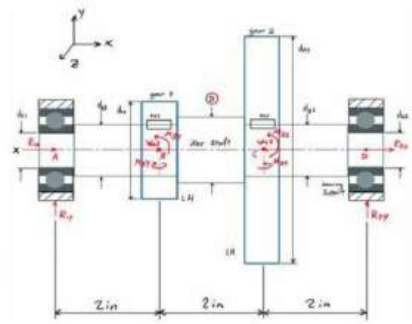


Figure 15. FBD to show "D"

$K_{ts} := 1.5$ $\quad K_t := 1.75$

$q := 0.9$ $\quad q_s := 0.925$

Sut = 155.9 kpsi

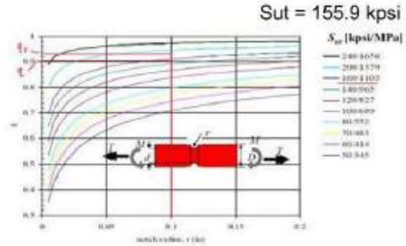


Figure 16. Notch Sensitivity (to solve for q and qs) [4]

Step SCF: Keyway SCF:

$K_{f2} := 1 + q \cdot (K_t - 1) = 1.675 \quad \leq \quad K_f = 1.9$

$K_{fs2} := 1 + q_s \cdot (K_{ts} - 1) = 1.4625 \quad \leq \quad K_{fs} = 2.64$

Therefore assuming the SCF of the keyway is greater than the SCF of the shoulder is valid

Gear 2

assume keyway is the main failure point

$M_{mean} := \frac{M_{g2} + (-M_{g2})}{2} = 0 \text{ lbf ft}$ $\qquad M_{alt} := \frac{M_{g2} - (-M_{g2})}{2} = 56.8137 \text{ lbf ft}$

Minimum Diameter Iteration Calculation

Iterate using MATLAB code to find minimum shaft diameter, d. The MATLAB function takes an initial diameter and checks if it passes DE Elliptic Failure Criteria, if not increase diameter and try again, if yes then minimum diameter has been found.

Plug in an intial diameter guess of d = 0.8210 in (minimum diameter for input shaft which experiences less loads) into the MATLAB function minShaftDiameter.m, as well as other required input values.
The MATLAB function iteratively tests for a minimum shaft diameter that meets DE Elliptic Failure Criteria using the above formulas.

Computed minimum shaft diameter at gear 2: $d_{g2} := 0.9923\ \text{in}$

Calculate final endurance strength using minimum diameter:

$$k_b := 0.897 \cdot \left(\frac{d_{g2}}{\text{in}} \right)^{-0.107} = 0.8977 \qquad S_e := k_a \cdot k_b \cdot k_c \cdot k_d \cdot k_e \cdot k_f \cdot S'_e = 43.2392\ \text{kpsi}$$

Check Yield Failure Criteria

DE Failure criteria states: $\sigma' < \frac{S_y}{n}$

$$\sigma' := \left(\left(\frac{32 \cdot K_f \cdot M_{g2'max}}{\pi \cdot d_{g2}^{3}} + \frac{4 \cdot K_f \cdot F_{a'max}}{\pi \cdot d_{g2}^{2}} \right)^2 + 3 \cdot \left(\frac{16 \cdot K_{fs} \cdot T_{max}}{\pi \cdot d_{g2}^{3}} \right)^2 \right)^{\frac{1}{2}} = 41.275\ \text{kpsi}$$

$\sigma' = 41.275\ \text{kpsi} \quad \le \quad \frac{S_y}{n} = 76.3889\ \text{kpsi}$ Therefore, first cycle yield passes.

Check the assumptions

$$k_b = \begin{cases} 0.879 d^{-0.107} & 0.11 \le d \le 2\ \text{in} \\ 0.91 d^{-0.157} & 2 < d \le 10\ \text{in} \\ 1.24 d^{-0.107} & 2.79 \le d \le 51\ \text{mm} \\ 1.51 d^{-0.157} & 51 < d \le 254\ \text{mm} \end{cases}$$

0.11 ≤ dg2 = 0.9923 ≤ 2 in

Assumption is valid

Moment of Inertia: $I := \pi \cdot \frac{d_{g2}^{4}}{32} = 0.0952\ \text{in}^4$

Polar Moment of Inertia: $J := 2 \cdot I = 0.1904\ \text{in}^4$

Cross-Sectional Area: $A := \frac{1}{4} \cdot \pi \cdot d_{g2}^{2} = 0.7733\ \text{in}^2$

$$\sigma'_{max} := \sqrt{ \left(\frac{M_{g2} \cdot \frac{d_{g2}}{2}}{I} + \frac{W_{a2}}{A} \right)^2 + 3 \cdot \left(\frac{T \cdot \frac{d_{g2}}{2}}{J} \right)^2 } = 7.7053\ \text{kpsi}$$

$K_f \cdot \sigma'_{max} = 14.64\ \text{kpsi} \quad \le \quad S_y = 143\ \text{kpsi}$ Therefore, Kfm = Kf was a valid assumption

Check the SCF of the keyway is greater than the SCF of the step

$\frac{D}{d_{g2}} = 1.5116$ $\quad \frac{r_{shoulder}}{d_{g2}} = 0.1008$

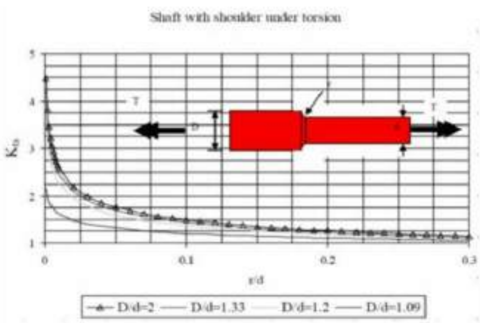


Figure 17. Step Shaft = Shaft with shoulder under torsion [4]

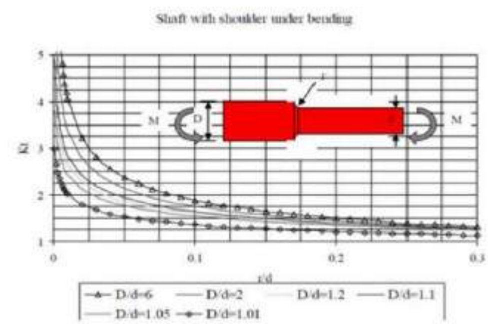


Figure 18. Step Shaft = Shaft with shoulder under bending [4]

$K_{ts} := 1.5$ $\quad K_t := 1.7$

Step SCF: Keyway SCF:

$K_{f2} := 1 + q \cdot (K_t - 1) = 1.63 \quad \leq \quad K_f = 1.9$

$K_{fs2} := 1 + q_s \cdot (K_{ts} - 1) = 1.4625 \quad \leq \quad K_{fs} = 2.64$

Therefore assuming the SCF of the keyway is greater than the SCF of the shoulder is valid

Minimum diameter of shaft where gears are located:

$d_{g3} = 1.1059\ in$ $\quad d_{g2} = 0.9923\ in$

$d_{g3} := 32\ mm = 1.2598\ in$ $\quad d_{g2} := 32\ mm = 1.2598\ in$ Size up to possible gear bore diameter

Assume the diameters at the bearings

$d_{b1} := 30\ mm = 1.1811\ in$ $\quad d_{b2} := 30\ mm = 1.1811\ in$ Chosen bearing bore diameter

Slope and Deflection Calculations

$I_1 := \pi \cdot \frac{d_{b1}^4}{64} = 0.0955\ in^4$ $\quad I_2 := \pi \cdot \frac{d_{g3}^4}{64} = 0.1237\ in^4$ $\quad I_3 := \pi \cdot \frac{d_{g2}^4}{64} = 0.1237\ in^4$ $\quad I_4 := \pi \cdot \frac{d_{b2}^4}{64} = 0.0955\ in^4$

$J_1 := 2 \cdot I_1 = 0.1911\ in^4$ $\quad J_2 := 2 \cdot I_2 = 0.2473\ in^4$ $\quad J_3 := 2 \cdot I_3 = 0.2473\ in^4$ $\quad J_4 := 2 \cdot I_4 = 0.1911\ in^4$

Singularity Function Equations for Angular Deflection and Deflection of Shaft

$$\theta_z(x) = \int \frac{M_z(x)}{EI} dx + C_1 \quad \text{Slope}$$

$$y(x) = \int \left[\int \frac{M_z(x)}{EI} dx + C_1 \right] dx + C_2 \quad \text{Deflection}$$

$$\theta_z(x) := \frac{1}{E \cdot I_1} \cdot \frac{R_{1y}}{2} \cdot s(x, 0, 2) + \frac{1}{E \cdot I_2} \cdot \left(\frac{\left(W_{r3y} + W_{t3y}\right)}{2} \cdot s\left(x, l_{g3}, 2\right) - M_{3z} \cdot s\left(x, l_{g3}, 1\right) \right) + \\ + \frac{1}{E \cdot I_3} \cdot \left(- \frac{\left(W_{r2y} + W_{t2y}\right)}{2} \cdot s\left(x, l_{g2}, 2\right) - M_{2z} \cdot s\left(x, l_{g2}, 1\right) \right) + \frac{1}{E \cdot I_4} \cdot \frac{R_{2y}}{2} \cdot s(x, l, 2) + c_{1y}$$

$$y(x) := \frac{1}{E \cdot I_1} \cdot \frac{R_{1y}}{6} \cdot s(x, 0, 3) + \frac{1}{E \cdot I_2} \cdot \left(\frac{\left(W_{r3y} + W_{t3y}\right)}{6} \cdot s\left(x, l_{g3}, 3\right) - \frac{M_{3z}}{2} \cdot s\left(x, l_{g3}, 2\right) \right) + \\ + \frac{1}{E \cdot I_3} \cdot \left(- \frac{\left(W_{r2y} + W_{t2y}\right)}{6} \cdot s\left(x, l_{g2}, 3\right) - \frac{M_{2z}}{2} \cdot s\left(x, l_{g2}, 2\right) \right) + \frac{1}{E \cdot I_4} \cdot \frac{R_{2y}}{6} \cdot s(x, l, 3) + c_{1y} \cdot x + c_{2y}$$

$$\theta_y(x) := \frac{1}{E \cdot I_1} \cdot \frac{R_{1z}}{2} \cdot s(x, 0, 2) + \frac{1}{E \cdot I_2} \cdot \left(\frac{\left(W_{r3z} - W_{t3z}\right)}{2} \cdot s\left(x, l_{g3}, 2\right) - M_{3y} \cdot s\left(x, l_{g3}, 1\right) \right) + \\ + \frac{1}{E \cdot I_3} \cdot \left(\frac{\left(W_{r2z} - W_{t2z}\right)}{2} \cdot s\left(x, l_{g2}, 2\right) + M_{2y} \cdot s\left(x, l_{g2}, 1\right) \right) + \frac{1}{E \cdot I_4} \cdot \frac{R_{2z}}{2} \cdot s(x, l, 2) + c_{1z}$$

$$z(x) := \frac{1}{E \cdot I_1} \cdot \frac{R_{1z}}{6} \cdot s(x, 0, 3) + \frac{1}{E \cdot I_2} \cdot \left(\frac{\left(W_{r3z} - W_{t3z}\right)}{6} \cdot s\left(x, l_{g3}, 3\right) - \frac{M_{3y}}{2} \cdot s\left(x, l_{g3}, 2\right) \right) + \\ + \frac{1}{E \cdot I_3} \cdot \left(\frac{\left(W_{r2z} - W_{t2z}\right)}{6} \cdot s\left(x, l_{g2}, 3\right) + \frac{M_{2y}}{2} \cdot s\left(x, l_{g2}, 2\right) \right) + \frac{1}{E \cdot I_4} \cdot \frac{R_{2z}}{6} \cdot s(x, l, 3) + c_{1z} \cdot x + c_{2z}$$

Solve for constants using boundary conditions: (No linear deflection at bearings)

Y-Direction

$y(0) = 0 \text{ in}$ $\qquad$ $y(l) = 0 \text{ in}$

$c_{2y} := 0$

$$c_{1y} := \frac{\frac{1}{E \cdot I_1} \cdot \frac{R_{1y}}{6} \cdot s(l, 0, 3) + \frac{1}{E \cdot I_2} \cdot \left(\frac{\left(W_{r3y} + W_{t3y}\right)}{6} \cdot s\left(l, l_{g3}, 3\right) - \frac{M_{3z}}{2} \cdot s\left(l, l_{g3}, 2\right) \right) + \frac{1}{E \cdot I_3} \cdot \left(\left(- \frac{\left(W_{r2y} + W_{t2y}\right)}{6} \right) \cdot s\left(l, l_{g2}, 3\right) - \frac{M_{2z}}{2} \cdot s\left(l, l_{g2}, 2\right) \right)}{-l}$$

$c_{1y} = 0.0006$

Z-Direction

$z(0) = 0 \text{ in}$ $\qquad$ $z(l) = 0 \text{ in}$

$c_{2z} := 0$

$$c_{1z} := \frac{\frac{1}{E \cdot I_1} \cdot \frac{R_{1z}}{6} \cdot s(l, 0, 3) + \frac{1}{E \cdot I_2} \cdot \left(\frac{\left(W_{r3z} - W_{t3z}\right)}{6} \cdot s\left(l, l_{g3}, 3\right) - \frac{M_{3y}}{2} \cdot s\left(l, l_{g3}, 2\right) \right) + \frac{1}{E \cdot I_3} \cdot \left(\frac{\left(W_{r2z} - W_{t2z}\right)}{6} \cdot s\left(l, l_{g2}, 3\right) + \frac{M_{2y}}{2} \cdot s\left(l, l_{g2}, 2\right) \right)}{-l}$$

$c_{1z} = -0.0005$

$$\theta_z(x) := \frac{1}{E \cdot I_1} \cdot \frac{R_{1y}}{2} \cdot s(x, 0, 2) + \frac{1}{E \cdot I_2} \cdot \left(\frac{\left(W_{r3y} + W_{t3y}\right)}{2} \cdot s\left(x, l_{g3}, 2\right) - M_{3z} \cdot s\left(x, l_{g3}, 1\right) \right) + \\ + \frac{1}{E \cdot I_3} \cdot \left(- \frac{\left(W_{r2y} + W_{t2y}\right)}{2} \cdot s\left(x, l_{g2}, 2\right) - M_{2z} \cdot s\left(x, l_{g2}, 1\right) \right) + \frac{1}{E \cdot I_4} \cdot \frac{R_{2y}}{2} \cdot s(x, l, 2) + c_{1y}$$

$$y(x) := \frac{1}{E \cdot I_1} \cdot \frac{R_{1y}}{6} \cdot s(x, 0, 3) + \frac{1}{E \cdot I_2} \cdot \left(\frac{\left(W_{r3y} + W_{t3y}\right)}{6} \cdot s\left(x, l_{g3}, 3\right) - \frac{M_{3z}}{2} \cdot s\left(x, l_{g3}, 2\right) \right) + \\ + \frac{1}{E \cdot I_3} \cdot \left(- \frac{\left(W_{r2y} + W_{t2y}\right)}{6} \cdot s\left(x, l_{g2}, 3\right) - \frac{M_{2z}}{2} \cdot s\left(x, l_{g2}, 2\right) \right) + \frac{1}{E \cdot I_4} \cdot \frac{R_{2y}}{6} \cdot s(x, l, 3) + c_{1y} \cdot x + c_{2y}$$

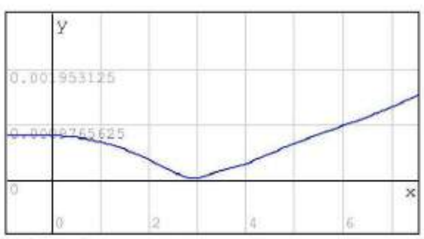
$$\theta_y(x) := \frac{1}{E \cdot I_1} \cdot \frac{R_{1z}}{2} \cdot s(x, 0, 2) + \frac{1}{E \cdot I_2} \cdot \left(\frac{\left(W_{r3z} - W_{t3z}\right)}{2} \cdot s\left(x, l_{g3}, 2\right) - M_{3y} \cdot s\left(x, l_{g3}, 1\right) \right) + \\ + \frac{1}{E \cdot I_3} \cdot \left(\frac{\left(W_{r2z} - W_{t2z}\right)}{2} \cdot s\left(x, l_{g2}, 2\right) + M_{2y} \cdot s\left(x, l_{g2}, 1\right) \right) + \frac{1}{E \cdot I_4} \cdot \frac{R_{2z}}{2} \cdot s(x, l, 2) + c_{1z}$$

$$z(x) := \frac{1}{E \cdot I_1} \cdot \frac{R_{1z}}{6} \cdot s(x, 0, 3) + \frac{1}{E \cdot I_2} \cdot \left(\frac{\left(W_{r3z} - W_{t3z}\right)}{6} \cdot s\left(x, l_{g3}, 3\right) - \frac{M_{3y}}{2} \cdot s\left(x, l_{g3}, 2\right) \right) + \\ + \frac{1}{E \cdot I_3} \cdot \left(\frac{\left(W_{r2z} - W_{t2z}\right)}{6} \cdot s\left(x, l_{g2}, 3\right) + \frac{M_{2y}}{2} \cdot s\left(x, l_{g2}, 2\right) \right) + \frac{1}{E \cdot I_4} \cdot \frac{R_{2z}}{6} \cdot s(x, l, 3) + c_{1z} \cdot x + c_{2z}$$

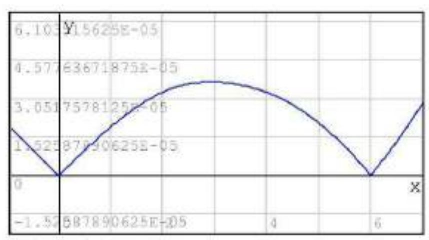
Total Angular Deflection and Deflection

$$\theta(x) := \sqrt{\theta_z(x)^2 + \theta_y(x)^2}$$

$$\delta(x) := \sqrt{y(x)^2 + z(x)^2}$$



$\theta(x \text{ in})$



$\delta(x \text{ in})$

Angular Deflection at Bearings — **Criteria** ≤ 0.004 rad

$\theta_{R1} := \theta(0) = 0.0008 \text{ rad}$ Angular deflection at beginning of shaft

$\theta_{R2} := \theta(l) = 0.001 \text{ rad}$ Angular deflection at end of shaft

Both supports angular deflection does not exceed the allowable value. Therefore, criteria is met!

PASS

Angular Deflection at Gears — **Criteria** ≤ 0.0005 rad

$\theta_{G3} := \theta\left(l_{g3}\right) = 0.0004 \text{ rad}$ Angular deflection at gear 3

$\theta_{G2} := \theta\left(l_{g2}\right) = 0.0003 \text{ rad}$ Angular deflection at gear 2

Both gears angular deflection exceeds the allowable value. Therefore, criteria does pass.

PASS

Linear Deflection at Gears

$\delta_{G3} := \delta\left(l_{g3}\right) = 0.0326\ mm$ Deflection at gear 3

$\delta_{G2} := \delta\left(l_{g2}\right) = 0.0325\ mm$ Deflection at gear 2

Criteria ≤ 0.127 mm

Both gears deflection does not exceed the allowable value. Therefore, criteria is met!

PASS

Angle of Twist

Only need to check where J is the lowest and there are the most SCFs

$\phi_{twist} := \frac{T}{G \cdot J_1} = 1.5133\ \frac{deg}{m}$

Criteria ≤ 3 deg/m

Angle of twist exceeds the allowable value. Therefore, criteria does pass.

PASS

Summary of Results

As a reminder, the evaluation criteria we need to satisfy are:

2. Shaft twist	≤ 3 deg/m
3. Linaer deflection at gears	≤ 0.127 mm
4. Angular deflection at gears	≤ 0.03 deg (0.0005 rad)
5. Angular deflection at bearings	≤ 0.004 rad

Minimum Diameters

At Gear 3:	At Gear 2:		At Bearing 1:	At Bearing 2:	
$d_{g3} = 1.2598\ in$	$d_{g2} = 1.2598\ in$	(=32mm)	$d_{b1} = 1.1811\ in$	$d_{b2} = 1.1811\ in$	(=30mm)

Evaluation Criteria

$\theta_{R1} = 0.0008$

$\theta_{R2} = 0.001$

$\theta_{G3} = 0.0004$

$\theta_{G2} = 0.0003$

$\delta_{G3} = 0.0326\ mm$

$\delta_{G2} = 0.0325\ mm$

$\phi_{twist} = 1.5133\ \frac{deg}{m}$

Initial Conclusions

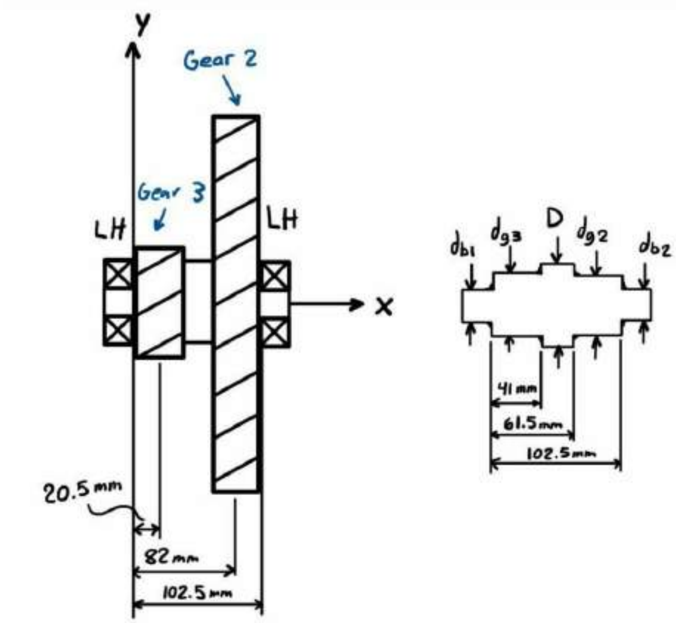
Calculations have been carried out, following a re-itteration method, a minimum diameter at both gears 2 and 3 of 1.378" (35 mm) and a minimum diameter around both bearings of 1.25" has been specified which meets all required conditions and defined evaluation criteria.

Optimized Shaft Length

Problem Statement

The objective of the following calculation is to confim a new design (new shaft length) meets evaluation criteria. Due to the intended use of the shafts to rotate in both directions (drive forward and reverse), need to consider a shaft design that supports the helical gears axially in both directions. Consider a design where the gears are "sandwiched" along the shaft to prevent any axial thrusts (shaft step on one side and support bearing on other).

Problem Diagram



From Gear Analysis, we are given that both of the gear face widths are 41 mm, use this to size the shaft appropriately.

Figure 19. Schematic of design problem: Simply supported shaft with two straddle mounted gears and bearings on each end (Note that both gears are helical and their mating gears are not in the same plane).

Evaluation Criteria

Same evaluation criteria as original shaft analysis

Assumptions

Same assumptions as orignial shaft analysis

New Shaft Lengths

Shaft Length $l := 102.5\ \text{mm} = 4.0354\ \text{in}$

Length to Gear 3 $l_{g3} := 20.5\ \text{mm} = 0.8071\ \text{in}$

Length to Gear 2 $l_{g2} := 82\ \text{mm} = 3.2283\ \text{in}$

Find the forces exerted on the idler shaft (gears 2 and 3):

Transverse Presure Angle $\phi_t := \operatorname{atan}\left(\frac{\tan(\phi)}{\cos(\psi)}\right) = 21.365\,°$

Loads on Gear 2

$W_{t2} := \frac{T}{0.5 \cdot d_{p2}} = 419.608\ \text{lbf}$

$W_{r2} := W_{t2} \cdot \tan(\phi_t) = 164.1465\ \text{lbf}$

$W_{a2} := W_{t2} \cdot \tan(\psi) = 165.288\ \text{lbf}$

$W_2 := \frac{W_{t2}}{\cos(\psi) \cdot \cos(\phi)} = 479.9324\ \text{lbf}$

$M_{2z} := W_{a2} \cdot \frac{d_{p2}}{2} \cdot \sin(\theta_1) = 414.1285\ \text{lbf in}$

$M_{2y} := W_{a2} \cdot \frac{d_{p2}}{2} \cdot \cos(\theta_1) = 414.1285\ \text{lbf in}$

Loads on Gear 3

$W_{t3} := \frac{T}{0.5 \cdot d_{p3}} = 1049.02\ \text{lbf}$

$W_{r3} := W_{t3} \cdot \tan(\phi_t) = 410.3663\ \text{lbf}$

$W_{a3} := W_{t3} \cdot \tan(\psi) = 413.22\ \text{lbf}$

$W_3 := \frac{W_{t3}}{\cos(\psi) \cdot \cos(\phi)} = 1199.8309\ \text{lbf}$

$M_{3z} := W_{a3} \cdot \frac{d_{p3}}{2} \cdot \sin(\theta_4) = 414.1285\ \text{lbf in}$

$M_{3y} := W_{a3} \cdot \frac{d_{p3}}{2} \cdot \cos(\theta_4) = 414.1285\ \text{lbf in}$

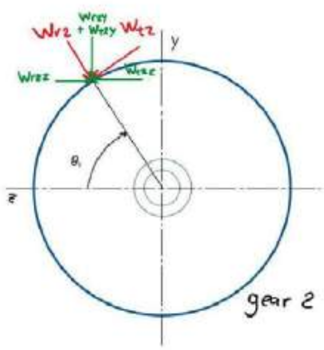


Figure 20. Snapshot of Figure 3.

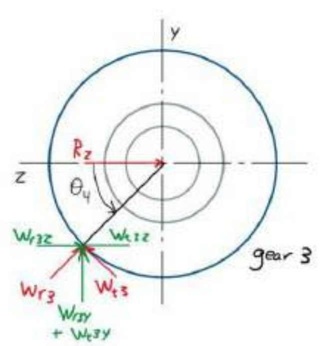


Figure 21. Snapshot of Figure 3.

$W_{r2z} := W_{r2} \cdot \cos(\theta_1) = 116.0691\ \text{lbf}$

$W_{r2y} := W_{r2} \cdot \sin(\theta_1) = 116.0691\ \text{lbf}$

$W_{t2z} := W_{t2} \cdot \cos(90\,° - \theta_1) = 296.7077\ \text{lbf}$

$W_{t2y} := W_{t2} \cdot \sin(90\,° - \theta_1) = 296.7077\ \text{lbf}$

$W_{r3z} := W_{r3} \cdot \cos(\theta_4) = 290.1728\ \text{lbf}$

$W_{r3y} := W_{r3} \cdot \sin(\theta_4) = 290.1728\ \text{lbf}$

$W_{t3z} := W_{t3} \cdot \cos(90\,° - \theta_4) = 741.7692\ \text{lbf}$

$W_{t3y} := W_{t3} \cdot \sin(90\,° - \theta_4) = 741.7692\ \text{lbf}$

Reaction Forces

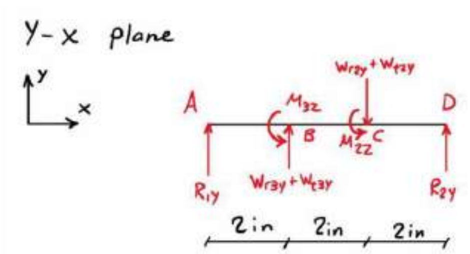


Figure 6. Reaction forces FBD in the y-x plane

$$\circlearrowleft + \Sigma M_A = 0 \Rightarrow 2(W_{r3y}+W_{t3y}) + M_{3z} - 4(W_{r2y}+W_{t2y}) + M_{2z} + 6R_{2y} = 0$$

$$R_{2y} := \frac{-M_{2z} - M_{3z} - l_{g3}\cdot\left(W_{r3y}+W_{t3y}\right) + l_{g2}\cdot\left(W_{r2y}+W_{t2y}\right)}{l}$$

$$R_{2y} = -81.4131 \text{ lbf}$$

$$+\uparrow \Sigma F_y = 0 \Rightarrow R_{1y} + W_{r3y} + W_{t3y} - W_{r2y} - W_{t2y} + R_{2y} = 0$$

$$R_{1y} := -W_{r3y} - W_{t3y} + W_{r2y} + W_{t2y} - R_{2y}$$

$$R_{1y} = -537.7521 \text{ lbf}$$

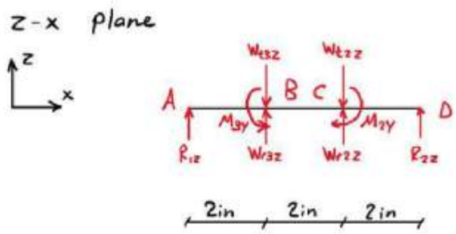


Figure 7. Reaction forces FBD in the z-x plane

$$\circlearrowleft + \Sigma M_A = 0 \Rightarrow 2(W_{r3z}-W_{t3z}) + M_{3y} + 4(W_{r2z}-W_{t2z}) - M_{2y} + 6R_{2z} = 0$$

$$R_{2z} := \frac{M_{2y} - M_{3y} - l_{g3}\cdot\left(W_{r3z}-W_{t3z}\right) - l_{g2}\cdot\left(W_{r2z}-W_{t2z}\right)}{l}$$

$$R_{2z} = 234.8301 \text{ lbf}$$

$$+\uparrow \Sigma F_y = 0 \Rightarrow R_{1z} + W_{r3z} - W_{t3z} + W_{r2z} - W_{t2z} + R_{2z} = 0$$

$$R_{1z} := W_{t3z} - W_{r3z} + W_{t2z} - W_{r2z} - R_{2z}$$

$$R_{1z} = 397.4048 \text{ lbf}$$

Assume all the axial load is on bearing **1**:

$R_{2x} := 0 \text{ lbf}$ $\quad R_{1x} := W_{a3} - W_{a2} = 247.932 \text{ lbf}$

Reaction Force Summary

$R_{1y} = -537.7521 \text{ lbf}$ $\quad R_{1z} = 397.4048 \text{ lbf}$ $\quad R_{1x} = 247.932 \text{ lbf}$

$R_{2y} = -81.4131 \text{ lbf}$ $\quad R_{2z} = 234.8301 \text{ lbf}$ $\quad R_{2x} = 0 \text{ lbf}$

Axial load:

$F_a := R_{1x} = 247.932 \text{ lbf}$

Singularity Function

On X-Y plane:

$$q_y(x) := R_{1y}\cdot s(x, 0, -1) + \left(W_{r3y}+W_{t3y}\right)\cdot s\left(x, l_{g3}, -1\right) - M_{3z}\cdot s\left(x, l_{g3}, -2\right) + \\ + \left(-\left(W_{r2y}+W_{t2y}\right)\right)\cdot s\left(x, l_{g2}, -1\right) - M_{2z}\cdot s\left(x, l_{g2}, -2\right) + R_{2y}\cdot s(x, l, -1)$$

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$$v_y(x) := R_{1y} \cdot s(x, 0, 0) + \left(W_{r3y} + W_{t3y}\right) \cdot s\left(x, l_{g3}, 0\right) - M_{3z} \cdot s\left(x, l_{g3}, -1\right) + \left(-\left(W_{r2y} + W_{t2y}\right)\right) \cdot s\left(x, l_{g2}, 0\right) - M_{2z} \cdot s\left(x, l_{g2}, -1\right) + R_{2y} \cdot s(x, l, 0)$$

$$M_z(x) := R_{1y} \cdot s(x, 0, 1) + \left(W_{r3y} + W_{t3y}\right) \cdot s\left(x, l_{g3}, 1\right) - M_{3z} \cdot s\left(x, l_{g3}, 0\right) + \left(-\left(W_{r2y} + W_{t2y}\right)\right) \cdot s\left(x, l_{g2}, 1\right) - M_{2z} \cdot s\left(x, l_{g2}, 0\right) + R_{2y} \cdot s(x, l, 1)$$

On X-Z plane:

$$q_z(x) := R_{1z} \cdot s(x, 0, -1) + \left(W_{r3z} - W_{t3z}\right) \cdot s\left(x, l_{g3}, -1\right) - M_{3y} \cdot s\left(x, l_{g3}, -2\right) + \left(W_{r2z} - W_{t2z}\right) \cdot s\left(x, l_{g2}, -1\right) + M_{2y} \cdot s\left(x, l_{g2}, -2\right) + R_{2z} \cdot s(x, l, -1)$$

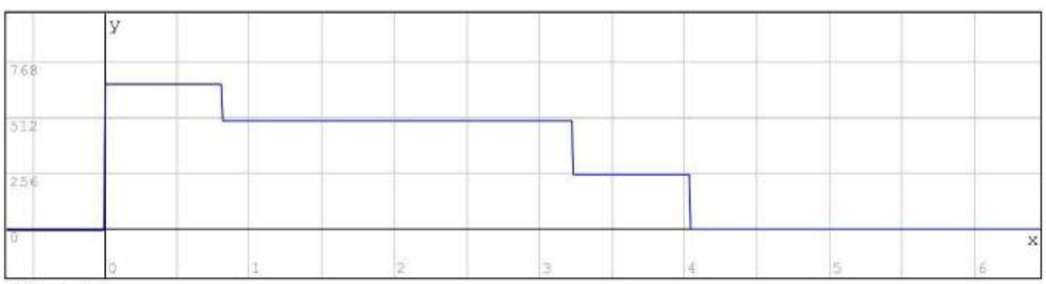
$$v_z(x) := R_{1z} \cdot s(x, 0, 0) + \left(W_{r3z} - W_{t3z}\right) \cdot s\left(x, l_{g3}, 0\right) - M_{3y} \cdot s\left(x, l_{g3}, -1\right) + \left(W_{r2z} - W_{t2z}\right) \cdot s\left(x, l_{g2}, 0\right) + M_{2y} \cdot s\left(x, l_{g2}, -1\right) + R_{2z} \cdot s(x, l, 0)$$

$$M_y(x) := R_{1z} \cdot s(x, 0, 1) + \left(W_{r3z} - W_{t3z}\right) \cdot s\left(x, l_{g3}, 1\right) - M_{3y} \cdot s\left(x, l_{g3}, 0\right) + \left(W_{r2z} - W_{t2z}\right) \cdot s\left(x, l_{g2}, 1\right) + M_{2y} \cdot s\left(x, l_{g2}, 0\right) + R_{2z} \cdot s(x, l, 1)$$

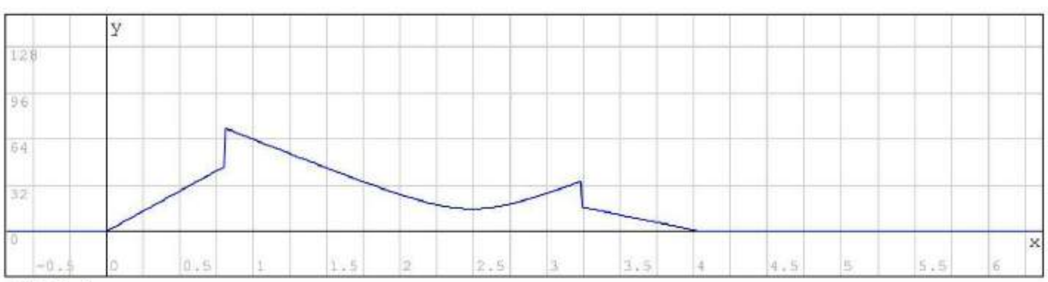
Total V and M:

$$V(x) := \sqrt{v_y(x)^2 + v_z(x)^2}$$

$$M(x) := \sqrt{M_y(x)^2 + M_z(x)^2}$$



$\frac{V(x \text{ in})}{\text{lbf}}$



$\frac{M(x \text{ in})}{\text{lbf ft}}$

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The resultant moment at both gears

$M\left(l_{g3}\right) = 71.1056 \text{ lbf ft}$ $\qquad$ $M\left(l_{g2}\right) = 16.7163 \text{ lbf ft}$

Torque on idler shaft

Mean and alternating torque from before:

$T_{mean} = 61.95 \text{ lbf ft}$ $\qquad$ $T_{alt} = 61.95 \text{ lbf ft}$

Bending moment will be fully reversed loading $\qquad$ Only need to consider where moment is a maximum

$$M_{mean} := \frac{M\left(l_{g3}\right) + \left(-M\left(l_{g3}\right)\right)}{2} = 0 \text{ lbf ft} \qquad M_{alt} := \frac{M\left(l_{g3}\right) - \left(-M\left(l_{g3}\right)\right)}{2} = 71.1056 \text{ lbf ft}$$

Axial force will be constant throughout the input shaft, but fluctuate between 0 (when not in use) and operating shaft torque.

$$F_{a'mean} := \frac{F_a + 0}{2} = 123.966 \text{ lbf} \qquad F_{a'alt} := \frac{F_a - 0}{2} = 123.966 \text{ lbf}$$

Minimum diameter of shaft where gears are located:

$d_{g3} = 1.2598 \text{ in}$ $\qquad$ $d_{g2} = 1.2598 \text{ in}$ $\qquad$ Previously determined minimum diameters

Diameters at the bearings:

$d_{b1} = 1.1811 \text{ in}$ $\qquad$ $d_{b2} = 1.1811 \text{ in}$ $\qquad$ Previously mentioned bearing diameters

Slope and Deflection Calculations

$$I_1 := \pi \cdot \frac{d_{b1}^{\,4}}{64} = 0.0955 \text{ in}^4 \quad I_2 := \pi \cdot \frac{d_{g3}^{\,4}}{64} = 0.1237 \text{ in}^4 \quad I_3 := \pi \cdot \frac{d_{g2}^{\,4}}{64} = 0.1237 \text{ in}^4 \quad I_4 := \pi \cdot \frac{d_{b2}^{\,4}}{64} = 0.0955 \text{ in}^4$$

$$J_1 := 2 \cdot I_1 = 0.1911 \text{ in}^4 \quad J_2 := 2 \cdot I_2 = 0.2473 \text{ in}^4 \quad J_3 := 2 \cdot I_3 = 0.2473 \text{ in}^4 \quad J_4 := 2 \cdot I_4 = 0.1911 \text{ in}^4$$

Solve for constants using boundary conditions: $\qquad$ (No linear deflection at bearings)

Y-Direction

$y(0) = 0 \text{ in}$ $\qquad$ $y(l) = 0 \text{ in}$

$c_{2y} := 0$

$$c_{1y} := \frac{\frac{1}{E \cdot I_1} \cdot \frac{R_{1y}}{6} \cdot s(l, 0, 3) + \frac{1}{E \cdot I_2} \cdot \left(\frac{\left(W_{r3y} + W_{t3y}\right)}{6} \cdot s\left(l, l_{g3}, 3\right) - \frac{M_{3z}}{2} \cdot s\left(l, l_{g3}, 2\right) \right) + \frac{1}{E \cdot I_3} \cdot \left(\left(-\frac{\left(W_{r2y} + W_{t2y}\right)}{6} \right) \cdot s\left(l, l_{g2}, 3\right) - \frac{M_{2z}}{2} \cdot s\left(l, l_{g2}, 2\right) \right)}{-l}$$

$c_{1y} = 0.0003$

Z-Direction

$$z(0) = 0 \text{ in} \qquad z(l) = 0 \text{ in}$$

$$c_{2z} := 0$$

$$c_{1z} := \frac{\frac{1}{E \cdot I_1} \cdot \frac{R_{1z}}{6} \cdot s(l, 0, 3) + \frac{1}{E \cdot I_2} \cdot \left(\frac{(W_{r3z} - W_{t3z})}{6} \cdot s(l, l_{g3}, 3) - \frac{M_{3y}}{2} \cdot s(l, l_{g3}, 2) \right) + \frac{1}{E \cdot I_3} \cdot \left(\frac{(W_{r2z} - W_{t2z})}{6} \cdot s(l, l_{g2}, 3) + \frac{M_{2y}}{2} \cdot s(l, l_{g2}, 2) \right)}{-l}$$

$$c_{1z} = -7.1735 \cdot 10^{-5}$$

$$\theta_z(x) := \frac{1}{E \cdot I_1} \cdot \frac{R_{1y}}{2} \cdot s(x, 0, 2) + \frac{1}{E \cdot I_2} \cdot \left(\frac{(W_{r3y} + W_{t3y})}{2} \cdot s(x, l_{g3}, 2) - M_{3z} \cdot s(x, l_{g3}, 1) \right) + \\ + \frac{1}{E \cdot I_3} \cdot \left(-\frac{(W_{r2y} + W_{t2y})}{2} \cdot s(x, l_{g2}, 2) - M_{2z} \cdot s(x, l_{g2}, 1) \right) + \frac{1}{E \cdot I_4} \cdot \frac{R_{2y}}{2} \cdot s(x, l, 2) + c_{1y}$$

$$y(x) := \frac{1}{E \cdot I_1} \cdot \frac{R_{1y}}{6} \cdot s(x, 0, 3) + \frac{1}{E \cdot I_2} \cdot \left(\frac{(W_{r3y} + W_{t3y})}{6} \cdot s(x, l_{g3}, 3) - \frac{M_{3z}}{2} \cdot s(x, l_{g3}, 2) \right) + \\ + \frac{1}{E \cdot I_3} \cdot \left(-\frac{(W_{r2y} + W_{t2y})}{6} \cdot s(x, l_{g2}, 3) - \frac{M_{2z}}{2} \cdot s(x, l_{g2}, 2) \right) + \frac{1}{E \cdot I_4} \cdot \frac{R_{2y}}{6} \cdot s(x, l, 3) + c_{1y} \cdot x + c_{2y}$$

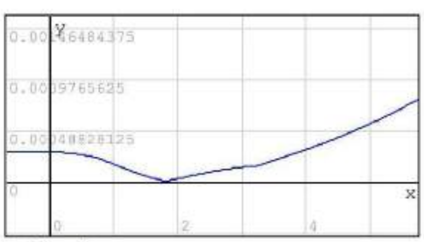
$$\theta_y(x) := \frac{1}{E \cdot I_1} \cdot \frac{R_{1z}}{2} \cdot s(x, 0, 2) + \frac{1}{E \cdot I_2} \cdot \left(\frac{(W_{r3z} - W_{t3z})}{2} \cdot s(x, l_{g3}, 2) - M_{3y} \cdot s(x, l_{g3}, 1) \right) + \\ + \frac{1}{E \cdot I_3} \cdot \left(\frac{(W_{r2z} - W_{t2z})}{2} \cdot s(x, l_{g2}, 2) + M_{2y} \cdot s(x, l_{g2}, 1) \right) + \frac{1}{E \cdot I_4} \cdot \frac{R_{2z}}{2} \cdot s(x, l, 2) + c_{1z}$$

$$z(x) := \frac{1}{E \cdot I_1} \cdot \frac{R_{1z}}{6} \cdot s(x, 0, 3) + \frac{1}{E \cdot I_2} \cdot \left(\frac{(W_{r3z} - W_{t3z})}{6} \cdot s(x, l_{g3}, 3) - \frac{M_{3y}}{2} \cdot s(x, l_{g3}, 2) \right) + \\ + \frac{1}{E \cdot I_3} \cdot \left(\frac{(W_{r2z} - W_{t2z})}{6} \cdot s(x, l_{g2}, 3) + \frac{M_{2y}}{2} \cdot s(x, l_{g2}, 2) \right) + \frac{1}{E \cdot I_4} \cdot \frac{R_{2z}}{6} \cdot s(x, l, 3) + c_{1z} \cdot x + c_{2z}$$

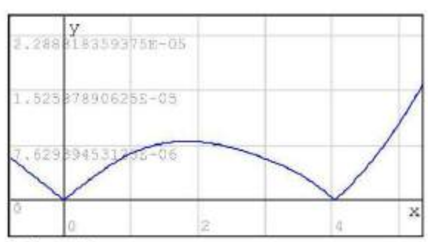
Total Angular Deflection and Deflection

$$\theta(x) := \sqrt{\theta_z(x)^2 + \theta_y(x)^2}$$

$$\delta(x) := \sqrt{y(x)^2 + z(x)^2}$$



$\theta(x \text{ in})$



$\delta(x \text{ in})$

Angular Deflection at Bearings

Criteria ≤ 0.004 rad

$\theta_{R1} := \theta(0) = 0.0003\ rad$ Angular deflection at beginning of shaft

$\theta_{R2} := \theta(l) = 0.0003\ rad$ Angular deflection at end of shaft

Both supports angular deflection does not exceed the allowable value. Therefore, criteria is met!

PASS

Angular Deflection at Gears

Criteria ≤ 0.0005 rad

$\theta_{G3} := \theta(l_{g3}) = 0.0002\ rad$ Angular deflection at gear 3

$\theta_{G2} := \theta(l_{g2}) = 0.0002\ rad$ Angular deflection at gear 2

Both gears angular deflection does not exceed the allowable value. Therefore, criteria is met!

PASS

Linear Deflection at Gears

Criteria ≤ 0.127 mm

$\delta_{G3} := \delta(l_{g3}) = 0.0055\ mm$ Deflection at gear 3

$\delta_{G2} := \delta(l_{g2}) = 0.0049\ mm$ Deflection at gear 2

Both gears deflection does not exceed the allowable value. Therefore, criteria is met!

PASS

Angle of Twist

Criteria ≤ 3 deg/m

Only need to check where J is the lowest and there are the most SCFs

$$\phi_{twist} := \frac{T}{G \cdot J_1} = 1.5133\ \frac{deg}{m}$$

Angle of twist does not exceeds the allowable value. Therefore, criteria is met!

PASS

Summary of Results

As a reminder, the evaluation criteria we need to satisfy are:

2. Shaft twist	≤ 3 deg/m
3. Linaer deflection at gears	≤ 0.127 mm
4. Angular deflection at gears	≤ 0.03 deg (0.0005 rad)
5. Angular deflection at bearings	≤ 0.004 rad

Minimum Diameters

At Gear 3:	At Gear 2:	At Bearing 1:	At Bearing 2:
$d_{g3} = 1.2598\ in$	$d_{g2} = 1.2598\ in$ (=32mm)	$d_{b1} = 1.1811\ in$	$d_{b2} = 1.1811\ in$ (=30mm)

Evaluation Criteria

$\theta_{R1} = 0.0003$

$\theta_{R2} = 0.0003$

$\theta_{G3} = 0.0002$

$\theta_{G2} = 0.0002$

$\delta_{G3} = 0.0055\ mm$

$\delta_{G2} = 0.0049\ mm$

$\phi_{twist} = 1.5133\ \frac{deg}{m}$

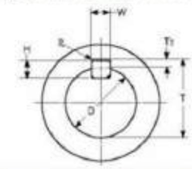
Given that all criteria are now satisfied, safety factors can be recalculated for new diameter. We need to find new notch radius and large diameter.

Key SCF

$r_{key} = 0.01\ in$ Our initial assumption still satisfies new diameter

$K_f = 1.9$ $K_{fs} = 2.64$

Metric Key Keyway Dimensions Per ISO/R773 - Js9 Width Tolerance



Key & Keyway Dimensions - Millimeters										
Shaft Diameter		Key Size		Keyway Width			Keyway Depth		Keyway Radius	
"D"		Nominal		Hub "W"			Hub "T2"		"R"	
Over	Thru	Width "W"	Height "H"	Nominal	Min	Max	Min	Max	Min	Max
22	30	8	7	8	-.0180	+0180	3.3	3.5	0.16	0.25
30	38	10	8	10	-.0180	+0180	3.3	3.5	0.25	0.40
38	44	12	8	12	-.0215	+.0215	3.3	3.5	0.25	0.40
44	50	14	9	14	-.0215	+.0215	3.8	4.0	0.25	0.40

Figure 16. Standard Key and Keyway dimension (used to find rkey) [5]

Check the SCF of the keyway is greater than the SCF of the step

$D := 40\ mm$ new large diameter

$r_{shoulder} := 0.1 \cdot d_{g2} = 0.126\ in$ assume ratio of r/d = 0.1

$\frac{D}{d_{g2}} = 1.25$

Shaft with shoulder under torsion

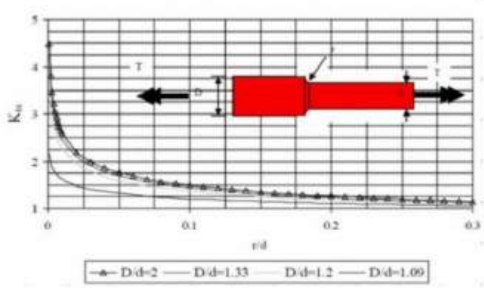


Figure 17. Step Shaft = Shaft with shoulder under torsion [4]

Shaft with shoulder under bending

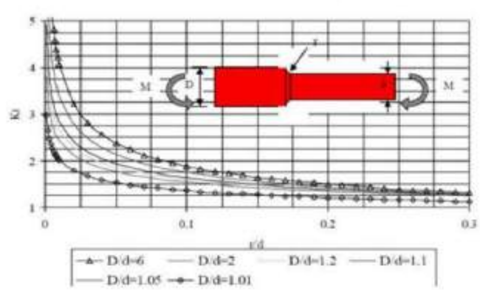


Figure 18. Step Shaft = Shaft with shoulder under bending [4]

$K_{ts} := 1.25$ $K_t := 1.6$

Step SCF: Keyway SCF:

$K_{f2} := 1 + q \cdot (K_t - 1) = 1.54 \quad \leq \quad K_f = 1.9$ Therefore, keyway SCF is still the main failure point

$K_{fs2} := 1 + q_s \cdot (K_{ts} - 1) = 1.2312 \quad \leq \quad K_{fs} = 2.64$

As previously stated in the initial length analysis, we can assume Kf = Kfm and Kfs = Kfsm

$K_{fm} := K_f = 1.9$ $K_{fsm} := K_{fs} = 2.64$

Fatigue and Yield Safety Factor Calculations

Although there are many failure theroies available for design criteria considerations, in this case the yielding safety factor will be calculated along with a fatigue safety factor. The fatigue safety factor will be found according to AGMA, guidelines suggest use of DE Elliptic criteria for evaluation of shaft failure:

DE Elliptic Fatigue Safety Factor

$$n_f = \sqrt{\frac{1}{\left(\frac{\sigma'_{alt}}{S_e}\right)^2 + \left(\frac{\sigma'_{mean}}{S_Y}\right)^2}}$$

Yield Safety Factor

$$n_y = \frac{1}{\frac{\sigma'_{alt}}{S_y} + \frac{\sigma'_{mean}}{S_y}}$$

$$d_{min} := d_{g2} = 1.2598 \text{ in}$$

Alternating and Mean Von Mises Stresses

$$\sigma'_{alt} := \left(\left(\frac{32 \cdot K_f \cdot M_{alt}}{\pi \cdot d_{min}^3} + \frac{4 \cdot K_f \cdot F_{a'alt}}{\pi \cdot d_{min}^2}\right)^2 + 3 \cdot \left(\frac{16 \cdot K_{fs} \cdot T_{alt}}{\pi \cdot d_{min}^3}\right)^2\right)^{\frac{1}{2}}$$

$$\sigma'_{mean} := \left(\left(\frac{32 \cdot K_{fm} \cdot M_{mean}}{\pi \cdot d_{min}^3} + \frac{4 \cdot K_{fm} \cdot F_{a'mean}}{\pi \cdot d_{min}^2}\right)^2 + 3 \cdot \left(\frac{16 \cdot K_{fsm} \cdot T_{mean}}{\pi \cdot d_{min}^3}\right)^2\right)^{\frac{1}{2}}$$

$$n_f := \sqrt{\frac{1}{\left(\frac{\sigma'_{alt}}{S_e}\right)^2 + \left(\frac{\sigma'_{mean}}{S_y}\right)^2}}$$

$$n_y := \frac{1}{\frac{\sigma'_{alt}}{S_y} + \frac{\sigma'_{mean}}{S_y}}$$

$$n_f = 3.4937$$

$$n_y = 6.8896$$

Conclusion

Calculations have been carried out, following a re-itteration method, a minimum diameter at both gear 2 and 3 of 1.2598" (32 mm) and a minimum diameter around both bearings of 1.1811" (30 mm) has been specified which meets all required conditions and defined evaluation criteria. Additionally, the fatigue and yield safety factors were recalculated to be 3.49 and 6.89, respectively for this shaft diameter. It can be noted that the obtatined safety factors are both greater than the design safety factor of 1.872, indicating the shaft is safe for use.

References

[1] "AISI 4140 Steel, oil quenched, 25 mm round [845C quench, 540C tempered]," MatWeb. Accessed on: Nov 1, 2024. [Online]. Available: https://www.matweb.com/search/DataSheet.aspx?MatGUID=07d1795c3f034c97b52cccda78ae1409

[2] D. Romanyk, Class Lecture, Topic: "Singularity Functions." Faculty of Engineering, University of Alberta, Edmonton, Alberta, 2024

[3] D. Romanyk, Class Lecture, Topic: "Shaft Analysis." Faculty of Engineering, University of Alberta, Edmonton, Alberta, 2024

[4] D. Romanyk, Class Lecture, Topic: "Stress Concentration Factors." Faculty of Engineering, University of Alberta, Edmonton, Alberta, 2024

[5] "Keyway Chart," Hallite. Accessed on Nov 7, 2024. [Online]. Available: https://hallite.com/au/hallite-transeals/transmission-products/keyway-chart/

C14 *- Shaft Analysis - Output Shaft*

MECE 360: Birdie Boys Design Calculations

MECE 360: Shaft Analysis, Output Shaft

Annotation Legend
Blue: Notes
Yellow: Final Answers

Problem Statement

The intention of the following calculation is to aid the design of a golf cart transmission. The simply supported shaft made of AISI 4140 Steel below represents the third shaft in a three shaft transmission, the output shaft. For this application, the gear on the shaft is helical and is driven by a gear on the idler shaft. The shaft will be connected to a differential that will provide power to the wheels of the golf cart. The gear is straddle mounted and the shaft is supported on either side by bearings (axial load to be carried on the left).

The objective of this design calculation code is to solve for the minimum required diameter for the shaft show in Figure 1 below. Additional calculations are presented to ensure that the evaluation criterial (including shaft twist angle, linear deflection at the gears, and angular deflection between gears) is satisfied. As well, a final calculation for fatigue and yeild safety factor safety factors are included.

Problem Diagram

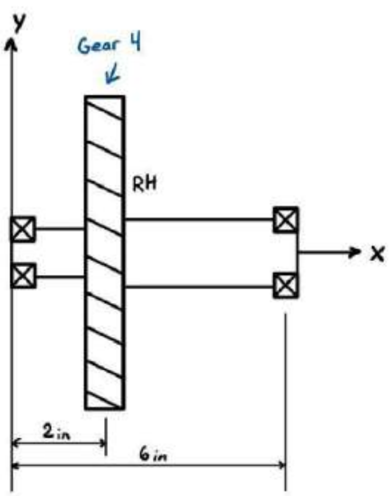
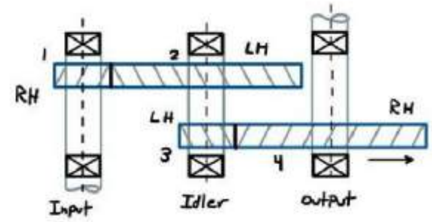


Figure 1. Schematic of design problem: Simply supported shaft with one straddle mounted gear and bearings on each end.



Note the shafts are not in the same plane

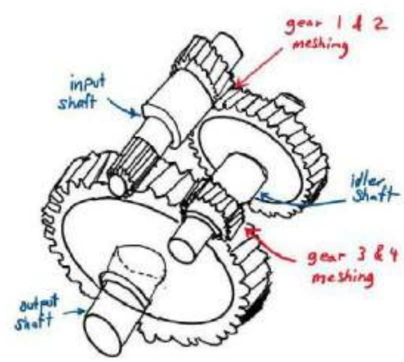


Figure 2. Full layout for all three shafts and gears

Evaluation Criteria

Based on the problem objective, the following criteria must be satisfied:

1. Strength Failure (DE Elliptic Criteria)

- First Cycle Yield: $\sigma'_{max} < \frac{S_y}{n}$
- Fatigue Failure: $\left(\frac{n\sigma'_a}{S_e}\right)^2 + \left(\frac{n\sigma'_m}{S_y}\right)^2 < 1$

Based on the problem objective, the following criteria will be compared with to ensure it is satisfied:

1. Shaft twist	≤ 3 deg/m
2. Linaer deflection at gears	≤ 0.127 mm
3. Angular deflection at gears	≤ 0.03 deg (0.0005 rad)
4. Angular deflection at bearings	≤ 0.004 rad

Page 1

Assumptions

The following assumptions were make for the given problem:

1. Assume machined shaft
2. Assume combined loading, but bending is dominant
3. Assume the operating temperature is 40 °C (104 °F)
4. Assume there are no miscellaneous correction factors
5. Assume lifespan ≥ 10^6 cycles therefore, infinite life
6. Assume 90% reliaitliy
7. Assume constant torque, but will fluctuate between 0 (when not in use) and operating torque
8. Assume forces on shaft are fully reversed, due to shaft rotation causing a critical element to switch between compression and tension of equal magnitudes
9. Assume that bearings and gears act on shaft as point loads
10. Assume that axial load from helical gears goes to bearing with highest radial load
11. Assume that for the stress concentration factors, Kfm = Kf and Kfms = Kfs
12. Assume gears are mounted using parallel keyways

Sketches

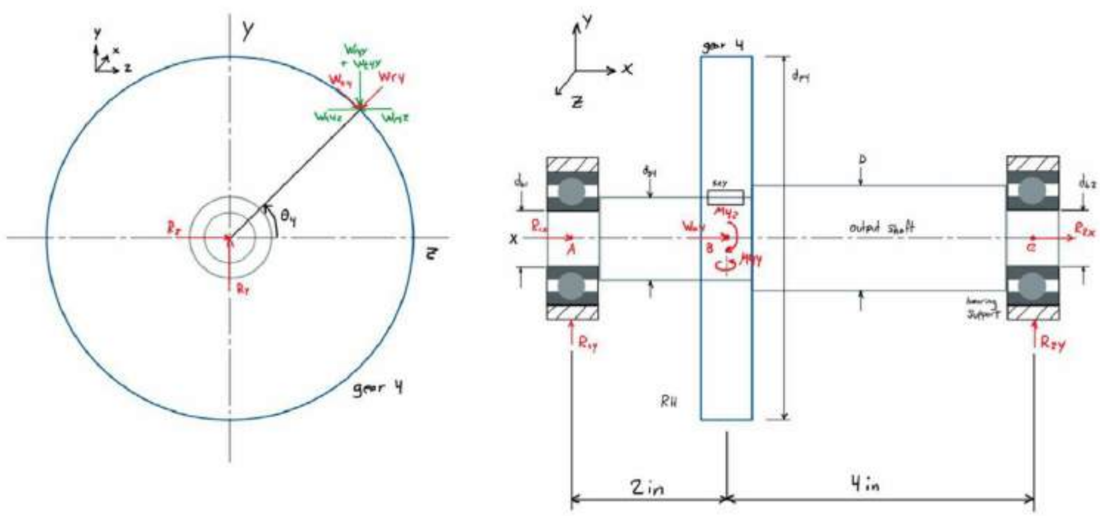


Figure 3. Free Body Diagram (FBD) of design problem with labelled reaction forces on shaft

FBD Variables

At Left Bearing (A)
R1x - Axial Reaction Force
R1y - Y-Direction Reaction Force
R1z - Z-Direction Reaction Force

At Right Bearing (C)
R2x - Axial Reaction Force
R2y - Y-Direction Reaction Force
R2z - Z-Direction Reaction Force

At Gear 4 (B)
Wa4 - Axial Force
Wr4 - Radial Force
Wt4 - Tangential Force
M4z - Z-Direction Moment
M4y - Y-Direction Moment

Misc.

θ4 - Angle between idler shaft and the output shaft

Given for Motor:

$RPM_{in} := 3000\ rpm$ $T_{run} := 41.3\ lbf\ ft$ $T_{max} := 44.69\ lbf\ ft$ (max torque on startup)

Gear Analysis Summary: Pressure Angle $\phi := 20\ °$ Helix Angle $\psi := 21.5\ °$

Number of Teeth	Module	Pitch Diameter
$N_1 := 20\ teeth$	$m_1 := 3\ \frac{mm}{teeth}$	$d_{p1} := m_1 \cdot N_1 = 60\ mm$
$N_2 := 60\ teeth$	$m_2 := 3\ \frac{mm}{teeth}$	$d_{p2} := m_2 \cdot N_2 = 180\ mm$
$N_3 := 24\ teeth$	$m_3 := 3\ \frac{mm}{teeth}$	$d_{p3} := m_3 \cdot N_3 = 72\ mm$
$N_4 := 60\ teeth$	$m_4 := 3\ \frac{mm}{teeth}$	$d_{p4} := m_4 \cdot N_4 = 180\ mm$

Output Shaft Inputs

Shaft Velocity $\omega := RPM_{in} \cdot \frac{N_1 \cdot N_3}{N_2 \cdot N_4} = 41.8879\ \frac{rad}{s}$

Shaft Torque $T := T_{run} \cdot \frac{N_2 \cdot N_4}{N_1 \cdot N_3} = 309.75\ lbf\ ft$

Max Torque $T_{max} := T_{max} \cdot \frac{N_2 \cdot N_4}{N_1 \cdot N_3} = 335.175\ lbf\ ft$

Ultimate Strength $S_{ut} := 155.9\ kpsi$

Yield Strength $S_y := 143\ kpsi$

Young's Modulus $E := 29700\ kpsi$

Shear Modulus $G := 11600\ kpsi$

Design Factor of Safety $n := 1.872$

Operation Temperature $T_{oper} := 104\ °F$

Shaft Length $l := 6\ in$

Length to Gear 4 $l_{g4} := 2\ in$

Angle Gear 2 to Gear 1 $\theta_1 := 45\ °$

Angle Gear 3 to Gear 4 $\theta_4 := 45\ °$

Material properties taken from matweb.com [1]

Find the forces exerted on the output shaft (gears 4):

Transverse Presure Angle $\phi_t := atan\left(\frac{\tan(\phi)}{\cos(\psi)}\right) = 21.365\ °$

Loads on Gear 4

$$W_{t4} := \frac{T}{0.5 \cdot d_{p4}} = 1049.02\ lbf$$

$$W_{r4} := W_{t4} \cdot \tan(\phi_t) = 410.3663\ lbf$$

$$W_{a4} := W_{t4} \cdot \tan(\psi) = 413.22\ lbf$$

$$W_4 := \frac{W_{t4}}{\cos(\psi) \cdot \cos(\phi)} = 1199.8309\ lbf$$

$$M_{4z} := W_{a4} \cdot \frac{d_{p4}}{2} \cdot \sin\left(\theta_1\right) = 86.2768 \text{ lbf ft}$$

$$M_{4y} := W_{a4} \cdot \frac{d_{p4}}{2} \cdot \cos\left(\theta_1\right) = 86.2768 \text{ lbf ft}$$

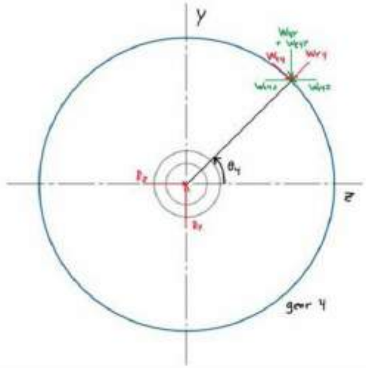


Figure 5. Snapshot of Figure 3.

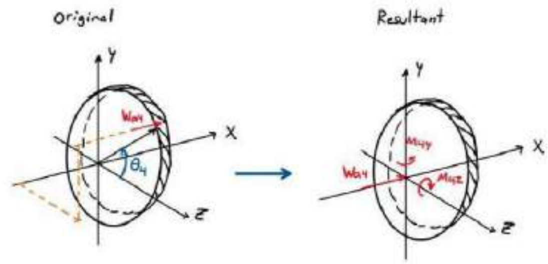


Figure 4. Visualization of two resulting moments, M4z and M4y, due to axial load.

$$W_{r4z} := W_{r4} \cdot \cos\left(\theta_4\right) = 290.1728 \text{ lbf}$$

$$W_{r4y} := W_{r4} \cdot \sin\left(\theta_4\right) = 290.1728 \text{ lbf}$$

$$W_{t4z} := W_{t4} \cdot \cos\left(90\,° - \theta_4\right) = 741.7692 \text{ lbf}$$

$$W_{t4y} := W_{t4} \cdot \sin\left(90\,° - \theta_4\right) = 741.7692 \text{ lbf}$$

Reaction Forces

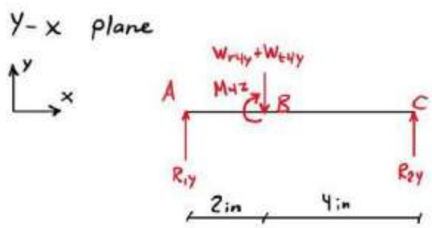


Figure 5. Reaction forces FBD in the y-x plane

$$\circlearrowleft + \sum M_A = 0 \Rightarrow \quad -2\left(W_{r4y} + W_{t4y}\right) - M_{4z} + 6R_{2y} = 0$$

$$R_{2y} := \frac{M_{4z} + l_{g4} \cdot \left(W_{r4y} + W_{t4y}\right)}{l}$$

$$R_{2y} = 516.5342 \text{ lbf}$$

$$+\uparrow \sum F_y = 0 \Rightarrow \quad R_{1y} - W_{r4y} - W_{t4y} + R_{2y} = 0$$

$$R_{1y} := W_{r4y} + W_{t4y} - R_{2y}$$

$$R_{1y} = 515.4078 \text{ lbf}$$

z-x plane

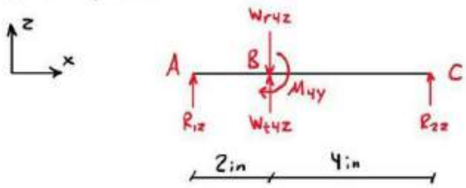


Figure 6. Reaction forces FBD in the z-x plane

$$\circlearrowleft + \sum M_A = 0 \Rightarrow 2(W_{t4z} - W_{r4z}) - M_{4y} + 6R_{2z} = 0$$

$$R_{2z} := \frac{M_{4y} - l_{g4} \cdot (W_{t4z} - W_{r4z})}{l}$$

$$R_{2z} = 22.0214 \text{ lbf}$$

$$+\uparrow \sum F_y = 0 \Rightarrow R_{1z} + W_{t4z} - W_{r4z} + R_{2z} = 0$$

$$R_{1z} := W_{r4z} - W_{t4z} - R_{2z}$$

$$R_{1z} = -473.6177 \text{ lbf}$$

Assume all the axial load is on bearing 1:

$R_{2x} := 0 \text{ lbf}$ $\quad$ $R_{1x} := -W_{a4} = -413.22 \text{ lbf}$

Reaction Force Summary

$R_{1y} = 515.4078 \text{ lbf}$	$R_{1z} = -473.6177 \text{ lbf}$	$R_{1x} = -413.22 \text{ lbf}$	Axial load: $F_a := -R_{1x} = 413.22 \text{ lbf}$
$R_{2y} = 516.5342 \text{ lbf}$	$R_{2z} = 22.0214 \text{ lbf}$	$R_{2x} = 0 \text{ lbf}$	

Singularity Function

```
s(x, a, n) := if (((x − a) > 0) ∧ (n ≥ 0))
                 (x − a)^n
              else
                 if (((x − a) = 0) ∧ (n = 0))
                    1
                 else
                    0
```

Function	$q(x)$	Evaluation
Ramp	$<x-a>^1$	$=\begin{cases} 0, \text{if } x < a \\ x-a, \text{if } x \geq a \end{cases}$
Shear flow/ distributed load	$<x-a>^0$	$=\begin{cases} 0, \text{if } x < a \\ 1, \text{if } x \geq a \end{cases}$
Shear force/ support reactions	$<x-a>^{-1}$	$=\begin{cases} 0, \text{if } x \neq a \\ +\infty, \text{if } x = a \end{cases}$
Moment/ couple (internal!)	$<x-a>^{-2}$	$=\begin{cases} 0, \text{if } x \neq a \\ \pm\infty, \text{if } x = a \end{cases}$

Figure 8. Singularity Function (to solve for Vy(x) and Mz(x)) [2]

Singularity Function Equations for shear force and bending moment

Shear Force $\quad V_y(x) = \int q_y(x)dx$

Bending Moment $\quad M_z(x) = \int V_y(x)dx$

On X-Y plane:

$$q_y(x) := R_{1y} \cdot s(x, 0, -1) - (W_{r4y} + W_{t4y}) \cdot s(x, l_{g4}, -1) + M_{4z} \cdot s(x, l_{g4}, -2) + R_{2y} \cdot s(x, l, -1)$$

$$V_y(x) := R_{1y} \cdot s(x, 0, 0) - (W_{r4y} + W_{t4y}) \cdot s(x, l_{g4}, 0) + M_{4z} \cdot s(x, l_{g4}, -1) + R_{2y} \cdot s(x, l, 0)$$

$$M_z(x) := R_{1y} \cdot s(x, 0, 1) - (W_{r4y} + W_{t4y}) \cdot s(x, l_{g4}, 1) + M_{4z} \cdot s(x, l_{g4}, 0) + R_{2y} \cdot s(x, l, 1)$$

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On X-Z plane:

$$q_z(x) := R_{1z} \cdot s(x, 0, -1) + (W_{t4z} - W_{r4z}) \cdot s(x, l_{g4}, -1) + M_{4y} \cdot s(x, l_{g4}, -2) + R_{2z} \cdot s(x, l, -1)$$

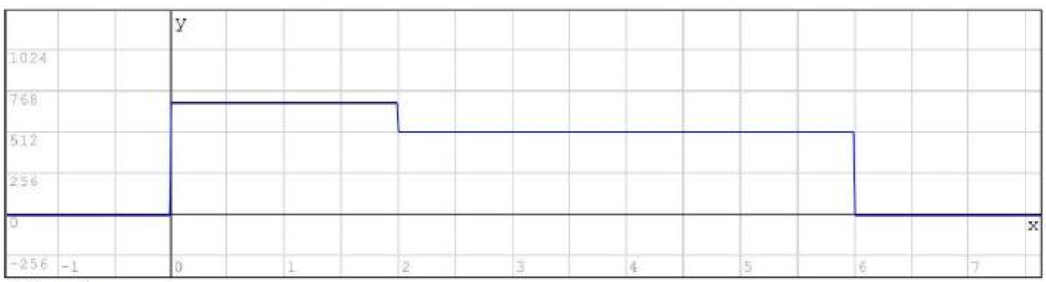
$$v_z(x) := R_{1z} \cdot s(x, 0, 0) + (W_{t4z} - W_{r4z}) \cdot s(x, l_{g4}, 0) + M_{4y} \cdot s(x, l_{g4}, -1) + R_{2z} \cdot s(x, l, 0)$$

$$M_y(x) := R_{1z} \cdot s(x, 0, 1) + (W_{t4z} - W_{r4z}) \cdot s(x, l_{g4}, 1) + M_{4y} \cdot s(x, l_{g4}, 0) + R_{2z} \cdot s(x, l, 1)$$

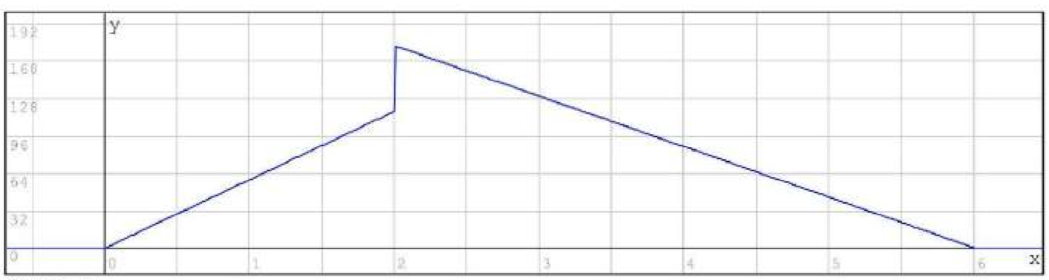
Total V and M:

$$V(x) := \sqrt{v_y(x)^2 + v_z(x)^2}$$

$$M(x) := \sqrt{M_y(x)^2 + M_z(x)^2}$$



$\frac{V(x\ \text{in})}{\text{lbf}}$



$\frac{M(x\ \text{in})}{\text{lbf ft}}$

The resultant moment at gear 4

$$M_{g4} := M(l_{g4}) = 172.3345\ \text{lbf ft}$$

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Consider Loads on Startup (Max Torque)

Loads on Gear 4

$$W_{t4} := \frac{T_{max}}{0.5 \cdot d_{p4}} = 1135.126\ \text{lbf}$$

$$W_{r4} := W_{t4} \cdot \tan\left(\phi_t\right) = 444.0502\ \text{lbf}$$

$$W_{a4} := W_{t4} \cdot \tan\left(\psi\right) = 447.138\ \text{lbf}$$

$$M_{4z} := W_{a4} \cdot \frac{d_{p4}}{2} \cdot \sin\left(\theta_1\right) = 93.3586\ \text{lbf ft}$$

$$M_{4y} := W_{a4} \cdot \frac{d_{p4}}{2} \cdot \cos\left(\theta_1\right) = 93.3586\ \text{lbf ft}$$

$$W_{r4z} := W_{r4} \cdot \cos\left(\theta_4\right) = 313.9909\ \text{lbf}$$

$$W_{r4y} := W_{r4} \cdot \sin\left(\theta_4\right) = 313.9909\ \text{lbf}$$

$$W_{t4z} := W_{t4} \cdot \cos\left(90\ ° - \theta_4\right) = 802.6553\ \text{lbf}$$

$$W_{t4y} := W_{t4} \cdot \sin\left(90\ ° - \theta_4\right) = 802.6553\ \text{lbf}$$

Reaction Forces

y-x plane:

$$R_{2y} := \frac{M_{4z} + l_{g4} \cdot \left(W_{r4y} + W_{t4y}\right)}{l} = 558.9325\ \text{lbf}$$

$$R_{1y} := W_{r4y} + W_{t4y} - R_{2y} = 557.7137\ \text{lbf}$$

z-x plane:

$$R_{2z} := \frac{M_{4y} - l_{g4} \cdot \left(W_{t4z} - W_{r4z}\right)}{l} = 23.829\ \text{lbf}$$

$$R_{1z} := W_{r4z} - W_{t4z} - R_{2z} = -512.4934\ \text{lbf}$$

Axial load is on bearing 1:

$R_{2x} := 0\ \text{lbf}$ $\quad R_{1x} := -W_{a4} = -447.138\ \text{lbf}$

max axial load:

$$F_{a'max} := -R_{1x} = 447.138\ \text{lbf}$$

Singularity Function

On X-Y plane:

$$q_y\left(x\right) := R_{1y} \cdot s\left(x, 0, -1\right) - \left(W_{r4y} + W_{t4y}\right) \cdot s\left(x, l_{g4}, -1\right) + M_{4z} \cdot s\left(x, l_{g4}, -2\right) + R_{2y} \cdot s\left(x, l, -1\right)$$

$$v_y\left(x\right) := R_{1y} \cdot s\left(x, 0, 0\right) - \left(W_{r4y} + W_{t4y}\right) \cdot s\left(x, l_{g4}, 0\right) + M_{4z} \cdot s\left(x, l_{g4}, -1\right) + R_{2y} \cdot s\left(x, l, 0\right)$$

$$M_z(x) := R_{1y} \cdot s(x, 0, 1) - \left(W_{r4y} + W_{t4y}\right) \cdot s\left(x, l_{g4}, 1\right) + M_{4z} \cdot s\left(x, l_{g4}, 0\right) + R_{2y} \cdot s(x, l, 1)$$

On X-Z plane:

$$q_z(x) := R_{1z} \cdot s(x, 0, -1) + \left(W_{t4z} - W_{r4z}\right) \cdot s\left(x, l_{g4}, -1\right) + M_{4y} \cdot s\left(x, l_{g4}, -2\right) + R_{2z} \cdot s(x, l, -1)$$

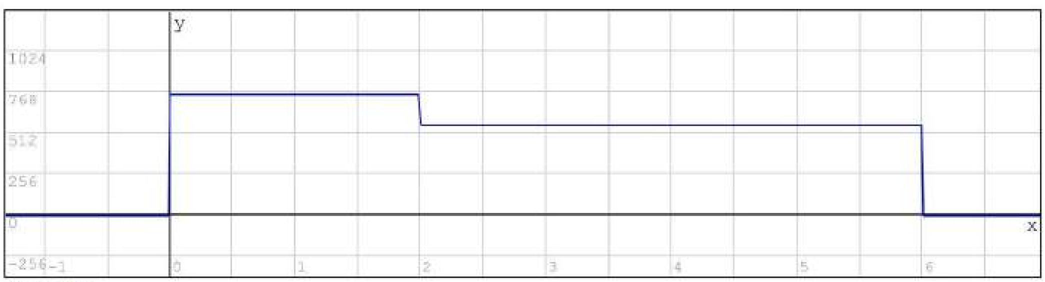
$$v_z(x) := R_{1z} \cdot s(x, 0, 0) + \left(W_{t4z} - W_{r4z}\right) \cdot s\left(x, l_{g4}, 0\right) + M_{4y} \cdot s\left(x, l_{g4}, -1\right) + R_{2z} \cdot s(x, l, 0)$$

$$M_y(x) := R_{1z} \cdot s(x, 0, 1) + \left(W_{t4z} - W_{r4z}\right) \cdot s\left(x, l_{g4}, 1\right) + M_{4y} \cdot s\left(x, l_{g4}, 0\right) + R_{2z} \cdot s(x, l, 1)$$

Total V and M:

$$V(x) := \sqrt{v_y(x)^2 + v_z(x)^2}$$

$$M(x) := \sqrt{M_y(x)^2 + M_z(x)^2}$$



$\frac{V(x \text{ in})}{\text{lbf}}$

$\frac{M(x \text{ in})}{\text{lbf ft}}$

The resultant moment at gear 4:

$M_{g4'max} := M\left(l_{g4}\right) = 186.4801 \text{ lbf ft}$ Max moment for startup torque

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Sizing the Shaft Based on Strength

Gear 4

Determine minimum diameter at gear 4 assuming keyway has greatest SCF

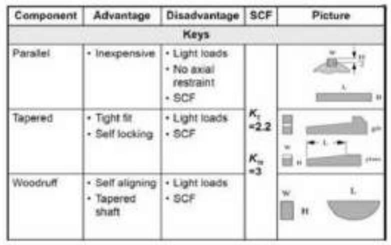
$K_t := 2.2$

SCF for key

$K_{ts} := 3$

$r_{key} := 0.01\ \text{in}$

$r_{shoulder} := 0.1\ \text{in}$



Component	Advantage	Disadvantage	SCF	Picture
Keys				
Parallel	• Inexpensive	• Light loads • No axial restraint • SCF	K_t =2.2 K_{ts} =3	
Tapered	• Tight fit • Self locking	• Light loads • SCF		
Woodruff	• Self aligning • Tapered shaft	• Light loads • SCF		

Figure 9. Keys SCF [3]

Sut = 155.9 kpsi

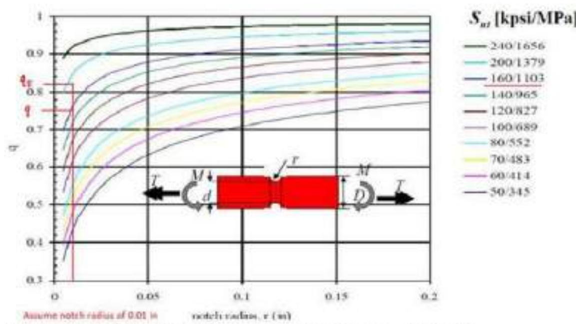


Figure 10. Notch Sensitivity (to solve for q and qs) [4]

$q := 0.75 \qquad q_s := 0.82$

$K_f := 1 + q \cdot (K_t - 1) = 1.9$

$K_{fs} := 1 + q_s \cdot (K_{ts} - 1) = 2.64$

Torque on output shaft

The torque will be constant throughout the output shaft, but fluctuate between 0 (when not in use) and operating shaft torque.

$T_{mean} := \dfrac{T + 0}{2} = 154.875\ \text{lbf ft} \qquad T_{alt} := \dfrac{T - 0}{2} = 154.875\ \text{lbf ft}$

Bending moment will be fully reversed loading.

$M_{mean} := \dfrac{M_{g4} + (-M_{g4})}{2} = 0\ \text{lbf ft} \qquad M_{alt} := \dfrac{M_{g4} - (-M_{g4})}{2} = 172.3345\ \text{lbf ft}$

Axial force will be constant throughout the output shaft, but fluctuate between 0 (when not in use) and operating shaft torque.

$F_{a'mean} := \dfrac{F_a + 0}{2} = 206.61\ \text{lbf} \qquad F_{a'alt} := \dfrac{F_a - 0}{2} = 206.61\ \text{lbf}$

Endurance Strength

Uncorrected endurance strength

```
S'e := if Sut ≤ 1463 MPa
         0.504 · Sut
       else
         737 MPa
```

$S'_e = 78573.6\ \text{psi}$

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Surface Condition Factor, ka:

Assuming the shaft is machined

$a := 2.7 \qquad b := -0.265$

$k_a := a \cdot \left(S_{ut} \cdot \frac{1}{kpsi} \right)^b = 0.7084$

	MPa		kpsi	
Surface finish	**a**	**b**	**a**	**B**
Ground (standard unless otherwise indicated)	*1.58*	*-0.085*	*1.34*	*-0.085*
Machined or Cold drawn	4.51	-0.265	2.7	-0.265
Hot-rolled	57.7	-0.718	14.4	-0.718
As-forged	272	-0.995	39.9	-0.995

Size Correction Factor, kb:

Assuming bending and torsion with rotation

Assume dg4 is between 0.11 and 2 in:

kb = 0.897*d^(-0.107)

$$k_b = \begin{cases} 0.879 d^{-0.107} & 0.11 \le d \le 2 \text{ in} \\ 0.91 d^{-0.157} & 2 < d \le 10 \text{ in} \\ 1.24 d^{-0.107} & 2.79 \le d \le 51 \text{ mm} \\ 1.51 d^{-0.157} & 51 < d \le 254 \text{ mm} \end{cases}$$

We must come back to check this

Load Correction Factor, kc:

Combined loading, bending is considered dominant

$k_c := 1$

$$k_c = \begin{cases} 1 & \text{Bending} \\ 0.85 & \text{Axial} \\ 0.59 & \text{Pure torsion} \end{cases}$$

Temperature Correction Factor, kd:

Temperature Correction for operating temperature of 104 F

$$k_d := 0.975 + 0.00032 \cdot \left(\frac{T_{oper}}{°F} \right) - 0.115 \cdot 10^{-5} \cdot \left(\frac{T_{oper}}{°F} \right)^2 + 0.104 \cdot 10^{-8} \cdot \left(\frac{T_{oper}}{°F} \right)^3 - 0.595 \cdot 10^{-12} \cdot \left(\frac{T_{oper}}{°F} \right)^4 = 0.9969$$

Reliability Factor, ke:

Assume reliability to be 95%

$k_e := 0.868$

Reliability	Ke
50	1
90	0.897
95	0.868
99	0.814
99.9	0.753
99.99	0.702
99.999	0.659
99.9999	0.62

Miscellaneous Correction Factor, kf:

Assume no miscellaneous correction

$k_f := 1$

Result of all correction factors without kb:

$k := k_a \cdot k_c \cdot k_d \cdot k_e \cdot k_f = 0.613$

Minimum Diameter Iteration Calculation

Iterate using MATLAB code to find minimum shaft diameter, d. The MATLAB function takes an initial diameter and checks if it passes DE Elliptic Failure Criteria, if not increase diameter and try again, if yes then minimum diameter has been found.

MECE 360: Birdie Boys Design Calculations

DE Elliptic Failure Criteria: $\left(\frac{n\sigma'_a}{S_e}\right)^2 + \left(\frac{n\sigma'_m}{S_y}\right)^2 < 1$

Von Mises Stresses accounting for bending stress, axial stress, and torsional stress:

$$\sigma'_{alt} = \left(\left(\frac{32 \cdot K_f \cdot M_{alt}}{\pi \cdot d^3} + \frac{4 \cdot K_f \cdot F_{a'alt}}{\pi \cdot d^2}\right)^2 + 3 \cdot \left(\frac{16 \cdot K_{fs} \cdot T_{alt}}{\pi \cdot d^3}\right)^2\right)^{\frac{1}{2}}$$

$$\sigma'_{mean} = \left(\left(\frac{32 \cdot K_{fm} \cdot M_{mean}}{\pi \cdot d^3} + \frac{4 \cdot K_{fm} \cdot F_{a'mean}}{\pi \cdot d^2}\right)^2 + 3 \cdot \left(\frac{16 \cdot K_{fsm} \cdot T_{mean}}{\pi \cdot d^3}\right)^2\right)^{\frac{1}{2}}$$

Corrected Endurance Strength:

$S_e = k_a \cdot k_b \cdot k_c \cdot k_d \cdot k_e \cdot k_f \cdot S'_e$ kb is a function of diameter

Assume $K_f \cdot \sigma'_{max} < S_y$ so $K_{fm} := K_f$ and $K_{fsm} := K_{fs}$

Plug in an intial diameter guess of d = 1.2598" (minimum diameter for idler shaft which experiences less loads) into the MATLAB function minShaftDiameter.m, as well as other required input values.
The MATLAB function iteratively tests for a minimum shaft diameter that meets DE Elliptic Failure Criteria using the above formulas.

Computed minimum shaft diameter at gear 1: $d_{g4} := 1.3974\ in$

Calculate final endurance strength using minimum diameter:

$k_b := 0.897 \cdot \left(\frac{d_{g4}}{in}\right)^{-0.107} = 0.8655$ $S_e := k_a \cdot k_b \cdot k_c \cdot k_d \cdot k_e \cdot k_f \cdot S'_e = 41.684\ kpsi$

Check Yield Failure Criteria

DE Failure criteria states: $\sigma' < \frac{S_y}{n}$

$$\sigma' := \left(\left(\frac{32 \cdot K_f \cdot M_{g4'max}}{\pi \cdot d_{g4}^3} + \frac{4 \cdot K_f \cdot F_{a'max}}{\pi \cdot d_{g4}^2}\right)^2 + 3 \cdot \left(\frac{16 \cdot K_{fs} \cdot T_{max}}{\pi \cdot d_{g4}^3}\right)^2\right)^{\frac{1}{2}} = 38.0535\ kpsi$$

$\sigma' = 38.0535\ kpsi \leq \frac{S_y}{n} = 76.3889\ kpsi$ Therefore, first cycle yield passes.

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Check the assumptions

$$k_b = \begin{cases} 0.879d^{-0.107} & 0.11 \le d \le 2\,\text{in} \\ 0.91d^{-0.157} & 2 < d \le 10\,\text{in} \\ 1.24d^{-0.107} & 2.79 \le d \le 51\,\text{mm} \\ 1.51d^{-0.157} & 51 < d \le 254\,\text{mm} \end{cases}$$

0.11 ≤ dg4 = 1.3974 ≤ 2 in

Assumption is valid

Moment of Inertia:

$$I := \pi \cdot \frac{d_{g4}^{\ 4}}{64} = 0.1872\ \text{in}^4$$

Polar Moment of Inertia:

$$J := 2 \cdot I = 0.3744\ \text{in}^4$$

Cross-Sectional Area:

$$A := \frac{1}{4} \cdot \pi \cdot d_{g4}^{\ 2} = 1.5337\ \text{in}^2$$

$$\sigma'_{max} := \sqrt{\left(\frac{M_{g4} \cdot \frac{d_{g4}}{2}}{I} + \frac{W_{a4}}{A} \right)^2 + 3 \cdot \left(\frac{T \cdot \frac{d_{g4}}{2}}{J} \right)^2} = 14.4417\ \text{kpsi}$$

$$K_f \cdot \sigma'_{max} = 27.4392\ \text{kpsi} \quad \le \quad S_y = 143\ \text{kpsi}$$

Therefore, Kfm = Kf was a valid assumption

Check the SCF of the keyway is greater than the SCF of the step

Assume Diameter of the center part is

$$D := 1.75\ \text{in} \qquad \frac{D}{d_{g4}} = 1.2523 \qquad \frac{r_{shoulder}}{d_{g4}} = 0.0716$$

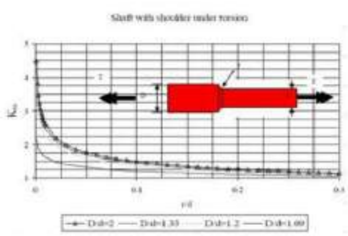


Figure 11. Step Shaft = Shaft with shoulder under torsion [4]

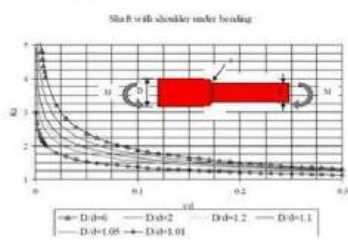


Figure 12. Step Shaft = Shaft with shoulder under bending [4]

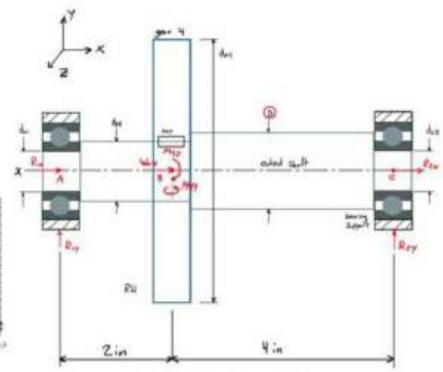


Figure 13. FBD to show "D'

$$K_{ts} := 1.5 \qquad K_t := 1.75$$

$$q := 0.9 \qquad q_s := 0.925$$

Step SCF:

Keyway SCF:

$$K_{f2} := 1 + q \cdot \left(K_t - 1 \right) = 1.675 \quad \le \quad K_f = 1.9$$

$$K_{fs2} := 1 + q_s \cdot \left(K_{ts} - 1 \right) = 1.4625 \quad \le \quad K_{fs} = 2.64$$

Sut = 155.9 kpsi

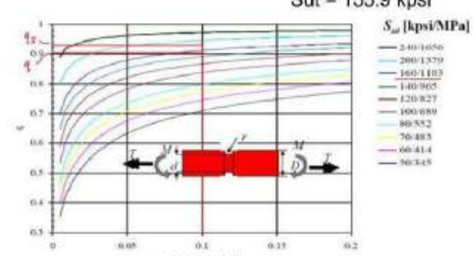


Figure 14. Notch Sensitivity (to solve for q and qs) [4]

Therefore assuming the SCF of the keyway is greater than the SCF of the shoulder is valid

Minimum diameter of shaft where gear is located:

$d_{g4} = 1.3974\ in$

$d_{g4} := 40\ mm = 1.5748\ in$ Size up to first available gear bore diameter

Assume the diameters at the bearings

$d_{b1} := 35\ mm = 1.378\ in$ $d_{b2} := 35\ mm = 1.378\ in$ Chosen bearing bore diameter

Slope and Deflection Calculations

$I_1 := \pi \cdot \frac{d_{b1}^4}{64} = 0.177\ in^4$ $I_2 := \pi \cdot \frac{d_{g4}^4}{64} = 0.3019\ in^4$ $I_3 := \pi \cdot \frac{d_{b2}^4}{64} = 0.177\ in^4$

$J_1 := 2 \cdot I_1 = 0.3539\ in^4$ $J_2 := 2 \cdot I_2 = 0.6038\ in^4$ $J_3 := 2 \cdot I_3 = 0.3539\ in^4$

Singularity Function Equations for Angular Deflection and Deflection of Shaft

$$\theta_z(x) = \int \frac{M_z(x)}{EI} dx + C_1 \quad \text{Slope}$$

$$y(x) = \int \left[\int \frac{M_z(x)}{EI} dx + C_1 \right] dx + C_2 \quad \text{Deflection}$$

$$\theta_z(x) := \frac{1}{E \cdot I_1} \cdot \frac{R_{1y}}{2} \cdot s(x, 0, 2) + \frac{1}{E \cdot I_2} \cdot \left(\frac{-\left(W_{r4y} + W_{t4y}\right)}{2} \cdot s\left(x, l_{g4}, 2\right) + M_{4z} \cdot s\left(x, l_{g4}, 1\right) \right) + \\ + \frac{1}{E \cdot I_3} \cdot \frac{R_{2y}}{2} \cdot s(x, l, 2) + c_{1y}$$

$$y(x) := \frac{1}{E \cdot I_1} \cdot \frac{R_{1y}}{6} \cdot s(x, 0, 3) + \frac{1}{E \cdot I_2} \cdot \left(\frac{-\left(W_{r4y} + W_{t4y}\right)}{6} \cdot s\left(x, l_{g4}, 3\right) + \frac{M_{4z}}{2} \cdot s\left(x, l_{g4}, 2\right) \right) + \\ + \frac{1}{E \cdot I_3} \cdot \frac{R_{2y}}{6} \cdot s(x, l, 3) + c_{1y} \cdot x + c_{2y}$$

$$\theta_y(x) := \frac{1}{E \cdot I_1} \cdot \frac{R_{1z}}{2} \cdot s(x, 0, 2) + \frac{1}{E \cdot I_2} \cdot \left(\frac{\left(W_{t4z} - W_{r4z}\right)}{2} \cdot s\left(x, l_{g4}, 2\right) + M_{4y} \cdot s\left(x, l_{g4}, 1\right) \right) + \\ + \frac{1}{E \cdot I_3} \cdot \frac{R_{2z}}{2} \cdot s(x, l, 2) + c_{1z}$$

$$z(x) := \frac{1}{E \cdot I_1} \cdot \frac{R_{1z}}{6} \cdot s(x, 0, 3) + \frac{1}{E \cdot I_2} \cdot \left(\frac{\left(W_{t4z} - W_{r4z}\right)}{6} \cdot s\left(x, l_{g4}, 3\right) + \frac{M_{4y}}{2} \cdot s\left(x, l_{g4}, 2\right) \right) + \\ + \frac{1}{E \cdot I_3} \cdot \frac{R_{2z}}{6} \cdot s(x, l, 3) + c_{1z} \cdot x + c_{2z}$$

Solve for constants using boundary conditions: (No linear deflection at bearings)

Y-Direction

$y(0) = 0\ \text{in} \qquad y(l) = 0\ \text{in}$

$c_{2y} := 0$

$$c_{1y} := \frac{\frac{1}{E \cdot I_1} \cdot \frac{R_{1y}}{6} \cdot s(l, 0, 3) + \frac{1}{E \cdot I_2} \cdot \left(\frac{-\left(W_{r4y} + W_{t4y}\right)}{6} \cdot s\left(l, l_{g4}, 3\right) + \frac{M_{4z}}{2} \cdot s\left(l, l_{g4}, 2\right) \right)}{-l}$$

$c_{1y} = -0.0006$

Z-Direction

$z(0) = 0\ \text{in} \qquad z(l) = 0\ \text{in}$

$c_{2z} := 0$

$$c_{1z} := \frac{\frac{1}{E \cdot I_1} \cdot \frac{R_{1z}}{6} \cdot s(l, 0, 3) + \frac{1}{E \cdot I_2} \cdot \left(\frac{\left(W_{t4z} - W_{r4z}\right)}{6} \cdot s\left(l, l_{g4}, 3\right) + \frac{M_{4y}}{2} \cdot s\left(l, l_{g4}, 2\right) \right)}{-l}$$

$c_{1z} = 0.0003$

$$\theta_z(x) := \frac{1}{E \cdot I_1} \cdot \frac{R_{1y}}{2} \cdot s(x, 0, 2) + \frac{1}{E \cdot I_2} \cdot \left(\frac{-\left(W_{r4y} + W_{t4y}\right)}{2} \cdot s\left(x, l_{g4}, 2\right) + M_{4z} \cdot s\left(x, l_{g4}, 1\right) \right) + \\ + \frac{1}{E \cdot I_3} \cdot \frac{R_{2y}}{2} \cdot s(x, l, 2) + c_{1y}$$

$$y(x) := \frac{1}{E \cdot I_1} \cdot \frac{R_{1y}}{6} \cdot s(x, 0, 3) + \frac{1}{E \cdot I_2} \cdot \left(\frac{-\left(W_{r4y} + W_{t4y}\right)}{6} \cdot s\left(x, l_{g4}, 3\right) + \frac{M_{4z}}{2} \cdot s\left(x, l_{g4}, 2\right) \right) + \\ + \frac{1}{E \cdot I_3} \cdot \frac{R_{2y}}{6} \cdot s(x, l, 3) + c_{1y} \cdot x + c_{2y}$$

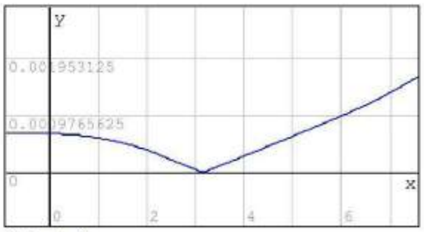
$$\theta_y(x) := \frac{1}{E \cdot I_1} \cdot \frac{R_{1z}}{2} \cdot s(x, 0, 2) + \frac{1}{E \cdot I_2} \cdot \left(\frac{\left(W_{t4z} - W_{r4z}\right)}{2} \cdot s\left(x, l_{g4}, 2\right) + M_{4y} \cdot s\left(x, l_{g4}, 1\right) \right) + \\ + \frac{1}{E \cdot I_3} \cdot \frac{R_{2z}}{2} \cdot s(x, l, 2) + c_{1z}$$

$$z(x) := \frac{1}{E \cdot I_1} \cdot \frac{R_{1z}}{6} \cdot s(x, 0, 3) + \frac{1}{E \cdot I_2} \cdot \left(\frac{\left(W_{t4z} - W_{r4z}\right)}{6} \cdot s\left(x, l_{g4}, 3\right) + \frac{M_{4y}}{2} \cdot s\left(x, l_{g4}, 2\right) \right) + \\ + \frac{1}{E \cdot I_3} \cdot \frac{R_{2z}}{6} \cdot s(x, l, 3) + c_{1z} \cdot x + c_{2z}$$

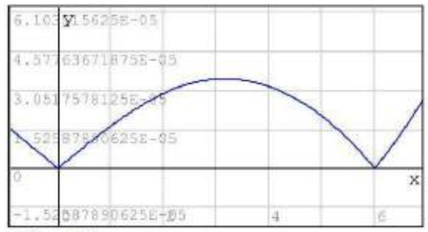
Total Angular Deflection and Deflection

$$\theta(x) := \sqrt{\theta_z(x)^2 + \theta_y(x)^2}$$

$$\delta(x) := \sqrt{y(x)^2 + z(x)^2}$$



$\theta(x\ \text{in})$



$\delta(x\ \text{in})$

Angular Deflection at Bearings

Criteria ≤ 0.004 rad

$\theta_{R1} := \theta(0) = 0.0007\ \text{rad}$ Angular deflection at beginning of shaft

$\theta_{R2} := \theta(l) = 0.001\ \text{rad}$ Angular deflection at end of shaft

Both supports angular deflection does not exceed the allowable value. Therefore, criteria is met!

PASS

Angular Deflection at Gear

Criteria ≤ 0.0005 rad

$\theta_{G4} := \theta(l_{g4}) = 0.0004\ \text{rad}$ Angular deflection at gear 1

Gear angular deflection is less than the allowable value. Therefore, criteria does pass.

PASS

Linear Deflection at Gear

Criteria ≤ 0.127 mm

$\delta_{G4} := \delta(l_{g4}) = 0.0291\ \text{mm}$ Deflection at gear 1

Both gears deflection does not exceed the allowable value. Therefore, criteria is met!

PASS

Angle of Twist

Criteria ≤ 3 deg/m

Only need to check where J is the lowest and there are the most SCFs

$$\phi_{twist} := \frac{T}{G \cdot J_1} = 2.0421\ \frac{\text{deg}}{\text{m}}$$

Angle of twist does not exceeds the allowable value. Therefore, criteria does pass.

PASS

Summary of Results

As a reminder, the evaluation criteria we need to satisfy are:

2. Shaft twist	≤ 3 deg/m
3. Linaer deflection at gears	≤ 0.127 mm
4. Angular deflection at gears	≤ 0.03 deg (0.0005 rad)
5. Angular deflection at bearings	≤ 0.004 rad

Minimum Diameters Summary

At Gear 4: $d_{g4} = 1.5748\ in$ (=40mm)

At Bearing 1: $d_{b1} = 1.378\ in$

At Bearing 2: $d_{b2} = 1.378\ in$ (=35mm)

Evaluation Criteria

$\theta_{R1} = 0.0007$

$\theta_{R2} = 0.001$

$\theta_{G4} = 0.0004$

$\delta_{G4} = 0.0291\ mm$

$\phi_{twist} = 2.0421\ \frac{deg}{m}$

Initial Conclusions

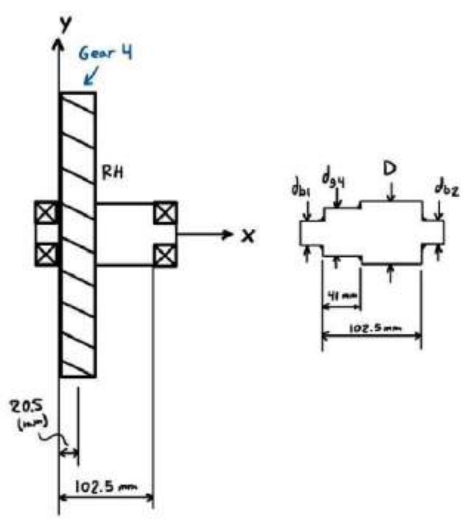
Calculations have been carried out, following a re-itteration method, a minimum diameter at gear 4 of 1.5748 in (40 mm) and a minimum diameter around both bearings of 1.35" has been specified which meets all required conditions and defined evaluation criteria.

Optimized Shaft Length

Problem Statement

The objective of the following calculation is to confim a new design (new shaft length) meets evaluation criteria. Due to the intended use of the shafts to rotate in both directions (drive forward and reverse), need to consider a shaft design that supports the helical gears axially in both directions. Consider a design where the gears are "sandwiched" along the shaft to prevent any axial thrusts (shaft step on one side and support bearing on other).

Problem Diagram



From Gear Analysis, we are given that the gear face width is 41 mm, use this to size the shaft appropriately.

Figure 15. Schematic of design problem: Simply supported shaft with two straddle mounted gears and bearings on each end.

Evaluation Criteria

Same evaluation criteria as orignial shaft analysis

Assumptions

Same assumptions as orignial shaft analysis

New Shaft Lengths

Shaft Length $l := 102.5\ \text{mm} = 4.0354\ \text{in}$

Length to Gear 4 $l_{g4} := 20.5\ \text{mm} = 0.8071\ \text{in}$

Find the forces exerted on the output shaft (gears 4):

Transverse Presure Angle $\phi_t := \text{atan}\left(\frac{\tan(\phi)}{\cos(\psi)}\right) = 21.365\ °$

Loads on Gear 4

$$W_{t4} := \frac{T}{0.5 \cdot d_{p4}} = 1049.02\ \text{lbf}$$

$$W_{r4} := W_{t4} \cdot \tan(\phi_t) = 410.3663\ \text{lbf}$$

$$W_{a4} := W_{t4} \cdot \tan(\psi) = 413.22\ \text{lbf}$$

$$W_4 := \frac{W_{t4}}{\cos(\psi) \cdot \cos(\phi)} = 1199.8309\ \text{lbf}$$

$$M_{4z} := W_{a4} \cdot \frac{d_{p4}}{2} \cdot \sin(\theta_1) = 1035.3212\ \text{lbf in}$$

$$M_{4y} := W_{a4} \cdot \frac{d_{p4}}{2} \cdot \cos(\theta_1) = 1035.3212\ \text{lbf in}$$

$$W_{r4z} := W_{r4} \cdot \cos(\theta_4) = 290.1728\ \text{lbf}$$

$$W_{r4y} := W_{r4} \cdot \sin(\theta_4) = 290.1728\ \text{lbf}$$

$$W_{t4z} := W_{t4} \cdot \cos(90\ ° - \theta_4) = 741.7692\ \text{lbf}$$

$$W_{t4y} := W_{t4} \cdot \sin(90\ ° - \theta_4) = 741.7692\ \text{lbf}$$

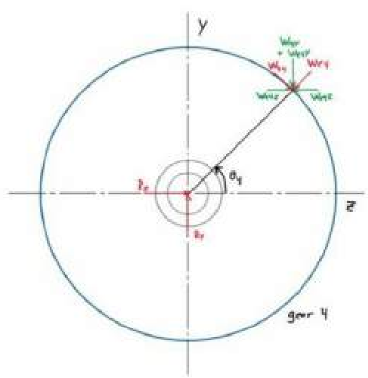


Figure 16. Snapshot of Figure 3.

Reaction Forces

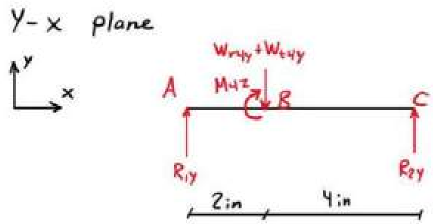


Figure 17. Reaction forces FBD in the y-x plane

$$\circlearrowleft + \sum M_A = 0 \Rightarrow -2(W_{r4y} + W_{t4y}) - M_{4z} + 6R_{2y} = 0$$

$$R_{2y} := \frac{M_{4z} + l_{g4} \cdot (W_{r4y} + W_{t4y})}{l}$$

$$R_{2y} = 462.946 \text{ lbf}$$

$$+\uparrow \sum F_y = 0 \Rightarrow R_{1y} - W_{r4y} - W_{t4y} + R_{2y} = 0$$

$$R_{1y} := W_{r4y} + W_{t4y} - R_{2y}$$

$$R_{1y} = 568.9959 \text{ lbf}$$

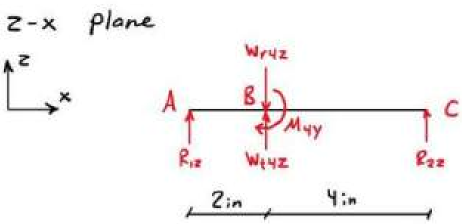


Figure 18. Reaction forces FBD in the z-x plane

$$\circlearrowleft + \sum M_A = 0 \Rightarrow 2(W_{t4z} - W_{r4z}) - M_{4y} + 6R_{2z} = 0$$

$$R_{2z} := \frac{M_{4y} - l_{g4} \cdot (W_{t4z} - W_{r4z})}{l}$$

$$R_{2z} = 166.2384 \text{ lbf}$$

$$+\uparrow \sum F_y = 0 \Rightarrow R_{1z} + W_{t4z} - W_{r4z} + R_{2z} = 0$$

$$R_{1z} := W_{r4z} - W_{t4z} - R_{2z}$$

$$R_{1z} = -617.8347 \text{ lbf}$$

Assume all the axial load is on bearing 1:

$R_{2x} := 0 \text{ lbf}$ $\quad R_{1x} := -W_{a4} = -413.22 \text{ lbf}$

Reaction Force Summary

$R_{1y} = 568.9959 \text{ lbf}$ $\quad R_{1z} = -617.8347 \text{ lbf}$ $\quad R_{1x} = -413.22 \text{ lbf}$

$R_{2y} = 462.946 \text{ lbf}$ $\quad R_{2z} = 166.2384 \text{ lbf}$ $\quad R_{2x} = 0 \text{ lbf}$

Axial load:

$F_a := -R_{1x} = 413.22 \text{ lbf}$

Singularity Function

On X-Y plane:

$$q_y(x) := R_{1y} \cdot s(x, 0, -1) - (W_{r4y} + W_{t4y}) \cdot s(x, l_{g4}, -1) + M_{4z} \cdot s(x, l_{g4}, -2) + R_{2y} \cdot s(x, l, -1)$$

$$v_y(x) := R_{1y} \cdot s(x, 0, 0) - (W_{r4y} + W_{t4y}) \cdot s(x, l_{g4}, 0) + M_{4z} \cdot s(x, l_{g4}, -1) + R_{2y} \cdot s(x, l, 0)$$

$$M_z(x) := R_{1y} \cdot s(x, 0, 1) - (W_{r4y} + W_{t4y}) \cdot s(x, l_{g4}, 1) + M_{4z} \cdot s(x, l_{g4}, 0) + R_{2y} \cdot s(x, l, 1)$$

On X-Z plane:

$$q_z(x) := R_{1z} \cdot s(x, 0, -1) + (W_{t4z} - W_{r4z}) \cdot s(x, l_{g4}, -1) + M_{4y} \cdot s(x, l_{g4}, -2) + R_{2z} \cdot s(x, l, -1)$$

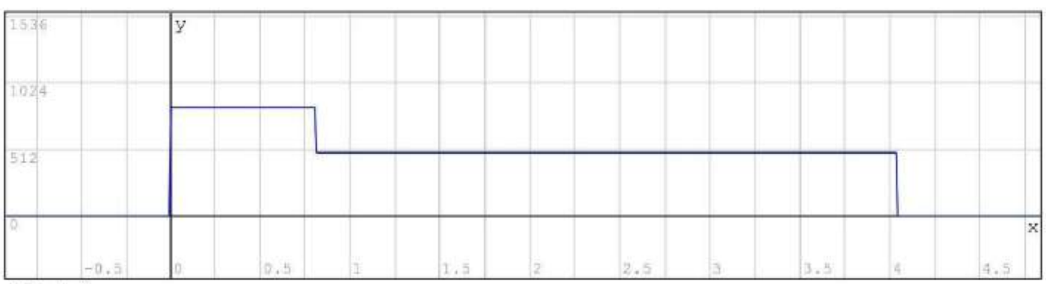
$$v_z(x) := R_{1z} \cdot s(x, 0, 0) + (W_{t4z} - W_{r4z}) \cdot s(x, l_{g4}, 0) + M_{4y} \cdot s(x, l_{g4}, -1) + R_{2z} \cdot s(x, l, 0)$$

$$M_y(x) := R_{1z} \cdot s(x, 0, 1) + (W_{t4z} - W_{r4z}) \cdot s(x, l_{g4}, 1) + M_{4y} \cdot s(x, l_{g4}, 0) + R_{2z} \cdot s(x, l, 1)$$

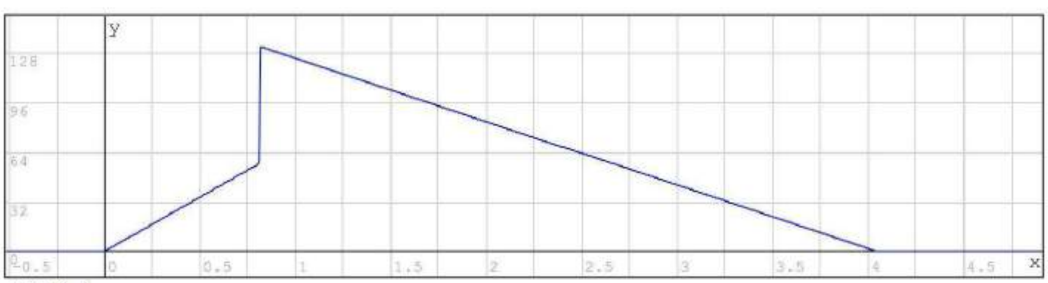
Total V and M:

$$V(x) := \sqrt{v_y(x)^2 + v_z(x)^2}$$

$$M(x) := \sqrt{M_y(x)^2 + M_z(x)^2}$$



$\frac{V(x\ in)}{lbf}$



$\frac{M(x\ in)}{lbf\ ft}$

The resultant moment at gear 4

$$M_{g4} := M(l_{g4}) = 132.3322\ lbf\ ft$$

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Torque on output shaft

Mean and alternating torque from before:

$T_{mean} = 154.875\ \text{lbf ft}$ $\qquad$ $T_{alt} = 154.875\ \text{lbf ft}$

Bending moment will be fully reversed loading.

$M_{mean} := \dfrac{M_{g4} + \left(-M_{g4}\right)}{2} = 0\ \text{lbf ft}$ $\qquad$ $M_{alt} := \dfrac{M_{g4} - \left(-M_{g4}\right)}{2} = 132.3322\ \text{lbf ft}$

Axial force will be constant throughout the output shaft, but fluctuate between 0 (when not in use) and operating shaft torque.

$F_{a'mean} := \dfrac{F_a + 0}{2} = 206.61\ \text{lbf}$ $\qquad$ $F_{a'alt} := \dfrac{F_a - 0}{2} = 206.61\ \text{lbf}$

Minimum diameter of shaft where gear is located:

$d_{g4} = 1.5748\ \text{in}$ $\qquad$ Previously determined minimum diameter

Assume the diameters at the bearings

$d_{b1} = 1.378\ \text{in}$ $\qquad$ $d_{b2} = 1.378\ \text{in}$ $\qquad$ Previously mentioned bearing diameter

Slope and Deflection Calculations

$I_1 := \pi \cdot \dfrac{{d_{b1}}^4}{64} = 0.177\ \text{in}^4$ $\qquad$ $I_2 := \pi \cdot \dfrac{{d_{g4}}^4}{64} = 0.3019\ \text{in}^4$ $\qquad$ $I_3 := \pi \cdot \dfrac{{d_{b2}}^4}{64} = 0.177\ \text{in}^4$

$J_1 := 2 \cdot I_1 = 0.3539\ \text{in}^4$ $\qquad$ $J_2 := 2 \cdot I_2 = 0.6038\ \text{in}^4$ $\qquad$ $J_3 := 2 \cdot I_3 = 0.3539\ \text{in}^4$

Solve for constants using boundary conditions: (No linear deflection at bearings)

Y-Direction

$y(0) = 0\ \text{in}$ $\qquad$ $y(l) = 0\ \text{in}$

$c_{2y} := 0$

$$c_{1y} := \dfrac{\dfrac{1}{E \cdot I_1} \cdot \dfrac{R_{1y}}{6} \cdot s(l, 0, 3) + \dfrac{1}{E \cdot I_2} \cdot \left(\dfrac{-\left(W_{r4y} + W_{t4y}\right)}{6} \cdot s\left(l, l_{g4}, 3\right) + \dfrac{M_{4z}}{2} \cdot s\left(l, l_{g4}, 2\right) \right)}{-l}$$

$c_{1y} = -0.0003$

Z-Direction

$z(0) = 0\ \text{in}$ $\qquad$ $z(l) = 0\ \text{in}$

$$c_{2z} := 0$$

$$c_{1z} := \frac{\frac{1}{E \cdot I_1} \cdot \frac{R_{1z}}{6} \cdot s(l, 0, 3) + \frac{1}{E \cdot I_2} \cdot \left(\frac{\left(W_{t4z} - W_{r4z}\right)}{6} \cdot s\left(l, l_{g4}, 3\right) + \frac{M_{4y}}{2} \cdot s\left(l, l_{g4}, 2\right) \right)}{-l}$$

$$c_{1z} = 9.9945 \cdot 10^{-5}$$

$$\theta_z(x) := \frac{1}{E \cdot I_1} \cdot \frac{R_{1y}}{2} \cdot s(x, 0, 2) + \frac{1}{E \cdot I_2} \cdot \left(\frac{-\left(W_{r4y} + W_{t4y}\right)}{2} \cdot s\left(x, l_{g4}, 2\right) + M_{4z} \cdot s\left(x, l_{g4}, 1\right) \right) + \\ + \frac{1}{E \cdot I_3} \cdot \frac{R_{2y}}{2} \cdot s(x, l, 2) + c_{1y}$$

$$y(x) := \frac{1}{E \cdot I_1} \cdot \frac{R_{1y}}{6} \cdot s(x, 0, 3) + \frac{1}{E \cdot I_2} \cdot \left(\frac{-\left(W_{r4y} + W_{t4y}\right)}{6} \cdot s\left(x, l_{g4}, 3\right) + \frac{M_{4z}}{2} \cdot s\left(x, l_{g4}, 2\right) \right) + \\ + \frac{1}{E \cdot I_3} \cdot \frac{R_{2y}}{6} \cdot s(x, l, 3) + c_{1y} \cdot x + c_{2y}$$

$$\theta_y(x) := \frac{1}{E \cdot I_1} \cdot \frac{R_{1z}}{2} \cdot s(x, 0, 2) + \frac{1}{E \cdot I_2} \cdot \left(\frac{\left(W_{t4z} - W_{r4z}\right)}{2} \cdot s\left(x, l_{g4}, 2\right) + M_{4y} \cdot s\left(x, l_{g4}, 1\right) \right) + \\ + \frac{1}{E \cdot I_3} \cdot \frac{R_{2z}}{2} \cdot s(x, l, 2) + c_{1z}$$

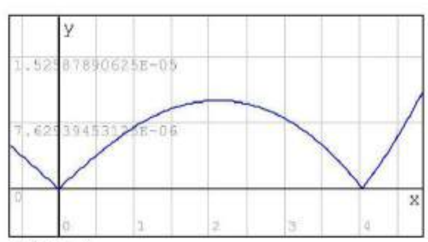
$$z(x) := \frac{1}{E \cdot I_1} \cdot \frac{R_{1z}}{6} \cdot s(x, 0, 3) + \frac{1}{E \cdot I_2} \cdot \left(\frac{\left(W_{t4z} - W_{r4z}\right)}{6} \cdot s\left(x, l_{g4}, 3\right) + \frac{M_{4y}}{2} \cdot s\left(x, l_{g4}, 2\right) \right) + \\ + \frac{1}{E \cdot I_3} \cdot \frac{R_{2z}}{6} \cdot s(x, l, 3) + c_{1z} \cdot x + c_{2z}$$

Total Angular Deflection and Deflection

$$\theta(x) := \sqrt{\theta_z(x)^2 + \theta_y(x)^2}$$

$$\delta(x) := \sqrt{y(x)^2 + z(x)^2}$$

$\theta(x \text{ in})$



$\delta(x \text{ in})$

Angular Deflection at Bearings

Criteria ≤ 0.004 rad

$\theta_{R1} := \theta(0) = 0.0003\ rad$ Angular deflection at beginning of shaft

$\theta_{R2} := \theta(l) = 0.0004\ rad$ Angular deflection at end of shaft

Both supports angular deflection does not exceed the allowable value. Therefore, criteria is met!

PASS

Angular Deflection at Gear

Criteria ≤ 0.0005 rad

$\theta_{G4} := \theta(l_{g4}) = 0.0003\ rad$ Angular deflection at gear 1

Gear angular deflection is less than the allowable value. Therefore, criteria does pass.

PASS

Linear Deflection at Gear

Criteria ≤ 0.127 mm

$\delta_{G4} := \delta(l_{g4}) = 0.0058\ mm$ Deflection at gear 1

Both gears deflection does not exceed the allowable value. Therefore, criteria is met!

PASS

Angle of Twist

Criteria ≤ 3 deg/m

Only need to check where J is the lowest and there are the most SCFs

$$\phi_{twist} := \frac{T}{G \cdot J_1} = 2.0421\ \frac{deg}{m}$$

Angle of twist does not exceeds the allowable value. Therefore, criteria does pass.

PASS

Summary of Results

As a reminder, the evaluation criteria we need to satisfy are:

1. Shaft twist	≤ 3 deg/m
2. Linaer deflection at gears	≤ 0.127 mm
3. Angular deflection at gears	≤ 0.03 deg (0.0005 rad)
4. Angular deflection at bearings	≤ 0.004 rad

Minimum Diameters Summary

At Gear 4: $d_{g4} = 1.5748\ in$ (=40mm)

At Bearing 1: $d_{b1} = 1.378\ in$

At Bearing 2: $d_{b2} = 1.378\ in$ (35mm)

Evaluation Criteria

$\theta_{R1} = 0.0003$

$\theta_{R2} = 0.0004$

$\theta_{G4} = 0.0003$

$\delta_{G4} = 0.0058\ mm$

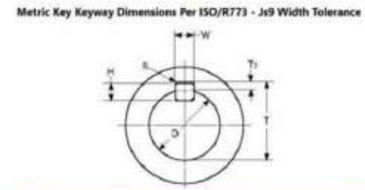
$\phi_{twist} = 2.0421\ \frac{deg}{m}$

Given that all criteria are now satisfied, safety factors can be recalculated for new diameter. We need to find new notch radius and large diameter.

Key SCF

$r_{key} = 0.01\ in$ Initial assumption still satisfies new diameter

$K_f = 1.9 \qquad K_{fs} = 2.64$



Key & Keyway Dimensions - Millimeters										
Shaft Diameter "D"		Key Size Nominal		Keyway Width Hub "W"			Keyway Depth Hub "T2"		Keyway Radius "R"	
Over	Thru	Width "W"	Height "H"	Nominal	Min	Max	Min	Max	Min	Max
22	30	8	7	8	-.0180	+0180	3.3	3.5	0.16	0.25
30	38	10	8	10	-.0180	+0180	3.3	3.5	0.25	0.40
38	44	12	8	12	-.0215	+.0215	3.3	3.5	0.25	0.40
44	50	14	9	14	-.0215	+.0215	3.8	4.0	0.25	0.40

Figure 19. Standard Key and Keyway dimension (used to find rkey) [4]

Check the SCF of the keyway is greater than the SCF of the step

$D := 50\ mm$ new large diameter

$r_{shoulder} := 0.1 \cdot d_{g4} = 4\ mm$ assume ratio of r/d = 0.1

$\frac{D}{d_{g4}} = 1.25$

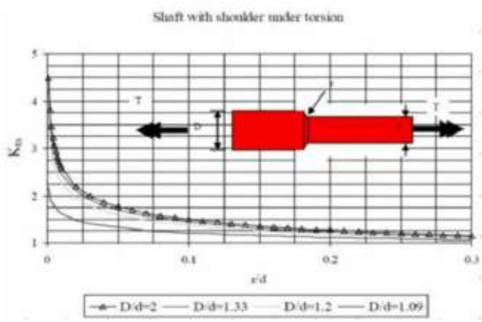


Figure 20. Step Shaft = Shaft with shoulder under torsion [4]

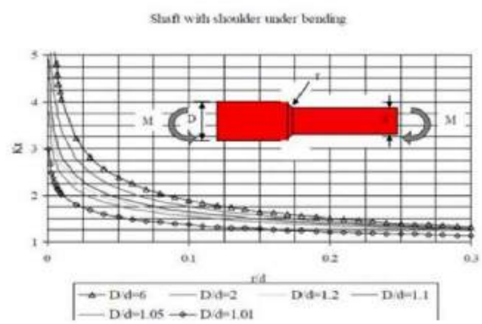


Figure 21. Step Shaft = Shaft with shoulder under bending [4]

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$K_{ts} := 1.2 \qquad K_t := 1.6$

Step SCF: Keyway SCF:

$K_{f2} := 1 + q \cdot (K_t - 1) = 1.54 \quad \le \quad K_f = 1.9$ Therefore, keyway SCF is still the main failure point

$K_{fs2} := 1 + q_s \cdot (K_{ts} - 1) = 1.185 \quad \le \quad K_{fs} = 2.64$

As previously stated in the initial length analysis, we can assume Kf = Kfm and Kfs = Kfsm

$K_{fm} := K_f = 1.9 \qquad K_{fsm} := K_{fs} = 2.64$

Fatigue and Yield Safety Factor Calculations

Although there are many failure theroies available for design criteria considerations, in this case the yielding safety factor will be calculated along with a fatigue safety factor. The fatigue safety factor will be found according to AGMA, guidelines suggest use of DE Elliptic criteria for evaluation of shaft failure:

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DE Elliptic Fatigue Safety Factor

$$n_f = \frac{1}{\sqrt{\left(\frac{\sigma'_{alt}}{S_e}\right)^2 + \left(\frac{\sigma'_{mean}}{S_y}\right)^2}}$$

Yield Safety Factor

$$n_y = \frac{1}{\frac{\sigma'_{alt}}{S_y} + \frac{\sigma'_{mean}}{S_y}}$$

$$d_{min} := d_{g4} = 1.5748 \text{ in}$$

Alternating and Mean Von Mises Stresses

$$\sigma'_{alt} := \left(\left(\frac{32 \cdot K_f \cdot M_{alt}}{\pi \cdot d_{min}^3} + \frac{4 \cdot K_f \cdot F_{a'alt}}{\pi \cdot d_{min}^2}\right)^2 + 3 \cdot \left(\frac{16 \cdot K_{fs} \cdot T_{alt}}{\pi \cdot d_{min}^3}\right)^2\right)^{\frac{1}{2}}$$

$$\sigma'_{mean} := \left(\left(\frac{32 \cdot K_{fm} \cdot M_{mean}}{\pi \cdot d_{min}^3} + \frac{4 \cdot K_{fm} \cdot F_{a'mean}}{\pi \cdot d_{min}^2}\right)^2 + 3 \cdot \left(\frac{16 \cdot K_{fsm} \cdot T_{mean}}{\pi \cdot d_{min}^3}\right)^2\right)^{\frac{1}{2}}$$

$$n_f := \frac{1}{\sqrt{\left(\frac{\sigma'_{alt}}{S_e}\right)^2 + \left(\frac{\sigma'_{mean}}{S_y}\right)^2}}$$

$$n_y := \frac{1}{\frac{\sigma'_{alt}}{S_y} + \frac{\sigma'_{mean}}{S_y}}$$

$$n_f = 2.9595$$

$$n_y = 5.7677$$

Conclusion

Calculations have been carried out, following a re-itteration method, a minimum diameter at gear 4 of 1.5748" (40 mm) and a minimum diameter around both bearings of 1.3780" (35 mm) has been specified which meets all required conditions and defined evaluation criteria. Additionally, the fatigue and yield safety factors were recalculated to be 2.96 and 5.77, respectively for this shaft diameter. It can be noted that the obtatined safety factors are both greater than the design safety factor of 1.872, indicating the shaft is safe for use.

References

[1] "AISI 4140 Steel, oil quenched, 25 mm round [845C quench, 540C tempered]," MatWeb. Accessed on: Nov 1, 2024. [Online]. Available: https://www.matweb.com/search/DataSheet.aspx?MatGUID=07d1795c3f034c97b52cccda78ae1409

[2] D. Romanyk, Class Lecture, Topic: "Singularity Functions." Faculty of Engineering, University of Alberta, Edmonton, Alberta, 2024

[3] D. Romanyk, Class Lecture, Topic: "Shaft Analysis." Faculty of Engineering, University of Alberta, Edmonton, Alberta, 2024

[4] D. Romanyk, Class Lecture, Topic: "Stress Concentration Factors." Faculty of Engineering, University of Alberta, Edmonton, Alberta, 2024

[5] "Keyway Chart," Hallite. Accessed on Nov 7, 2024. [Online]. Available: https://hallite.com/au/hallite-transeals/transmission-products/keyway-chart/

Keyway Analysis

The following analysis ensures that the keys and keyways in the Birdie Boys transmission can safely handle the applied forces. Keys play a critical role in preventing tangential slippage of gears on shafts, ensuring reliable and efficient power transmission.

Note: Keys will be purchased as keystock from McMASTER-CARR [1] and cut to the required length.

Shaft Diameter (in)	Nominal Key Width (in)	Setscrew Dia. (in)	Shaft Diameter (mm)	Key Width x Height (mm)
$0.312 < d \le 0.437$	0.093	#10	$8 < d \le 10$	3 x 3
$0.437 < d \le 0.562$	0.125	#10	$10 < d \le 12$	4 x 4
$0.562 < d \le 0.875$	0.187	0.250	$12 < d \le 17$	5 x 5
$0.875 < d \le 1.250$	0.250	0.312	$17 < d \le 22$	6 x 6
$1.250 < d \le 1.375$	0.312	0.375	$22 < d \le 30$	8 x 7
$1.375 < d \le 1.750$	0.375	0.375	$30 < d \le 38$	10 x 8
$1.750 < d \le 2.250$	0.500	0.500	$38 < d \le 44$	12 x 8
$2.250 < d \le 2.750$	0.625	0.500	$44 < d \le 50$	14 x 9
$2.750 < d \le 3.250$	0.750	0.625	$50 < d \le 58$	16 x 10
$3.250 < d \le 3.750$	0.875	0.750	$58 < d \le 65$	18 x 11
$3.750 < d \le 4.500$	1.000	0.750	$65 < d \le 75$	20 x 12
$4.500 < d \le 5.500$	1.250	0.875	$75 < d \le 85$	22 x 14
$5.500 < d \le 6.500$	1.500	1.000	$85 < d \le 95$	25 x 14

Table 9-2: Machine Design, Norton RL, 3rd ed.

Figure 1: This table is used to size keys for all three shafts of the Birdie Boys transmission [2]

Inputs from Shaft Analysis: Transverse Pressure Angle: $\phi_t := 21.365\ °$ Helix Angle: $\psi := 21.5\ °$

Gear 1:

$d_{p1} := 60\ mm = 2.3622\ in$ $\quad T_1 := 41.3\ lbf\ ft$ $\quad$ (shaft diameter) $d_1 := 30\ mm = 1.1811\ in$

$$W_{t1} := \frac{T_1}{0.5 \cdot d_{p1}} = 419.608\ lbf \qquad W_{r1} := W_{t1} \cdot \tan(\phi_t) = 152.7248\ lbf \qquad W_{a1} := W_{t1} \cdot \tan(\psi) = 165.288\ lbf$$

Gear 2:

$d_{p2} := 180\ mm = 7.0866\ in$ $\quad T_2 := 123.9\ lbf\ ft$ $\quad$ (shaft diameter) $d_2 := 32\ mm = 1.2598\ in$

$$W_{t2} := \frac{T_2}{0.5 \cdot d_{p2}} = 419.608\ lbf \qquad W_{r2} := W_{t2} \cdot \tan(\phi_t) = 152.7248\ lbf \qquad W_{a2} := W_{t2} \cdot \tan(\psi) = 165.288\ lbf$$

Gear 3:

$d_{p3} := 72\ mm = 2.8346\ in$ $\quad T_3 := 123.9\ lbf\ ft$ $\quad$ (shaft diameter) $d_3 := 32\ mm = 1.2598\ in$

$$W_{t3} := \frac{T_3}{0.5 \cdot d_{p3}} = 1049.02\ lbf \qquad W_{r3} := W_{t3} \cdot \tan(\phi_t) = 381.8121\ lbf \qquad W_{a3} := W_{t3} \cdot \tan(\psi) = 413.22\ lbf$$

Gear 4:

$d_{p4} := 180\ mm = 7.0866\ in$ $\quad T_4 := 309.75\ lbf\ ft$ $\quad$ (shaft diameter) $d_4 := 40\ mm = 1.5748\ in$

$$W_{t4} := \frac{T_4}{0.5 \cdot d_{p4}} = 1049.02\ lbf \qquad W_{r4} := W_{t4} \cdot \tan(\phi_t) = 381.8121\ lbf \qquad W_{a4} := W_{t4} \cdot \tan(\psi) = 413.22\ lbf$$

Key dimensions for all gears (from figure 1):

Gear 1:	**Gear 2:**	**Gear 3:**	**Gear 4:**
$w_1 := 8\ mm = 0.315\ in$	$w_2 := 10\ mm = 0.3937\ in$	$w_3 := 10\ mm = 0.3937\ in$	$w_4 := 12\ mm = 0.4724\ in$
$h_1 := 7\ mm = 0.2756\ in$	$h_2 := 8\ mm = 0.315\ in$	$h_3 := 8\ mm = 0.315\ in$	$h_4 := 8\ mm = 0.315\ in$
$l_1 := 38\ mm = 1.4961\ in$	$l_2 := 38\ mm = 1.4961\ in$	$l_3 := 38\ mm = 1.4961\ in$	$l_4 := 38\ mm = 1.4961\ in$

Material Yield Strength (1045 Carbon Steel)

$S_y := 60.9\ kpsi$

Keys can fail in two ways: shear or bearing

Shear failure occurs when the key is split in half along its width at the interface between the shaft and the hub.

$\tau_{shear} := \frac{W_t}{A_{shear}}$ $A_{shear} := 2 \cdot (l \cdot w)$ There are two planes of shear, top of key (gear) as well as the bottom of key (shaft)

Bearing failure occurs when the key is compressed against the gear hub or shaft.

$\sigma_{bearing} := \frac{W_t + W_a}{A_{bearing}}$ $A_{bearing} := l \cdot \frac{h}{2}$ Key sits halfway into the shaft, the contact area of the key with the shaft and the hub will be equal

Both stresses will be compared to yield stress to check for failure (Von Mises).

Check for key failure at gear 1:

$A_{shear1} := 2 \cdot (l_1 \cdot w_1) = 0.9424\ in^2$ $A_{bearing1} := l_1 \cdot \left(\frac{h_1}{2}\right) = 0.2062\ in^2$

$\tau_{shear1} := \frac{W_{t1}}{A_{shear1}} = 0.4453\ kpsi$ $\sigma_{bearing1} := \frac{W_{t1} + W_{a1}}{A_{bearing1}} = 2.8372\ kpsi$

$\sigma'_1 := \sqrt{\sigma_{bearing1}^2 + 3 \cdot \tau_{shear1}^2} = 2.9402\ kpsi$

Since $\sigma'_1 = 2.9402\ kpsi$ is less than $S_y = 60.9\ kpsi$, the key at gear 1 will not fail.

Check for key failure at gear 2:

$A_{shear2} := 2 \cdot \left(l_2 \cdot w_2\right) = 1.178\ \text{in}^2$

$A_{bearing2} := l_2 \cdot \left(\frac{h_2}{2}\right) = 0.2356\ \text{in}^2$

$\tau_{shear2} := \frac{W_{t2}}{A_{shear2}} = 0.3562\ \text{kpsi}$

$\sigma_{bearing2} := \frac{W_{t2} + W_{a2}}{A_{bearing2}} = 2.4826\ \text{kpsi}$

$\sigma'_2 := \sqrt{\sigma_{bearing2}^2 + 3 \cdot \tau_{shear2}^2} = 2.5581\ \text{kpsi}$

Since $\sigma'_2 = 2.5581\ \text{kpsi}$ is less than $S_y = 60.9\ \text{kpsi}$, the key at gear 2 will not fail.

Check for key failure at gear 3:

$A_{shear3} := 2 \cdot \left(l_3 \cdot w_3\right) = 1.178\ \text{in}^2$

$A_{bearing3} := l_3 \cdot \left(\frac{h_3}{2}\right) = 0.2356\ \text{in}^2$

$\tau_{shear3} := \frac{W_{t3}}{A_{shear3}} = 0.8905\ \text{kpsi}$

$\sigma_{bearing3} := \frac{W_{t3} + W_{a3}}{A_{bearing3}} = 6.2064\ \text{kpsi}$

$\sigma'_3 := \sqrt{\sigma_{bearing3}^2 + 3 \cdot \tau_{shear3}^2} = 6.3952\ \text{kpsi}$

Since $\sigma'_3 = 6.3952\ \text{kpsi}$ is less than $S_y = 60.9\ \text{kpsi}$, the key at gear 3 will not fail.

Check for key failure at gear 4:

$A_{shear4} := 2 \cdot \left(l_4 \cdot w_4\right) = 1.4136\ \text{in}^2$

$A_{bearing4} := l_4 \cdot \left(\frac{h_4}{2}\right) = 0.2356\ \text{in}^2$

$\tau_{shear4} := \frac{W_{t4}}{A_{shear4}} = 0.7421\ \text{kpsi}$

$\sigma_{bearing4} := \frac{W_{t4} + W_{a4}}{A_{bearing4}} = 6.2064\ \text{kpsi}$

$\sigma'_4 := \sqrt{\sigma_{bearing4}^2 + 3 \cdot \tau_{shear4}^2} = 6.3381\ \text{kpsi}$

Since $\sigma'_4 = 6.3381\ \text{kpsi}$ is less than $S_y = 60.9\ \text{kpsi}$, the key at gear 4 will not fail.

References

[1] "Machine Key Stock," McMASTER-CARR. Accessed on: Dec 5, 2024. [Online]. Available: https://www.mcmaster.com/products/keys/machine-key-stock-2~/

[2] D. Romanyk, Class Lecture, Topic: "Shaft Analysis." Faculty of Engineering, University of Alberta, Edmonton, Alberta, 2024

Calculated the reaction forces independently from shaft analysis, but decided to take the resultant forces from shaft analysis for the idler and output bearings analysis to not repeat the work that's been done already

Given Variables:

$\omega := 3000\ \text{rpm} = 314.1593\ \frac{\text{rad}}{\text{s}}$ $\quad L_1 := 82\ \text{mm} = 3.2283\ \text{in}$ $\quad L_2 := 102.5\ \text{mm} = 4.0354\ \text{in}$

Gear Analysis Summary: Pressure Angle $\phi := 20\ °$ Helix Angle $\psi := 21.5\ °$ $\quad \phi_t := \text{atan}\left(\frac{\tan(\phi)}{\cos(\psi)}\right) = 21.365\ \text{deg}$

Number of Teeth	Module	Pitch Diameter	
$N_1 := 20\ \text{teeth}$	$m_1 := 3\ \frac{\text{mm}}{\text{teeth}}$	$d_{p1} := m_1 \cdot N_1 = 60\ \text{mm}$	$r_{p1} := \frac{d_{p1}}{2} = 0.03\ \text{m}$
$N_2 := 60\ \text{teeth}$	$m_2 := 3\ \frac{\text{mm}}{\text{teeth}}$	$d_{p2} := m_2 \cdot N_2 = 180\ \text{mm}$	$r_{g1} := \frac{d_{p2}}{2} = 0.09\ \text{m}$
$N_3 := 24\ \text{teeth}$	$m_3 := 3\ \frac{\text{mm}}{\text{teeth}}$	$d_{p3} := m_3 \cdot N_3 = 72\ \text{mm}$	$r_{p2} := \frac{d_{p3}}{2} = 0.036\ \text{m}$
$N_4 := 60\ \text{teeth}$	$m_4 := 3\ \frac{\text{mm}}{\text{teeth}}$	$d_{p4} := m_4 \cdot N_4 = 180\ \text{mm}$	$r_{g2} := \frac{d_{p4}}{2} = 0.09\ \text{m}$
$T_{in} := 56\ \text{N m}$			

$$W_{t1} := \frac{T_{in}}{r_{p1}} = 1866.6667\ \text{N}$$

Input Shaft Bearing Force Summary

Force from gear 1 on input shaft:

$W_{t1} = 419.6434\ \text{lbf}$ $\quad W_{r1} := W_{t1} \cdot \tan(\phi_t) = 164.1604\ \text{lbf}$ $\quad W_{a1} := W_{t1} \cdot \tan(\psi) = 165.3019\ \text{lbf}$

For conservative analysis, one of the bearings will solely take all the reaction axial force.

a) Left bearing taking all axial force

FBD

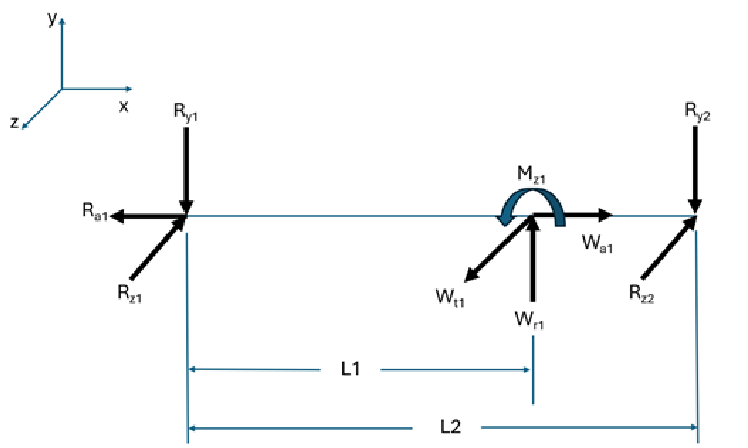

Figure 1. Free Body Diagram (FBD) of design problem with Left Bearing taking all reaction axial force

$M_{z1} := W_{a1} \cdot r_{p1}$ $\quad M_{z1} = 195.2385\ \text{lbf in}$

Sum of moment about z-axis:

$$R_{y2} := \frac{\left(M_{z1} + \left(W_{r1} \cdot L_1\right)\right)}{L_2}$$

$R_{y2} = 179.7093\ \text{lbf}$

[1] SK bearings and mounted products

Therefore, knowing sum of all forces in y-axis is equal to 0:

$$R_{y1} := W_{r1} - R_{y2}$$

$$R_{y1} = -15.549 \text{ lbf}$$

Sum of moment about y-axis:

$$R_{z2} := \frac{W_{t1} \cdot L_1}{L_2} \qquad R_{z2} = 335.7147 \text{ lbf}$$

Therefore, knowing sum of all forces in z-axis is equal to 0:

$$R_{z1} := W_{t1} - R_{z2}$$

$$R_{z1} = 83.9287 \text{ lbf}$$

Now calculating the Radial Force at the left bearing:

$$R_r := \sqrt{R_{z1}^{\ 2} + R_{y1}^{\ 2}}$$

$$R_r = 85.3569 \text{ lbf}$$

Axial Force at left bearing:

$$R_{a1} := W_{a1}$$

$$R_{a1} = 165.3019 \text{ lbf}$$

Radial Load Analysis Factors and Equation

$$L := 166.32$$

$$a := 3$$

$$P := R_r$$

$$K_r := 0.62$$

Reliability 95% (Consistent with Shaft. Assume Bearing replaced when shaft needs to be replaced

$$C_r := P \cdot \left(\frac{L}{K_r}\right)^{\frac{1}{3}}$$

$$C_r = 550.4983 \text{ lbf}$$ Dynamic Load

Thefore the static load is:

$$C_0 := 760 \text{ lbf}$$

Corresponding bearing is 6300

Designation	Principal dimensions						Basic load ratings				Speed rating[1]			Mass	
	Bore d		Outside dimeter D		Width B		Dynamic C		Static C_0		Reference speed	Limiting speed	Sealed Limiting speed		
	mm	in	mm	in	mm	in	N	lbf	N	lbf	r/min	r/min	r/min	kg	lb
634	4	0.1575	16	0.6299	5	0.1969	1 110	250	380	90	95 000	60 000	28 000	0.01	0.01
635	5	0.1969	19	0.7480	6	0.2362	2 340	530	950	210	80 000	50 000	24 000	0.01	0.02
6300	10	0.3937	35	1.3780	11	0.4331	8 520	1 910	3 400	760	50 000	32 000	15 000	0.05	0.12
6301	12	0.4724	37	1.4567	12	0.4724	10 100	2 270	4 150	930	45 000	28 000	14 000	0.06	0.13
6302	15	0.5906	42	1.6535	13	0.5118	11 900	2 670	5 400	1 210	38 000	24 000	12 000	0.08	0.18
6303	17	0.6693	47	1.8504	14	0.5512	14 300	3 210	6 550	1 470	34 000	22 000	11 000	0.11	0.25
6304	20	0.7874	52	2.0472	15	0.5906	16 800	3 780	7 800	1 750	30 000	19 000	9 500	0.14	0.32
6305	25	0.9843	62	2.4409	17	0.6693	23 400	5 260	11 600	2 610	24 000	16 000	7 500	0.23	0.50
6306	30	1.1811	72	2.8346	19	0.7480	29 600	6 650	16 000	3 600	20 000	13 000	6 300	0.35	0.77
6307	35	1.3780	80	3.1496	21	0.8268	35 100	7 890	19 000	4 270	19 000	12 000	6 000	0.46	1.01
6308	40	1.5748	90	3.5433	23	0.9055	42 300	9 510	24 000	5 390	17 000	11 000	5 000	0.63	1.38
6309	45	1.7717	100	3.9370	25	0.9843	55 300	12 430	31 500	7 080	15 000	9 500	4 500	0.84	1.84
6310	50	1.9685	110	4.3307	27	1.0630	65 000	14 610	38 000	8 540	13 000	8 500	4 300	1.08	2.38
6311	55	2.1654	120	4.7244	29	1.1417	74 100	16 650	45 000	10 110	12 000	8 000	3 800	1.37	3.03
6312	60	2.3622	130	5.1181	31	1.2205	85 200	19 150	52 000	11 690	11 000	7 000	3 400	1.72	3.80
6313	65	2.5591	140	5.5118	33	1.2992	97 500	21 910	60 000	13 480	10 000	6 700	3 200	2.11	4.64
6314	70	2.7559	150	5.9055	35	1.3780	111 000	24 940	68 000	15 280	9 500	6 300	3 000	2.55	5.62
6315	75	2.9528	160	6.2992	37	1.4567	119 000	26 740	76 500	17 190	9 000	5 600	2 800	3.06	6.74
6316	80	3.1496	170	6.6929	39	1.5354	130 000	29 210	86 500	19 440	8 500	5 300	2 600	3.63	8.00
6317	85	3.3465	180	7.0866	41	1.6142	140 000	31 460	96 500	21 690	8 000	5 000	2 400	4.25	9.38
6318	90	3.5433	190	7.4803	43	1.6929	151 000	33 930	108 000	24 270	7 500	4 800	2 400	4.97	10.96
6319	95	3.7402	200	7.8740	45	1.7717	159 000	35 730	118 000	26 520	7 000	4 500	2 200	5.75	12.67
6320	100	3.9370	215	8.4646	47	1.8504	174 000	39 100	140 000	31 460	6 700	4 300	2 000	7.08	15.60
6321	105	4.1339	225	8.8583	49	1.9291	182 000	40 900	153 000	34 380	6 300	4 000	–	8.18	18.03
6322	110	4.3307	240	9.4488	50	1.9685	203 000	45 620	180 000	40 450	6 000	3 800	1 800	9.66	21.31
6324	120	4.7244	260	10.2362	55	2.1654	208 000	46 740	186 000	41 800	5 600	3 400	1 700	12.66	27.92

Figure 2. Bearing Catalogue SKF [1]

[1] SKF bearings and mounted products

Calculate F_a/C_0:

$$\frac{R_{a1}}{C_0} = 0.2175$$

Therefore:

$$0.34 < e \quad \text{and} \quad e < 0.38$$

Check value of F_a/VF_r (V is 1):

$$\frac{R_{a1}}{R_r} = 1.9366$$ Much bigger than e

Now define X and Y to re-iterate:

$$X := 0.56$$

$$Y := 1.24090909091$$

$$F_{eff} := (X \cdot R_r) + (Y \cdot R_{a1}) = 252.9245 \text{ lbf}$$

Find new C_r:

$$C_r := F_{eff} \cdot \left(\frac{L}{K_r}\right)^{\frac{1}{3}} = 1631.2047 \text{ lbf}$$

Thefore the static load is:

$$C_0 := 760 \text{ lbf}$$

Bearing Type 3)	4) $\frac{F_a}{C_0}$	5) $\frac{F_a}{iZD_w^2}$	In Relation to the Load the Inner Ring is Rotating V	Stationary V	Single Row Bearings 1) $\frac{F_a}{VF_r} > e$ X	Y	Double Row Bearings 2) $\frac{F_a}{VF_r} \leq e$ X	Y	$\frac{F_a}{VF_r} > e$ X	Y	e
Radial Contact Groove Ball Bearings	0.014	25				2.30				2.30	0.19
	0.028	50				1.99				1.99	0.22
	0.056	100				1.71				1.71	0.26
	0.084	150				1.55				1.55	0.28
	0.11	200	1	1.2	0.56	1.45	1	0	0.56	1.45	0.30
	0.17	300				1.31				1.31	0.34
	0.28	500				1.15				1.15	0.38
	0.42	750				1.04				1.04	0.42
	0.56	1000				1.00				1.00	0.44
20°					0.43	1.00		1.09	0.70	1.63	0.57
25°					0.41	0.87		0.92	0.67	1.44	0.68
30°			1	1.2	0.39	0.76	1	0.78	0.63	1.24	0.80
35°					0.37	0.66		0.66	0.60	1.07	0.95
40°					0.35	0.57		0.55	0.57	0.93	1.14
Self-Aligning Ball Bearings			1	1	0.40	0.4 cot α	1	0.42 cot α	0.65	0.65 cot α	1.5 tan α
Self-Aligning and Tapered Roller Bearings			1	1.2	0.40	0.4 cot α	1	0.45 cot α	0.67	0.67 cot α	1.5 tan α

Figure 3. Equivalent Single Load Table for X and Y Factor [2]

Therefore, the Left bearing is <u>6300 but up the size to 6305 as it's the closest to gear bore diameter of 30mm</u>

b) Right bearing taking all axial force

FBD

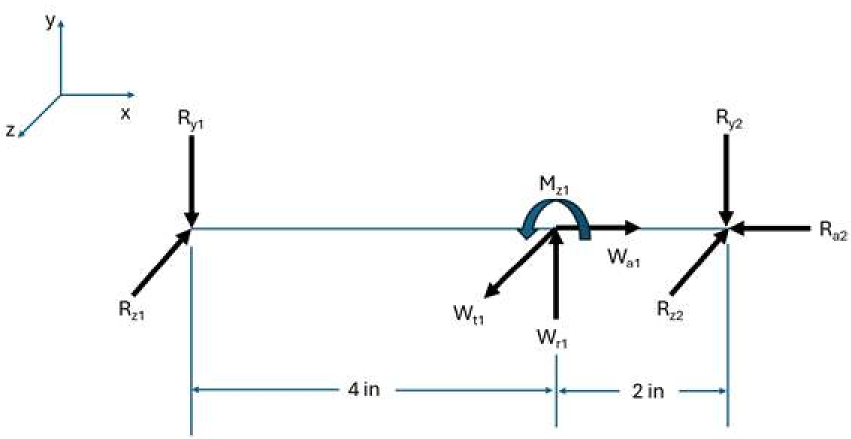

Figure 4. Free Body Diagram (FBD) of design problem with Right Bearing taking all reaction axial force

All reaction forces in y and z-direction would be the same, except the axial reaction force

[2] MecE 360: Mechanical Design II Rolling Bearing Elements and Analysis Sheet 24 of 28

$R_{a2} := W_{a1}$

$R_{a2} = 165.3019\ lbf$

Now calculating the Radial Force at the left bearing:

$R_{r2} := \sqrt{R_{y2}^{2} + R_{z2}^{2}}$

$R_{r2} = 380.7884\ lbf$

Radial Load Analysis Factors and Equation

$L := 166.32$

$P := R_{r2}$

$K_r := 0.62$ Reliability 95%

$a := 3$

$$C_r := P \cdot \left(\frac{L}{K_r}\right)^{\frac{1}{3}}$$

$C_r = 2455.847\ lbf$ Dynamic Load

Thefore the static load is:

$C_0 := 1210\ lbf$

Corresponding bearing is 6302

Calculate F_a/C_0:

$$\frac{R_{a2}}{C_0} = 0.1366$$

Therefore:

$0.30 < e$ and $e < 0.34$

Check value of F_a/VF_r (V is 1):

$$\frac{R_{a2}}{R_{r2}} = 0.4341$$

This is bigger than e

Now define X and Y to re-iterate:

$X := 0.56$

$Y := 1.387933333333$

$F_{eff} := \left(X \cdot R_{r2}\right) + \left(Y \cdot R_{a2}\right) = 442.6695\ lbf$

Find new C_r:

$$C_r := F_{eff} \cdot \left(\frac{L}{K_r}\right)^{\frac{1}{3}} = 2854.9416\ lbf$$

Thefore the static load is:

$C_0 := 1470\ lbf$

Corresponding bearing is 6303

Calculate F_a/C_0:

$$\frac{R_{a2}}{C_0} = 0.1125$$

Therefore:

$0.30 < e$ and $e < 0.34$

This is still smaller than F_a/VF_r

So define new X and Y to reiterate:

$Y := 1.444166666667$

$$F_{eff} := \left(X \cdot R_{r2}\right) + \left(Y \cdot R_{a2}\right) = 451.965\ lbf$$

Find new C_r:

$$C_r := F_{eff} \cdot \left(\frac{L}{K_r}\right)^{\frac{1}{3}} = 2914.8917\ lbf$$

Thefore the static load is:

$C_0 := 1470\ lbf$

Corresponding bearing is 6303

Therefore, the Right bearing is 6303 but up the size to 6305 as it's the closest to gear bore diameter of 30mm

Given Variables from idler shaft anaysis:

$\omega := 3000\ \text{rpm} = 314.1593\ \frac{\text{rad}}{\text{s}}$

$R_{y1} := -412.3262\ \text{lbf}$ $R_{y2} := -206.839\ \text{lbf}$ $R_a := 247.932\ \text{lbf}$

$R_{z1} := 361.2771\ \text{lbf}$ $R_{z2} := 270.9578\ \text{lbf}$

a) Left bearing taking all axial force

$R_{a1} := R_a$

$R_r := \sqrt{R_{y1}^2 + R_{z1}^2} = 548.2098\ \text{lbf}$

Radial Load Analysis Factors and Equation

$L := \frac{166.32}{3}$ one third of input shaft cycle

$a := 3$

$P := R_r$

$K_r := 0.62$ Reliability 95%

$$C_r := P \cdot \left(\frac{L}{K_r}\right)^{\frac{1}{3}}$$

$C_r = 2451.4555\ \text{lbf}$ Dynamic Load

Thefore the static load is:

$C_0 := 1210\ \text{lbf}$

Corresponding bearing is 6302

Designation	Principal dimensions						Basic load ratings				Speed rating[1]			Mass	
	Bore d		Outside dimeter D		Width B		Dynamic C		Static C0		Reference speed	Limiting speed	Sealed Limiting speed		
	mm	in	mm	in	mm	in	N	lbf	N	lbf	r/min	r/min	r/min	kg	lb
634	4	0.1575	16	0.6299	5	0.1969	1 110	250	380	90	95 000	60 000	28 000	0.01	0.01
635	5	0.1969	19	0.7480	6	0.2362	2 340	530	950	210	80 000	50 000	24 000	0.01	0.02
6300	10	0.3937	35	1.3780	11	0.4331	8 520	1 910	3 400	760	50 000	32 000	15 000	0.05	0.12
6301	12	0.4724	37	1.4567	12	0.4724	10 100	2 270	4 150	930	45 000	28 000	14 000	0.06	0.13
6302	15	0.5906	42	1.6535	13	0.5118	11 900	2 670	5 400	1 210	38 000	24 000	12 000	0.08	0.18
6303	17	0.6693	47	1.8504	14	0.5512	14 300	3 210	6 550	1 470	34 000	22 000	11 000	0.11	0.25
6304	20	0.7874	52	2.0472	15	0.5906	16 800	3 780	7 800	1 750	30 000	19 000	9 500	0.14	0.32
6305	25	0.9843	62	2.4409	17	0.6693	23 400	5 260	11 600	2 610	24 000	16 000	7 500	0.23	0.50
6306	30	1.1811	72	2.8346	19	0.7480	29 600	6 650	16 000	3 600	20 000	13 000	6 300	0.35	0.77
6307	35	1.3780	80	3.1496	21	0.8268	35 100	7 890	19 000	4 270	19 000	12 000	6 000	0.46	1.01
6308	40	1.5748	90	3.5433	23	0.9055	42 300	9 510	24 000	5 390	17 000	11 000	5 000	0.63	1.38
6309	45	1.7717	100	3.9370	25	0.9843	55 300	12 430	31 500	7 080	15 000	9 500	4 500	0.84	1.84
6310	50	1.9685	110	4.3307	27	1.0630	65 000	14 610	38 000	8 540	13 000	8 500	4 300	1.08	2.38
6311	55	2.1654	120	4.7244	29	1.1417	74 100	16 650	45 000	10 110	12 000	8 000	3 800	1.37	3.01
6312	60	2.3622	130	5.1181	31	1.2205	85 200	19 150	52 000	11 690	11 000	7 000	3 400	1.72	3.80
6313	65	2.5591	140	5.5118	33	1.2992	97 500	21 910	60 000	13 480	10 000	6 700	3 200	2.11	4.64
6314	70	2.7559	150	5.9055	35	1.3780	111 000	24 940	68 000	15 280	9 500	6 300	3 000	2.55	5.62
6315	75	2.9528	160	6.2992	37	1.4567	119 000	26 740	76 500	17 190	9 000	5 600	2 800	3.06	6.74
6316	80	3.1496	170	6.6929	39	1.5354	130 000	29 210	86 500	19 440	8 500	5 300	2 600	3.63	8.00
6317	85	3.3465	180	7.0866	41	1.6142	140 000	31 460	96 500	21 690	8 000	5 000	2 400	4.25	9.38
6318	90	3.5433	190	7.4803	43	1.6929	151 000	33 930	108 000	24 270	7 500	4 800	2 400	4.97	10.96
6319	95	3.7402	200	7.8740	45	1.7717	159 000	35 730	118 000	26 520	7 000	4 500	2 200	5.75	12.67
6320	100	3.9370	215	8.4646	47	1.8504	174 000	39 100	140 000	31 460	6 700	4 300	2 000	7.08	15.60
6321	105	4.1339	225	8.8583	49	1.9291	182 000	40 900	153 000	34 380	6 300	4 000	–	8.18	18.03
6322	110	4.3307	240	9.4488	50	1.9685	203 000	45 620	180 000	40 450	6 000	3 800	1 800	9.66	21.31
6324	120	4.7244	260	10.2362	55	2.1654	208 000	46 740	186 000	41 800	5 600	3 400	1 700	12.66	27.92

Figure 1. Bearing Catalogue SKF [1]

[1] SKF bearings and mounted products

Calculate F_a/C_0:

$$\frac{R_{a1}}{C_0} = 0.2049$$

Therefore:

$0.34 < e$ and $e < 0.38$

Check value of F_a/VF_r (V is 1):

$$\frac{R_{a1}}{R_r} = 0.4523$$ bigger than e

Now define X and Y to re-iterate:

$$X := 0.56$$

$$Y := 1.235645806$$

$$F_{eff} := \left(X \cdot R_r\right) + \left((Y) \cdot R_{a1}\right) = 613.3537 \text{ lbf}$$

Find new C_r:

$$C_r := F_{eff} \cdot \left(\frac{L}{K_r}\right)^{\frac{1}{3}} = 2742.7621 \text{ lbf}$$

Thefore the static load is:

$$C_0 := 1470 \text{ lbf}$$

Corresponding bearing is 6303

Calculate F_a/C_0:

$$\frac{R_{a1}}{C_0} = 0.1687$$

Therefore:

$0.28 < e$ and $e < 0.30$

This is still smaller than F_a/VF_r

So define new X and Y to reiterate:

$$Y := 1.31303333333333$$

$$F_{eff} := \left(X \cdot R_r\right) + \left((Y) \cdot R_{a1}\right) = 632.5405 \text{ lbf}$$

Find new C_r:

$$C_r := F_{eff} \cdot \left(\frac{L}{K_r}\right)^{\frac{1}{3}} = 2828.5608 \text{ lbf}$$

Thefore the static load is:

$$C_0 := 1470 \text{ lbf}$$

Corresponding bearing is 6303

Bearing Type 3)	4) $\frac{F_a}{C_0}$	5) $\frac{F_a}{i Z D_w^2}$	In Relation to the Load the Inner Ring is: Rotating V	Stationary V	Single Row Bearings 1) $\frac{F_a}{VF_r} > e$: X	Y	Double Row Bearings 2) $\frac{F_a}{VF_r} \le e$: X	Y	$\frac{F_a}{VF_r} > e$: X	Y	e
Radial Contact Groove Ball Bearings	0.014	25	1	1.2	0.56	2.30	1	0	0.56	2.30	0.1
	0.028	50				1.99				1.99	0.2
	0.056	100				1.71				1.71	0.2
	0.084	150				1.55				1.55	0.2
	0.11	200				1.45				1.45	0.3
	0.17	300				1.31				1.31	0.3
	0.28	500				1.15				1.15	0.3
	0.42	750				1.04				1.04	0.4
	0.56	1000				1.00				1.00	0.4
20°			1	1.2	0.43	1.00	1	1.09	0.70	1.63	0.5
25°					0.41	0.87		0.92	0.67	1.44	0.6
30°					0.39	0.76		0.78	0.63	1.24	0.8
35°					0.37	0.66		0.66	0.60	1.07	0.9
40°					0.35	0.57		0.55	0.57	0.93	1.1
Self-Aligning Ball Bearings			1	1	0.40	0.4 cot α	1	0.42 cot α	0.65	0.65 cot α	1.5 ta
Self-Aligning and Tapered Roller Bearings			1	1.2	0.40	0.4 cot α	1	0.45 cot α	0.67	0.67 cot α	1.5 ta

Figure 2. Equivalent Single Load Table for X and Y Factor [2]

Therefore, the Left bearing is up'd to 6306 as it's the closest to gear bore diameter of 32mm

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b) Right bearing taking all axial force

$R_{a2} := R_a$

Now calculating the Radial Force at the right bearing:

$$R_{r2} := \sqrt{R_{y2}^{\;2} + R_{z2}^{\;2}}$$

$R_{r2} = 340.8819$ lbf

Radial Load Analysis Factors and Equation

$P := R_{r2}$

$K_r := 0.62$ Reliability 95%

$a := 3$

$$C_r := P \cdot \left(\frac{L}{K_r}\right)^{\frac{1}{3}}$$

$C_r = 1524.3377$ lbf Dynamic Load

Thefore the static load is:

$C_0 := 760$ lbf

Corresponding bearing is 6300

Calculate F_a/C_0:

$$\frac{R_{a2}}{C_0} = 0.3262$$

Therefore:

$0.38 < e$ and $e < 0.42$

Check value of F_a/VF_r (V is 1):

$$\frac{R_{a2}}{R_{r2}} = 0.7273$$ This is bigger than e

Now define X and Y to re-iterate:

$X := 0.56$

$Y := 1.1137$

$F_{eff} := (X \cdot R_{r2}) + (Y \cdot R_{a2}) = 467.0158$ lbf

Find new C_r:

$$C_r := F_{eff} \cdot \left(\frac{L}{K_r}\right)^{\frac{1}{3}} = 2088.3761 \text{ lbf}$$

Thefore the static load is:

$C_0 := 930\ lbf$

Corresponding bearing is 6301

Calculate F_a/C_0:

$$\frac{R_{a2}}{C_0} = 0.2666$$

Therefore:

$0.34 < e$ and $e < 0.38$

This is still smaller than F_a/VF_r

So define new X and Y to reiterate:

$Y := 1.169490909090909$

$$F_{eff} := (X \cdot R_{r2}) + (Y \cdot R_{a2}) = 480.8481\ lbf$$

Find new C_r:

$$C_r := F_{eff} \cdot \left(\frac{L}{K_r}\right)^{\frac{1}{3}} = 2150.2309\ lbf$$

Thefore the static load is:

$C_0 := 930\ lbf$

Corresponding bearing is 6301

Therefore, the Right bearing is 6301 but up the size to 6306 as it's the closest to gear bore diameter of 32mm

Given Variables from output shaft anaysis:

$\omega := 3000\ rpm = 314.1593\ \frac{rad}{s}$

$R_{y1} := 515.4078\ lbf$ $\quad R_{y2} := 516.5342\ lbf$ $\quad R_a := (-413.22)\ lbf$

$R_{z1} := (-473.6177)\ lbf$ $\quad R_{z2} := 22.0214\ lbf$ $\quad R_a := |R_a|$

a) Left bearing taking all axial force

$R_{a1} := R_a$

$R_r := \sqrt{R_{y1}^2 + R_{z1}^2} = 699.9707\ lbf$

Radial Load Analysis Factors and Equation

$L := \frac{166.32}{7.5}$

$a := 3$

$P := R_r$

$K_r := 0.62$ Reliability 95%

$$C_r := P \cdot \left(\frac{L}{K_r}\right)^{\frac{1}{3}}$$

$C_r = 2306.271\ lbf$ Dynamic Load

Thefore the static load is:

$C_0 := 1210\ lbf$

Corresponding bearing is 6302

Designation	Principal dimensions						Basic load ratings				Speed rating[1]			Mass	
	Bore d		Outside dimeter D		Width B		Dynamic C		Static C_0		Reference speed	Limiting speed	Sealed Limiting speed		
	mm	in	mm	in	mm	in	N	lbf	N	lbf	r/min	r/min	r/min	kg	lb
634	4	0.1575	16	0.6299	5	0.1969	1 110	250	380	90	95 000	60 000	28 000	0.01	0.01
635	5	0.1969	19	0.7480	6	0.2362	2 340	530	950	210	80 000	50 000	24 000	0.01	0.02
6300	10	0.3937	35	1.3780	11	0.4331	8 520	1 910	3 400	760	50 000	32 000	15 000	0.05	0.12
6301	12	0.4724	37	1.4567	12	0.4724	10 100	2 270	4 150	930	45 000	28 000	14 000	0.06	0.13
6302	15	0.5906	42	1.6535	13	0.5118	11 900	2 670	5 400	1 210	38 000	24 000	12 000	0.08	0.18
6303	17	0.6693	47	1.8504	14	0.5512	14 300	3 210	6 550	1 470	34 000	22 000	11 000	0.11	0.25
6304	20	0.7874	52	2.0472	15	0.5906	16 800	3 780	7 800	1 750	30 000	19 000	9 500	0.14	0.32
6305	25	0.9843	62	2.4409	17	0.6693	23 400	5 260	11 600	2 610	24 000	16 000	7 500	0.23	0.50
6306	30	1.1811	72	2.8346	19	0.7480	29 600	6 650	16 000	3 600	20 000	13 000	6 300	0.35	0.77
6307	35	1.3780	80	3.1496	21	0.8268	35 100	7 890	19 000	4 270	19 000	12 000	6 000	0.46	1.01
6308	40	1.5748	90	3.5433	23	0.9055	42 300	9 510	24 000	5 390	17 000	11 000	5 000	0.63	1.38
6309	45	1.7717	100	3.9370	25	0.9843	55 300	12 430	31 500	7 080	15 000	9 500	4 500	0.84	1.84
6310	50	1.9685	110	4.3307	27	1.0630	65 000	14 610	38 000	8 540	13 000	8 500	4 300	1.08	2.38
6311	55	2.1654	120	4.7244	29	1.1417	74 100	16 650	45 000	10 110	12 000	8 000	3 800	1.37	3.03
6312	60	2.3622	130	5.1181	31	1.2205	85 200	19 150	52 000	11 690	11 000	7 000	3 400	1.72	3.80
6313	65	2.5591	140	5.5118	33	1.2992	97 500	21 910	60 000	13 480	10 000	6 700	3 200	2.11	4.64
6314	70	2.7559	150	5.9055	35	1.3780	111 000	24 940	68 000	15 280	9 500	6 300	3 000	2.55	5.62
6315	75	2.9528	160	6.2992	37	1.4567	119 000	26 740	76 500	17 190	9 000	5 600	2 800	3.06	6.74
6316	80	3.1496	170	6.6929	39	1.5354	130 000	29 210	86 500	19 440	8 500	5 300	2 600	3.63	8.00
6317	85	3.3465	180	7.0866	41	1.6142	140 000	31 460	96 500	21 690	8 000	5 000	2 400	4.25	9.38
6318	90	3.5433	190	7.4803	43	1.6929	151 000	33 930	108 000	24 270	7 500	4 800	2 400	4.97	10.96
6319	95	3.7402	200	7.8740	45	1.7717	159 000	35 730	118 000	26 520	7 000	4 500	2 200	5.75	12.67
6320	100	3.9370	215	8.4646	47	1.8504	174 000	39 100	140 000	31 460	6 700	4 300	2 000	7.08	15.60
6321	105	4.1339	225	8.8583	49	1.9291	182 000	40 900	153 000	34 380	6 300	4 000	-	8.18	18.03
6322	110	4.3307	240	9.4488	50	1.9685	203 000	45 620	180 000	40 450	6 000	3 800	1 800	9.66	21.31
6324	120	4.7244	260	10.2362	55	2.1654	208 000	46 740	186 000	41 800	5 600	3 400	1 700	12.66	27.92

Figure 2. Bearing Catalogue SKF [1]

[1] SKF bearings and mounted products

Calculate F_a/C_0:

$$\frac{R_{a1}}{C_0} = 0.3415$$

Therefore:

$0.38 < e$ and $e < 0.42$

Check value of F_a/VF_r (V is 1):

$$\frac{R_{a1}}{R_r} = 0.5903$$ bigger than e

Now define X and Y to re-iterate:

$X := 0.56$

$Y := 1.101678571$

$$F_{eff} := (X \cdot R_r) + ((Y) \cdot R_{a1}) = 847.2192 \text{ lbf}$$

Find new C_r:

$$C_r := F_{eff} \cdot \left(\frac{L}{K_r}\right)^{\frac{1}{3}} = 2791.4271 \text{ lbf}$$

Thefore the static load is:

$C_0 := 1470 \text{ lbf}$

Corresponding bearing is 6303

Calculate F_a/C_0:

$$\frac{R_{a1}}{C_0} = 0.2811$$

Therefore:

$0.38 < e$ and $e < 0.42$

This is still smaller than F_a/VF_r

So define new X and Y to reiterate:

$Y := 1.149135714$

$$F_{eff} := (X \cdot R_r) + ((Y) \cdot R_{a1}) = 866.8294 \text{ lbf}$$

Find new C_r:

$$C_r := F_{eff} \cdot \left(\frac{L}{K_r}\right)^{\frac{1}{3}} = 2856.0391 \text{ lbf}$$

Thefore the static load is:

$C_0 := 1470 \text{ lbf}$

Corresponding bearing is 6303

Bearing Type 3)	$\frac{F_a}{C_0}$ 4)	$\frac{F_a}{iZD_w^2}$ 5)	In Relation to the Load the Inner Ring is: Rotating V	Stationary V	Single Row Bearings 1) $\frac{F_a}{VF_r} > e$: X	Y	Double Row Bearings 2) $\frac{F_a}{VF_r} \leq e$: X	Y	$\frac{F_a}{VF_r} > e$: X	Y	e
Radial Contact Groove Ball Bearings	0.014	25				2.30				2.30	0.19
	0.028	50				1.99				1.99	0.22
	0.056	100				1.71				1.71	0.26
	0.084	150				1.55				1.55	0.28
	0.11	200	1	1.2	0.56	1.45	1	0	0.56	1.45	0.30
	0.17	300				1.31				1.31	0.34
	0.28	500				1.15				1.15	0.38
	0.42	750				1.04				1.04	0.42
	0.56	1000				1.00				1.00	0.44
20°					0.43	1.00		1.09	0.70	1.63	0.57
25°					0.41	0.87		0.92	0.67	1.44	0.68
30°			1	1.2	0.39	0.76	1	0.78	0.63	1.24	0.80
35°					0.37	0.66		0.66	0.60	1.07	0.95
40°					0.35	0.57		0.55	0.57	0.93	1.14
Self-Aligning Ball Bearings			1	1	0.40	0.4 cot α	1	0.42 cot α	0.65	0.65 cot α	1.5 tan α
Self-Aligning and Tapered Roller Bearings			1	1.2	0.40	0.4 cot α	1	0.45 cot α	0.67	0.67 cot α	1.5 tan α

Figure 3. Bearing Catalogue [2]

Therefore, the Left bearing is 6303 but choose 6307 as it's the closest to gear bore diameter of 40mm

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b) Right bearing taking all axial force

$R_{a2} := R_a$

Now calculating the Radial Force at the right bearing:

$$R_{r2} := \sqrt{R_{y2}^{\ 2} + R_{z2}^{\ 2}}$$

$R_{r2} = 517.0034$ lbf

Radial Load Analysis Factors and Equation

$L := \frac{166.32}{7.5}$

$P := R_{r2}$

$K_r := 0.62$ Reliability 95%

$a := 3$

$$C_r := P \cdot \left(\frac{L}{K_r}\right)^{\frac{1}{3}}$$

$C_r = 1703.4285$ lbf Dynamic Load

Thefore the static load is:

$C_0 := 760$ lbf

Corresponding bearing is 6300

Calculate F_a/C_0:

$$\frac{R_{a2}}{C_0} = 0.5437$$

Therefore:

$0.42 < e$ and $e < 0.44$

Check value of F_a/VF_r (V is 1):

$$\frac{R_{a2}}{R_{r2}} = 0.7993$$ This is bigger than e

Now define X and Y to re-iterate:

$X := 0.56$

$Y := 1.004657143$

$F_{eff} := (X \cdot R_{r2}) + (Y \cdot R_{a2}) = 704.6663$ lbf

Find new C_r:

$$C_r := F_{eff} \cdot \left(\frac{L}{K_r}\right)^{\frac{1}{3}} = 2321.7424 \text{ lbf}$$

Thefore the static load is:

$C_0 := 1210 \text{ lbf}$

Corresponding bearing is 6302

Calculate F_a/C_0:

$$\frac{R_{a2}}{C_0} = 0.3415$$

Therefore:

$0.38 < e$ and $e < 0.42$

This is still smaller than F_a/VF_r

So define new X and Y to reiterate:

$Y := 1.101678571$

$F_{eff} := (X \cdot R_{r2}) + (Y \cdot R_{a2}) = 744.7575 \text{ lbf}$

Find new C_r:

$$C_r := F_{eff} \cdot \left(\frac{L}{K_r}\right)^{\frac{1}{3}} = 2453.8353 \text{ lbf}$$

Thefore the static load is:

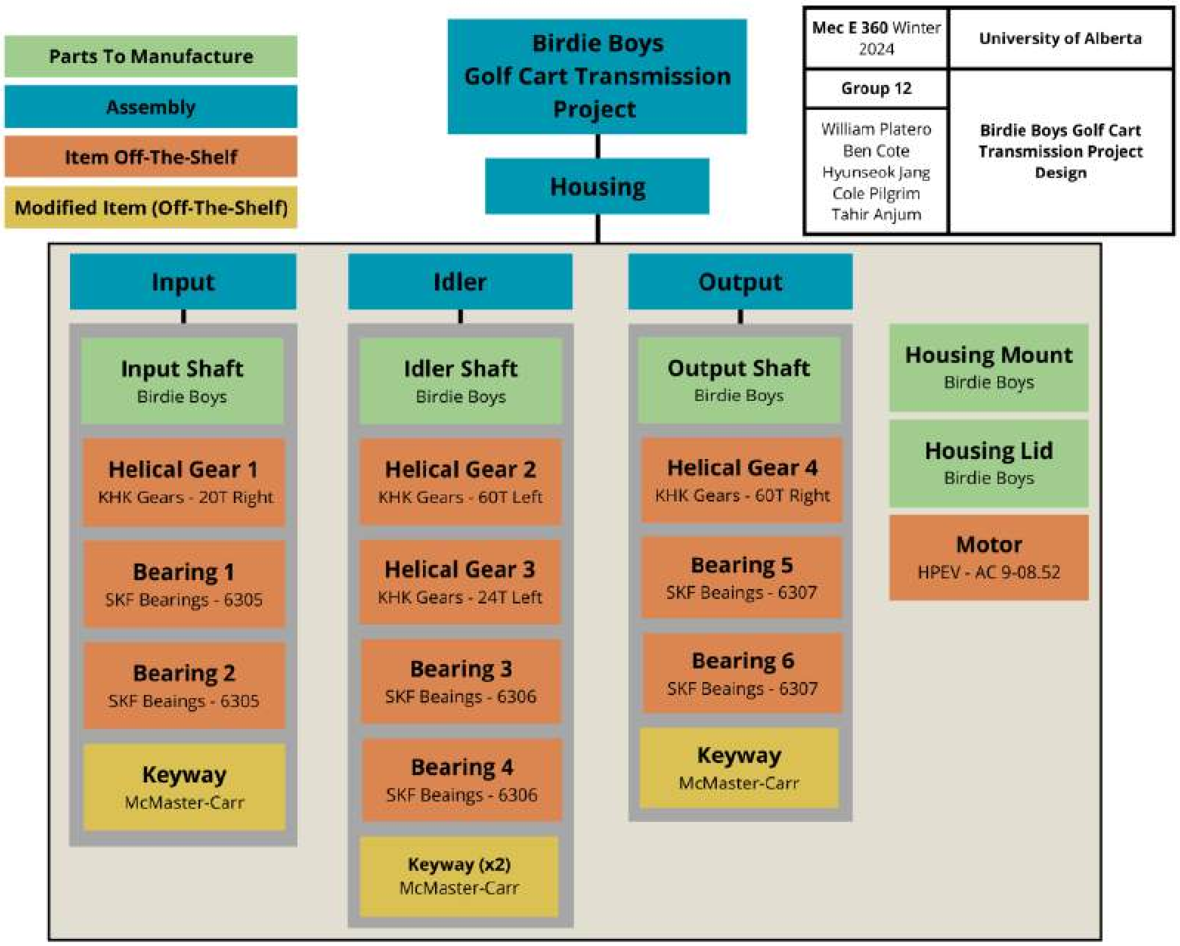
$C_0 := 1210 \text{ lbf}$

Corresponding bearing is 6302

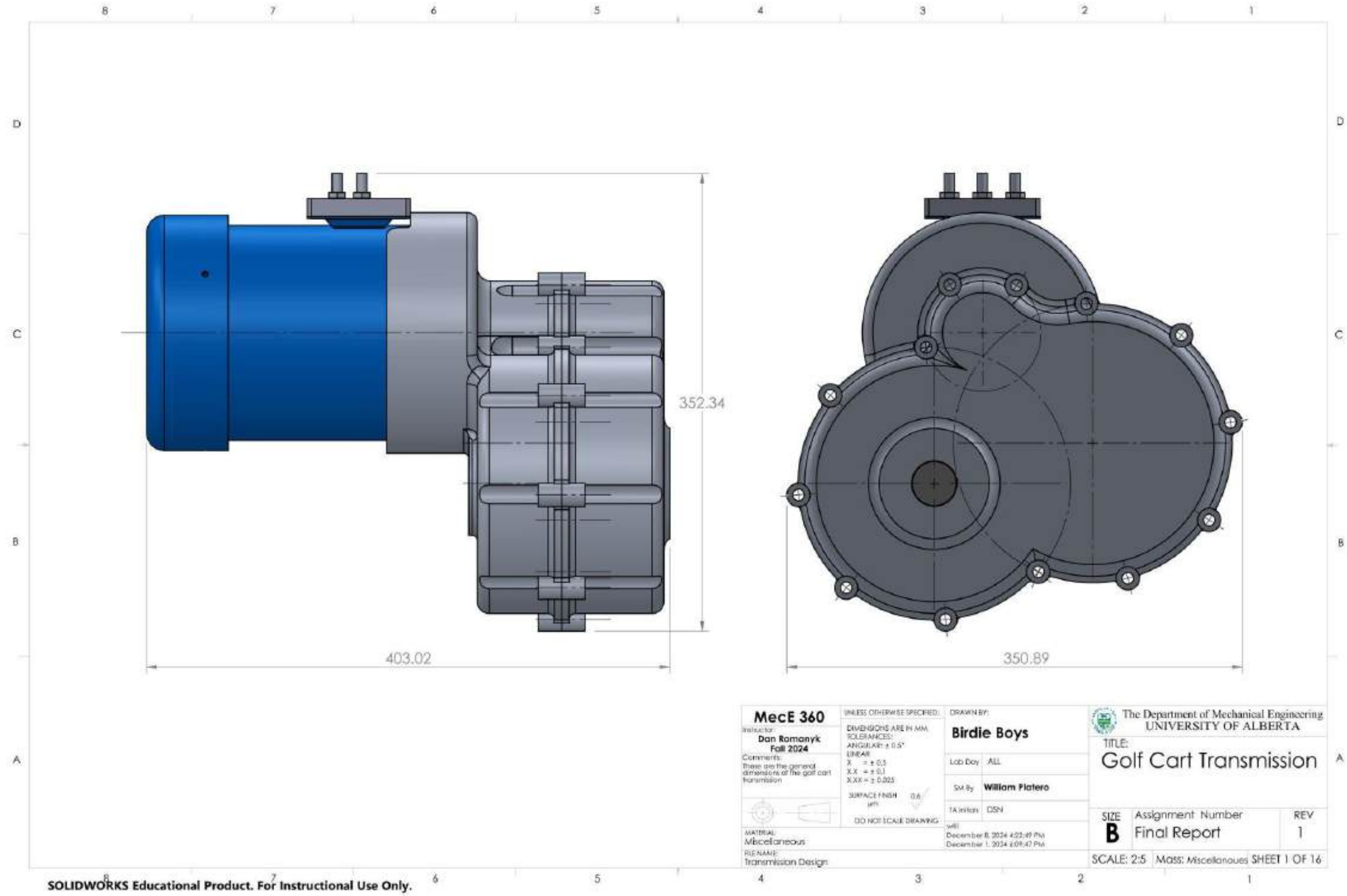
Therefore, the Right bearing is 6302 but choose 6307 as it's the closest to gear bore diameter of 40mm

Appendix D (Drawings)

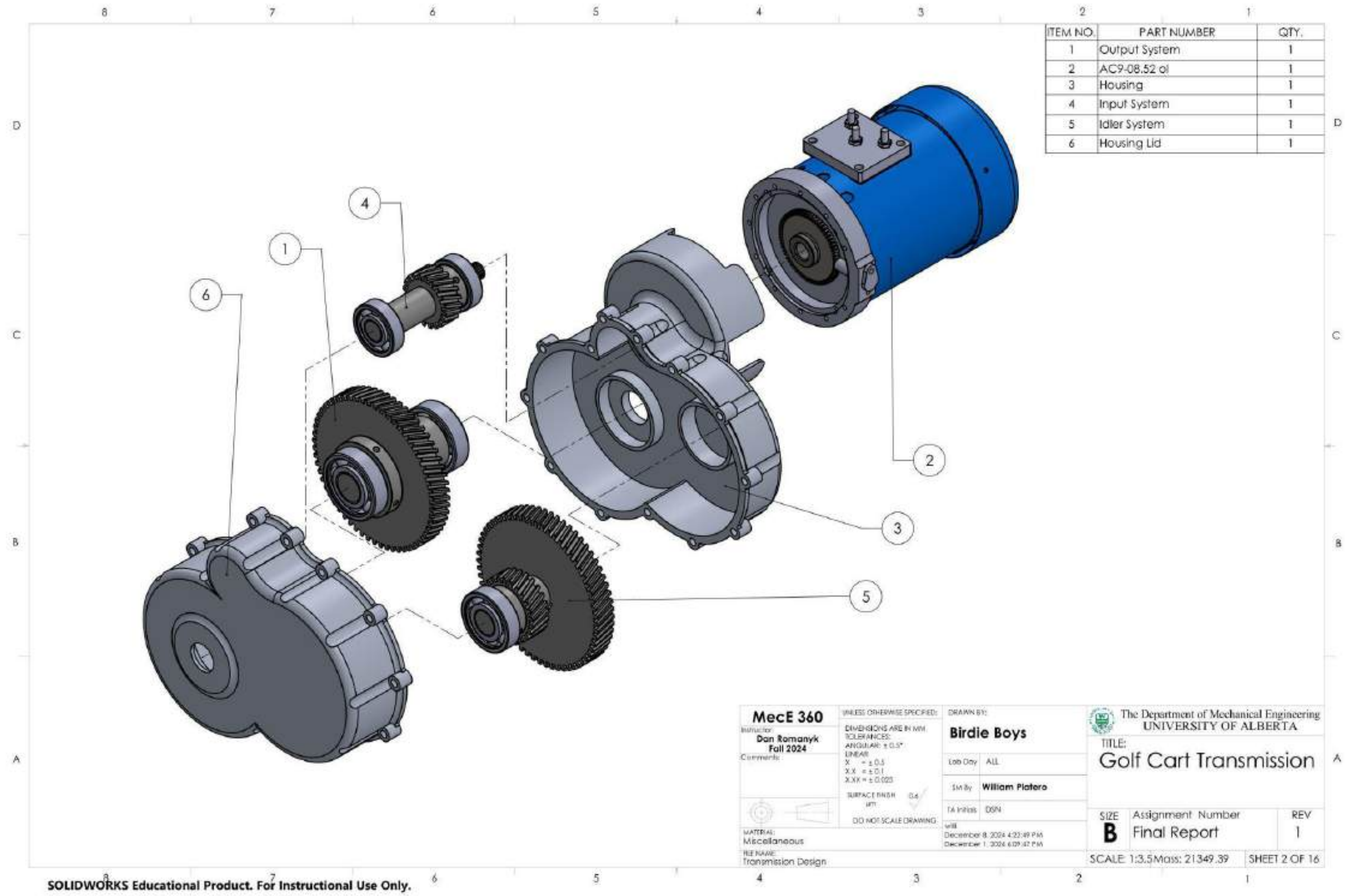
D1 - The following diagram shows the drawing tree for the golf cart transmission.



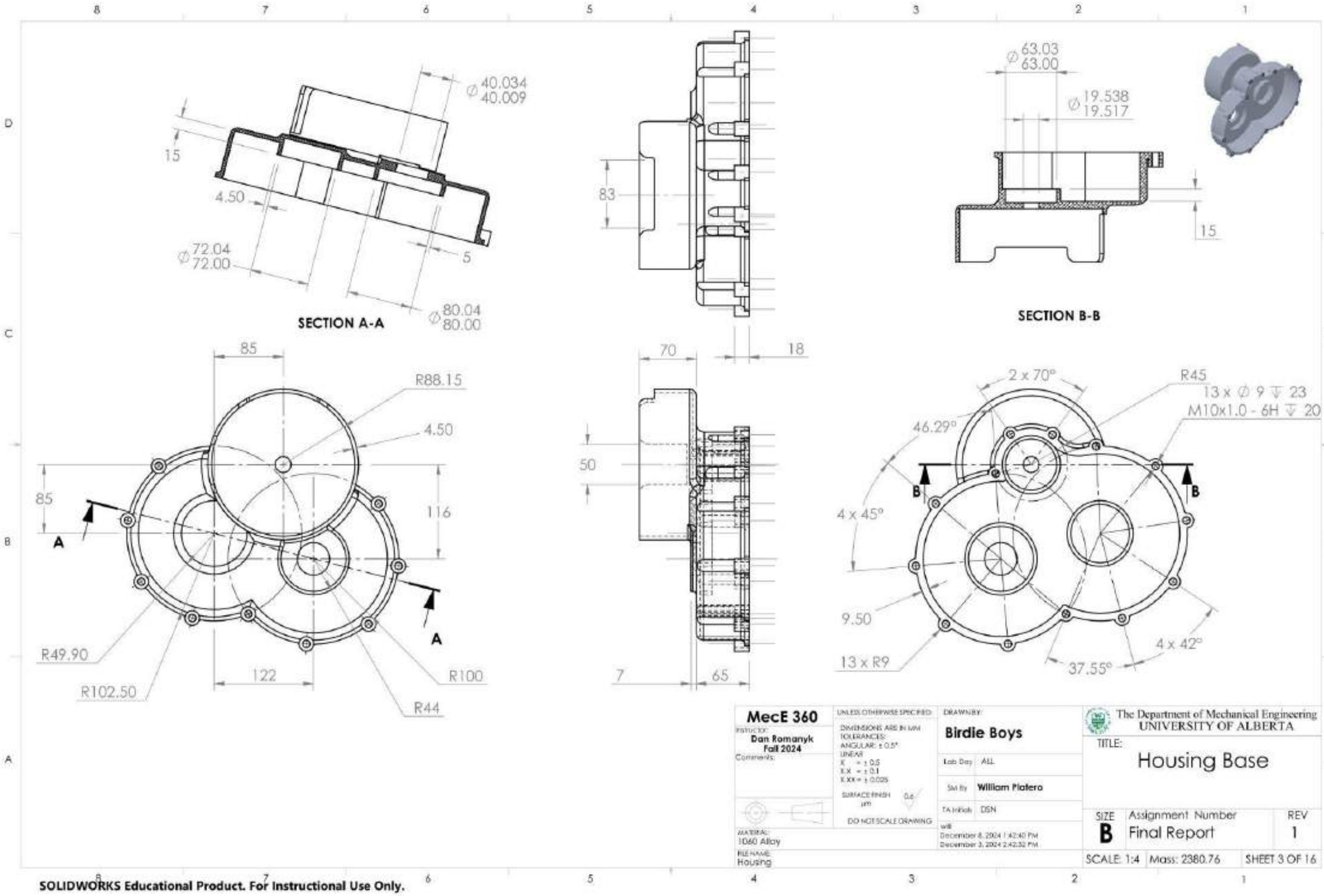
D2 - Golf cart transmission assembly drawing (shows the general dimensions).



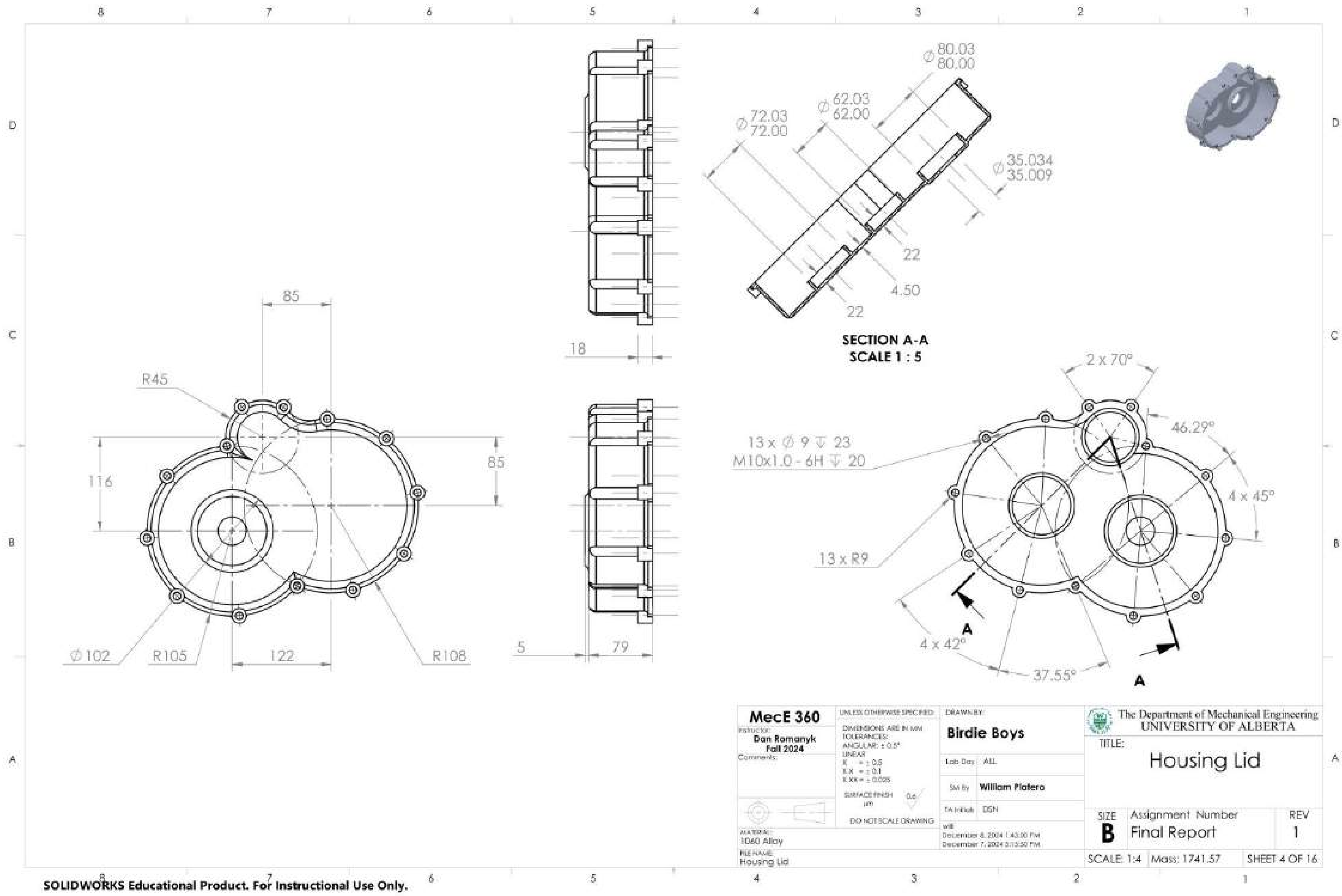
D3 - Exploded view of the transmission assembly.



D4 - Housing base engineering drawing (it also supports the motor to the transmission).

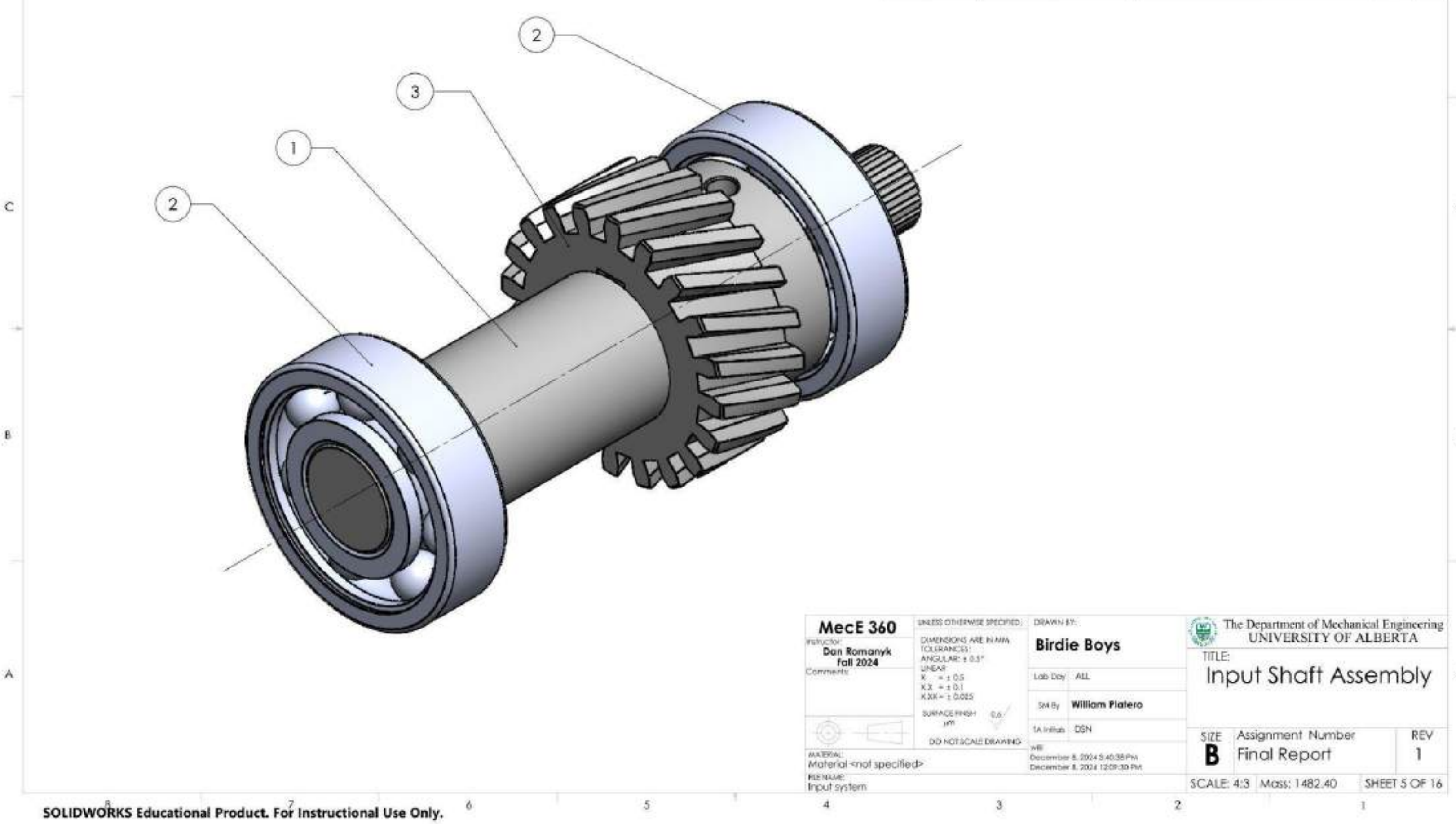


D5 - Housing lid engineering drawing.

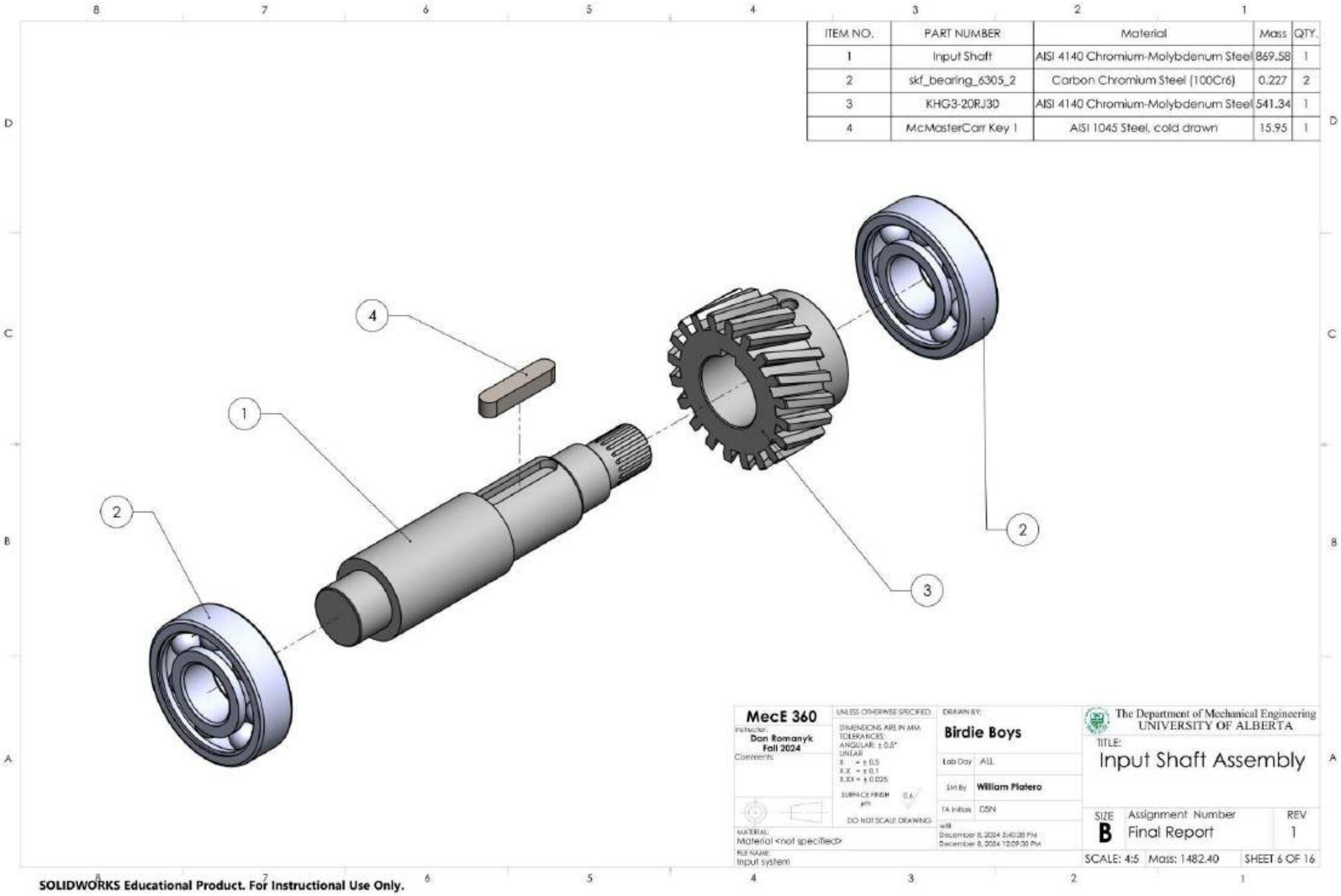


D6 - Input shaft assembly.

ITEM NO.	PART NUMBER	Material	Mass	QTY.
1	Input Shaft	AISI 4140 Chromium-Molybdenum Steel	869.58	1
2	skf_bearing_6305_2	Carbon Chromium Steel (100Cr6)	0.227	2
3	KHG3-20RJ30	AISI 4140 Chromium-Molybdenum Steel	541.34	1
4	McMasterCarr Key 1	AISI 1045 Steel, cold drawn	15.95	1

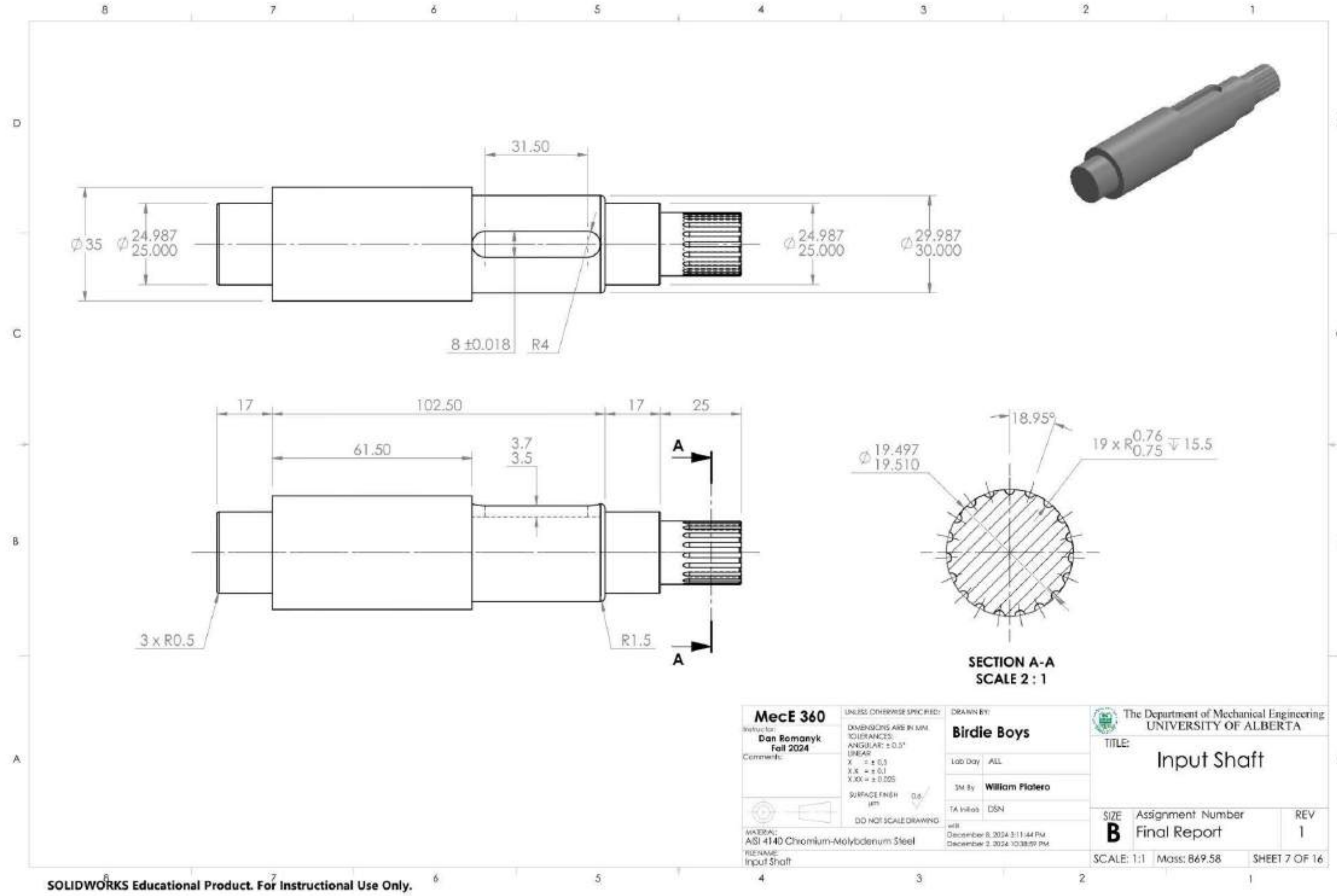


D7 - Input shaft assembly exploded view.

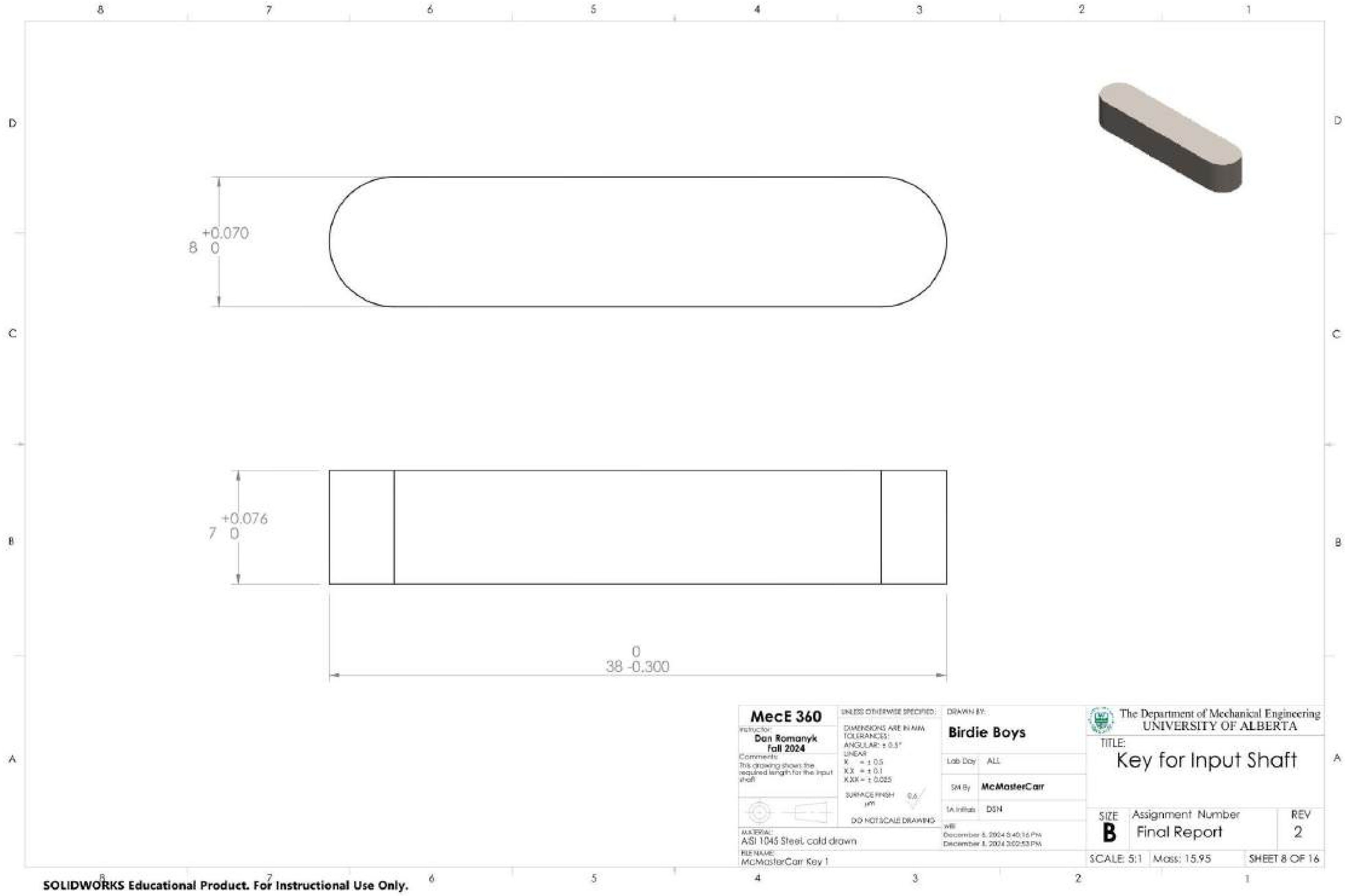


ITEM NO.	PART NUMBER	Material	Mass	QTY.
1	Input Shaft	AISI 4140 Chromium-Molybdenum Steel	869.58	1
2	skf_bearing_6305_2	Carbon Chromium Steel (100Cr6)	0.227	2
3	KHG3-20RJ30	AISI 4140 Chromium-Molybdenum Steel	541.34	1
4	McMasterCarr Key 1	AISI 1045 Steel, cold drawn	15.95	1

D8 - Input shaft engineering drawing for manufacturing.

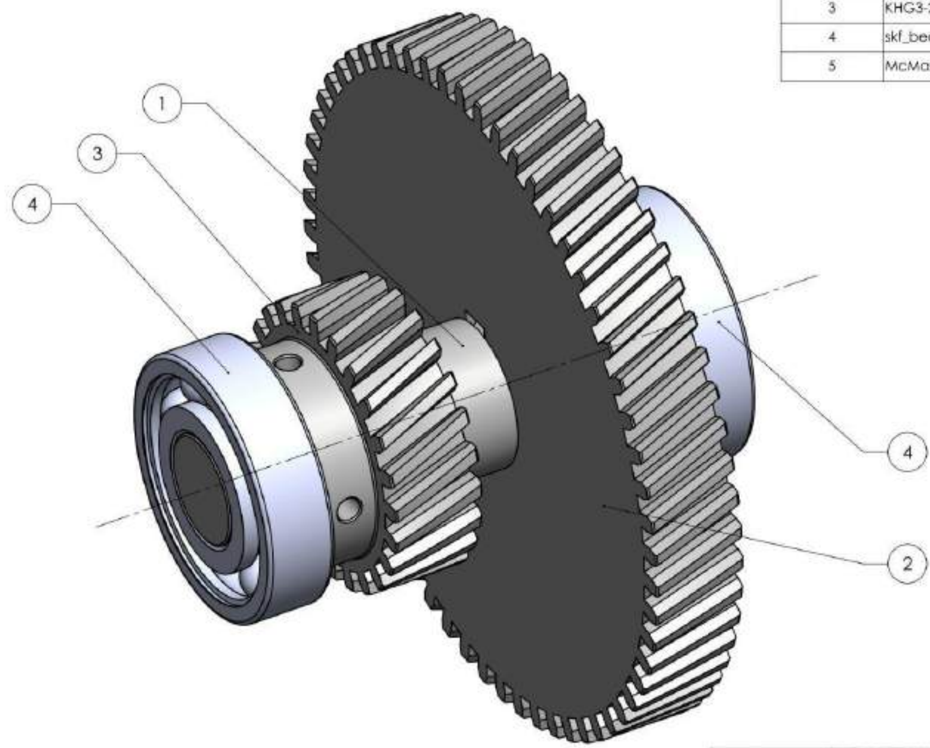


D9 - Key for input shaft engineering drawing, it's based on McMaster-Carr stock refined to an appropriate length.



D10 - Idler shaft assembly.

ITEM NO.	PART NUMBER	Material	MASS	QTY.
1	Idler Shaft	AISI 4140 Chromium-Molybdenum Steel	897.12	1
2	KHG3-60LJ32	AISI 4140 Chromium-Molybdenum Steel	5439.86	1
3	KHG3-24LJ32	AISI 4140 Chromium-Molybdenum Steel	830.99	1
4	skf_bearing_6306_2	Carbon Chromium Steel (100Cr6)	0.35	2
5	McMasterCarr Key 2-3	AISI 1045 Steel, cold drawn	22.52	2



MecE 360
Instructor:
Dan Romanyk
Fall 2024
Comments:

UNLESS OTHERWISE SPECIFIED:
DIMENSIONS ARE IN MM
TOLERANCES:
ANGULAR: ± 0.5°
LINEAR
X = ± 0.5
X.X = ± 0.1
X.XX = ± 0.025
SURFACE FINISH (µm) 0.6
DO NOT SCALE DRAWING

DRAWN BY:
Birdie Boys
Lab Day ALL
Sbt By William Platero
TA Initials DSN
December 8, 2024 4:05:35 PM
December 8, 2024 12:47:31 PM

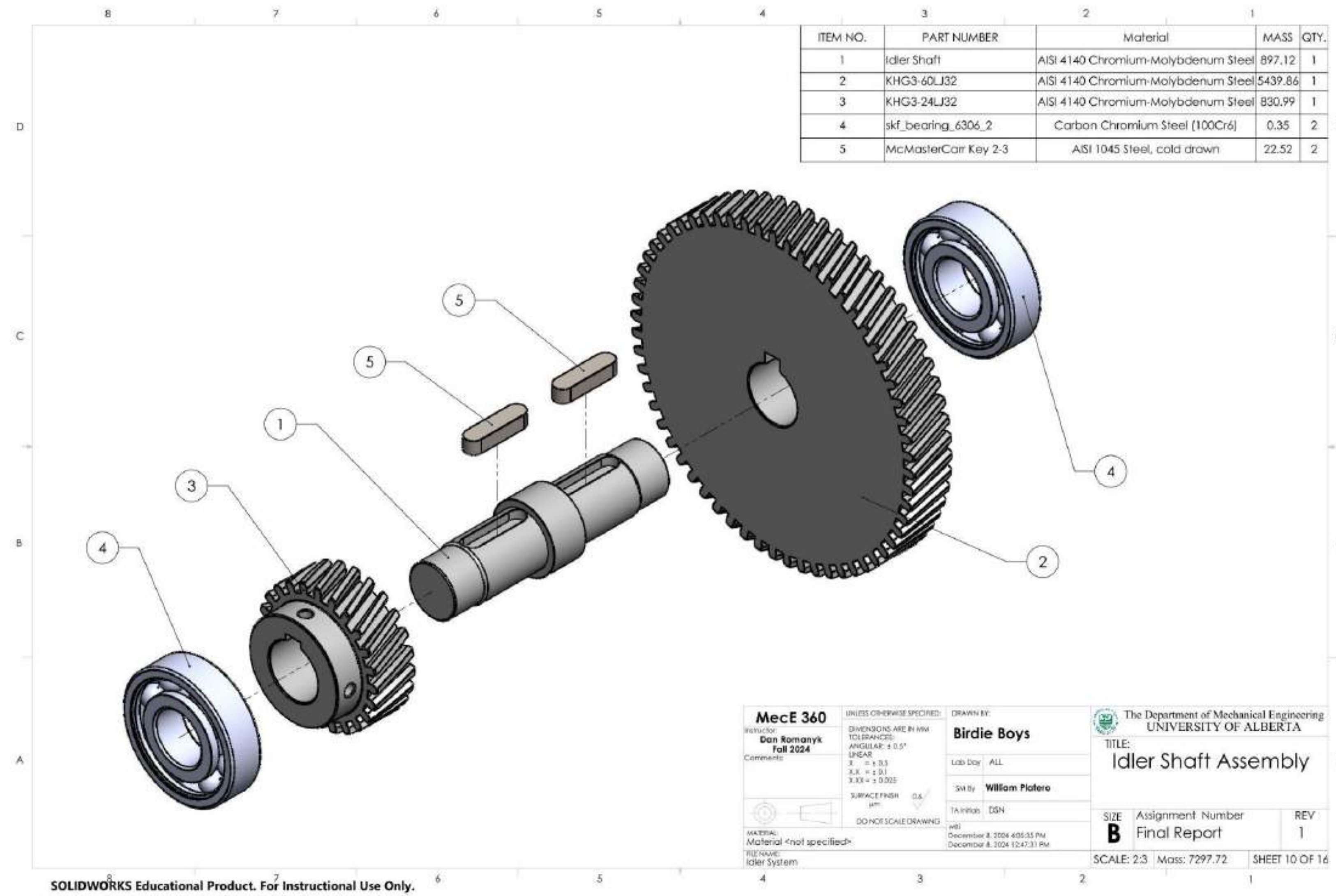
MATERIAL:
Material <not specified>
FILE NAME:
Idler System

The Department of Mechanical Engineering
UNIVERSITY OF ALBERTA
TITLE:
Idler Shaft Assembly

SIZE B | Assignment Number Final Report | REV 1

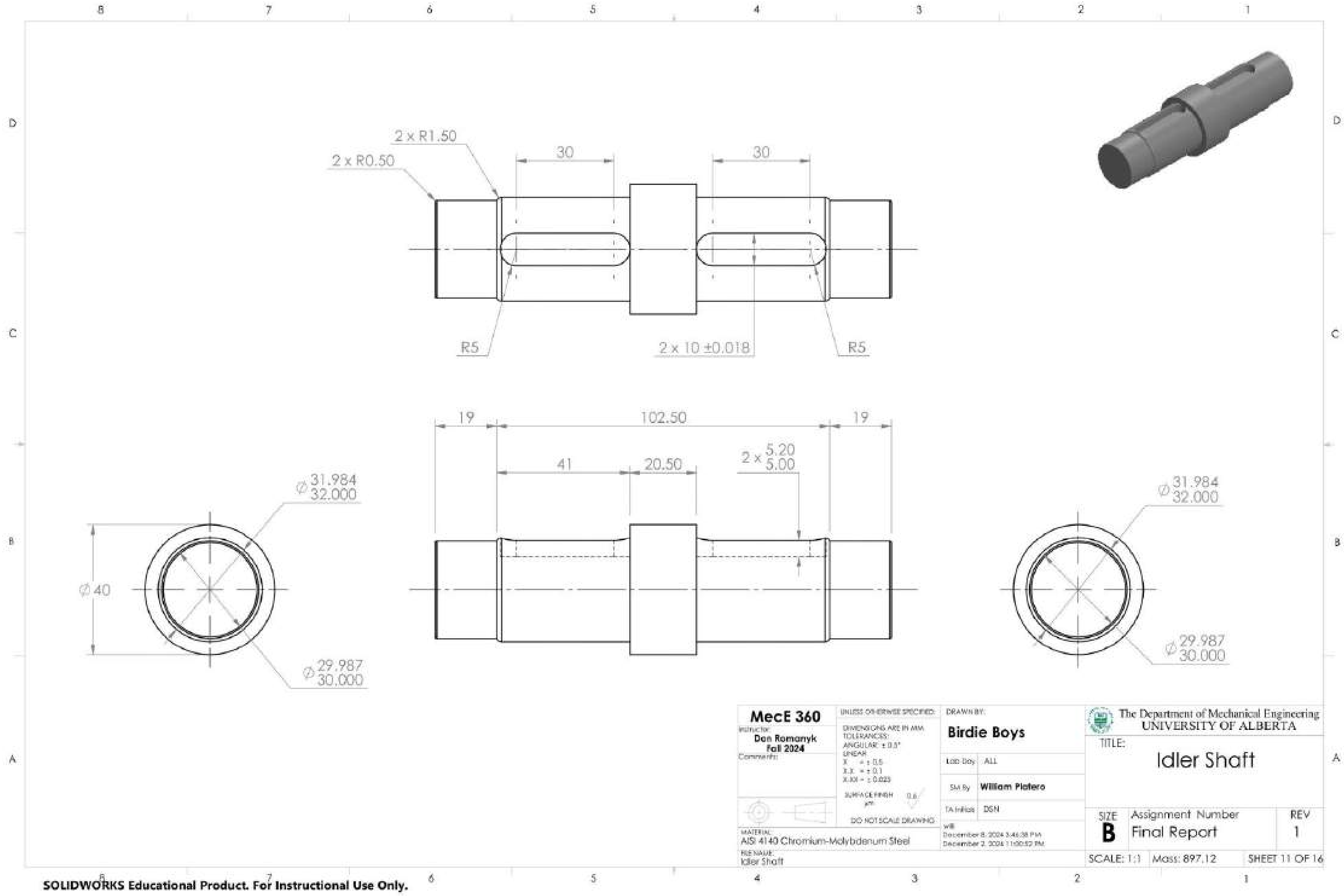
SCALE: 1:1 | Mass: 7297.72 | SHEET 9 OF 16

D11 - Idler shaft assembly exploded view.

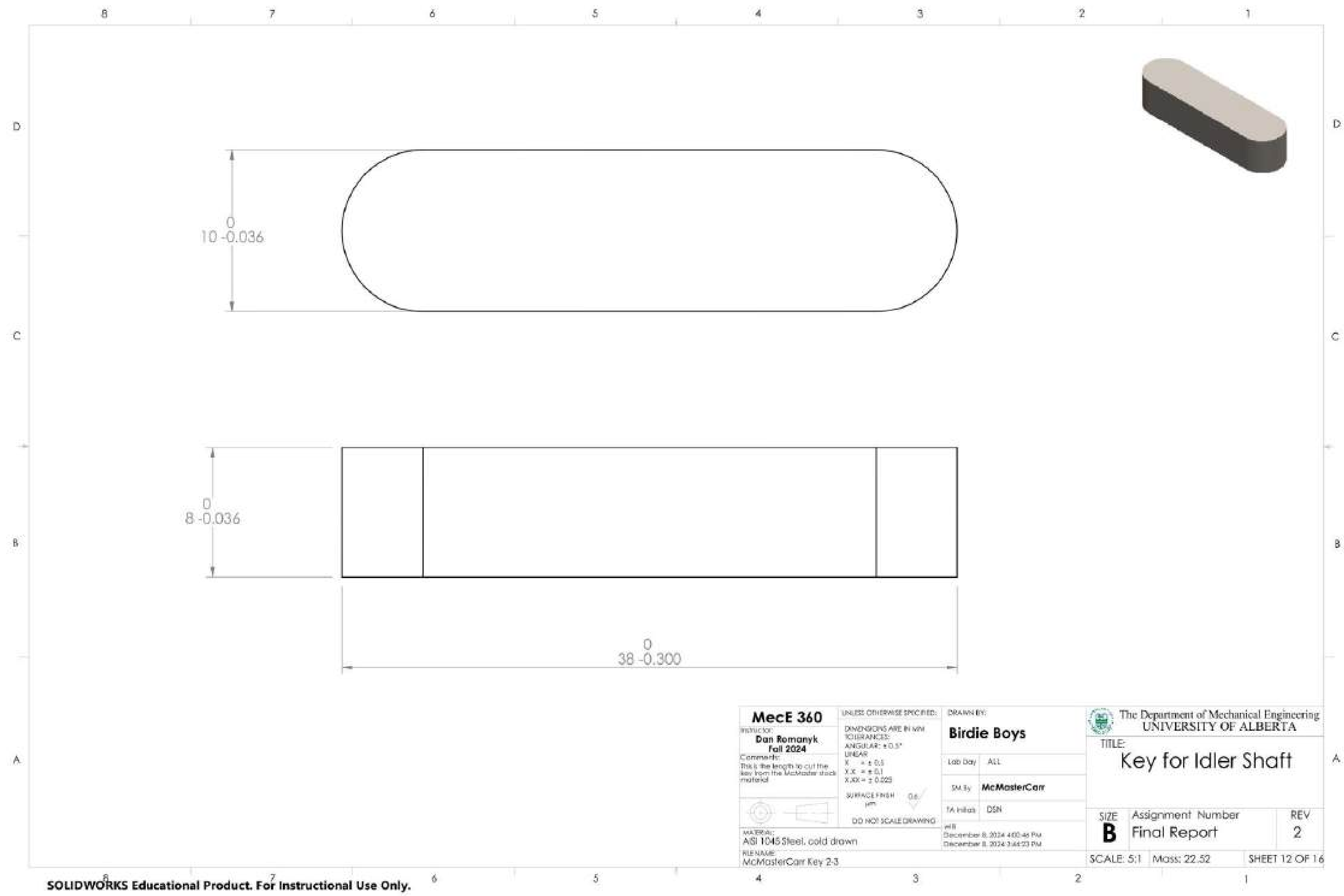


ITEM NO.	PART NUMBER	Material	MASS	QTY.
1	Idler Shaft	AISI 4140 Chromium-Molybdenum Steel	897.12	1
2	KHG3-60LJ32	AISI 4140 Chromium-Molybdenum Steel	5439.86	1
3	KHG3-24LJ32	AISI 4140 Chromium-Molybdenum Steel	830.99	1
4	skf_bearing_6306_2	Carbon Chromium Steel (100Cr6)	0.35	2
5	McMasterCarr Key 2-3	AISI 1045 Steel, cold drawn	22.52	2

D12 - Idler shaft engineering drawing for manufacturing.

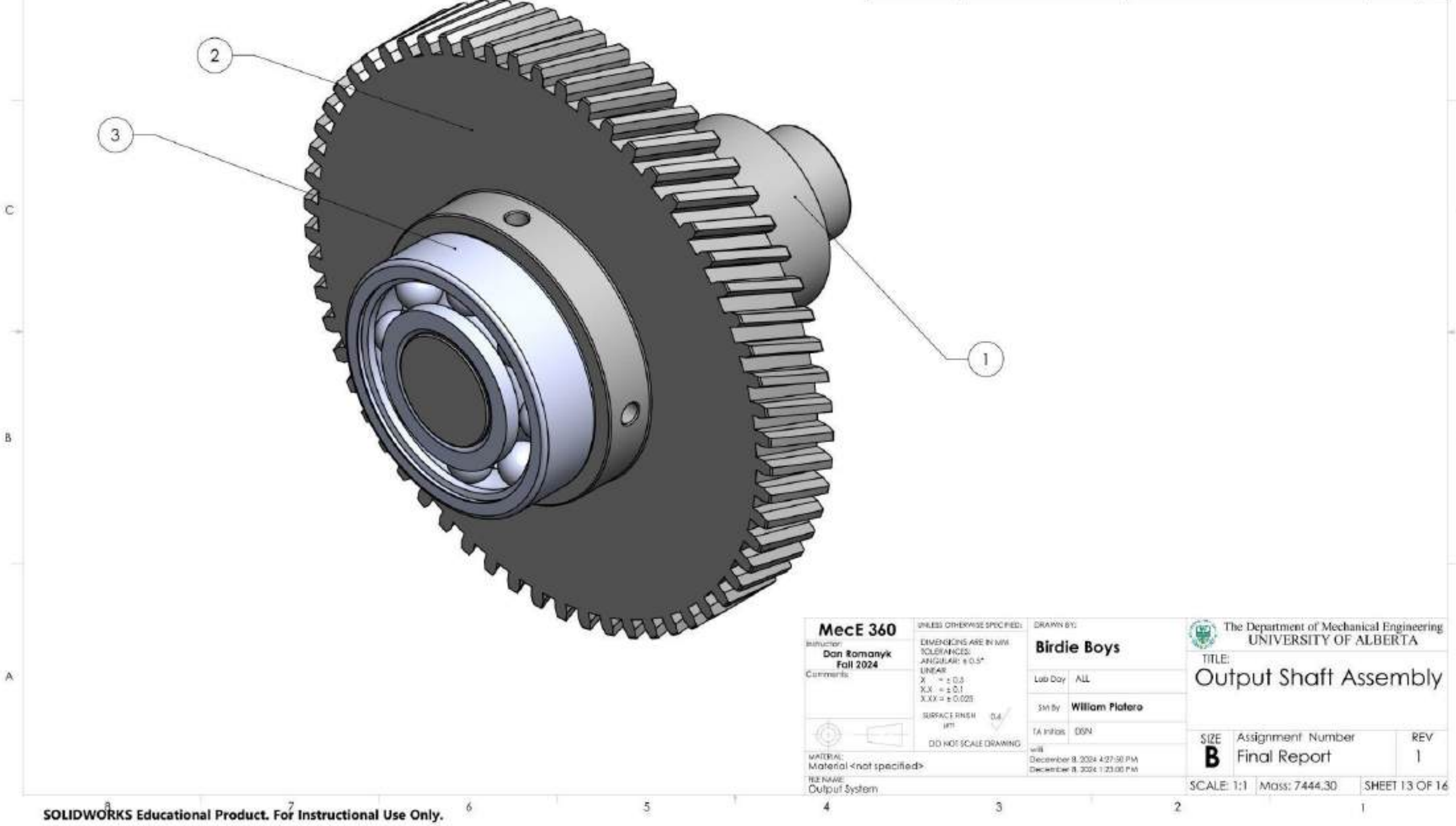


D13 - Key for idler shaft engineering drawing, it’s based on McMaster-Carr stock refined to an appropriate length.

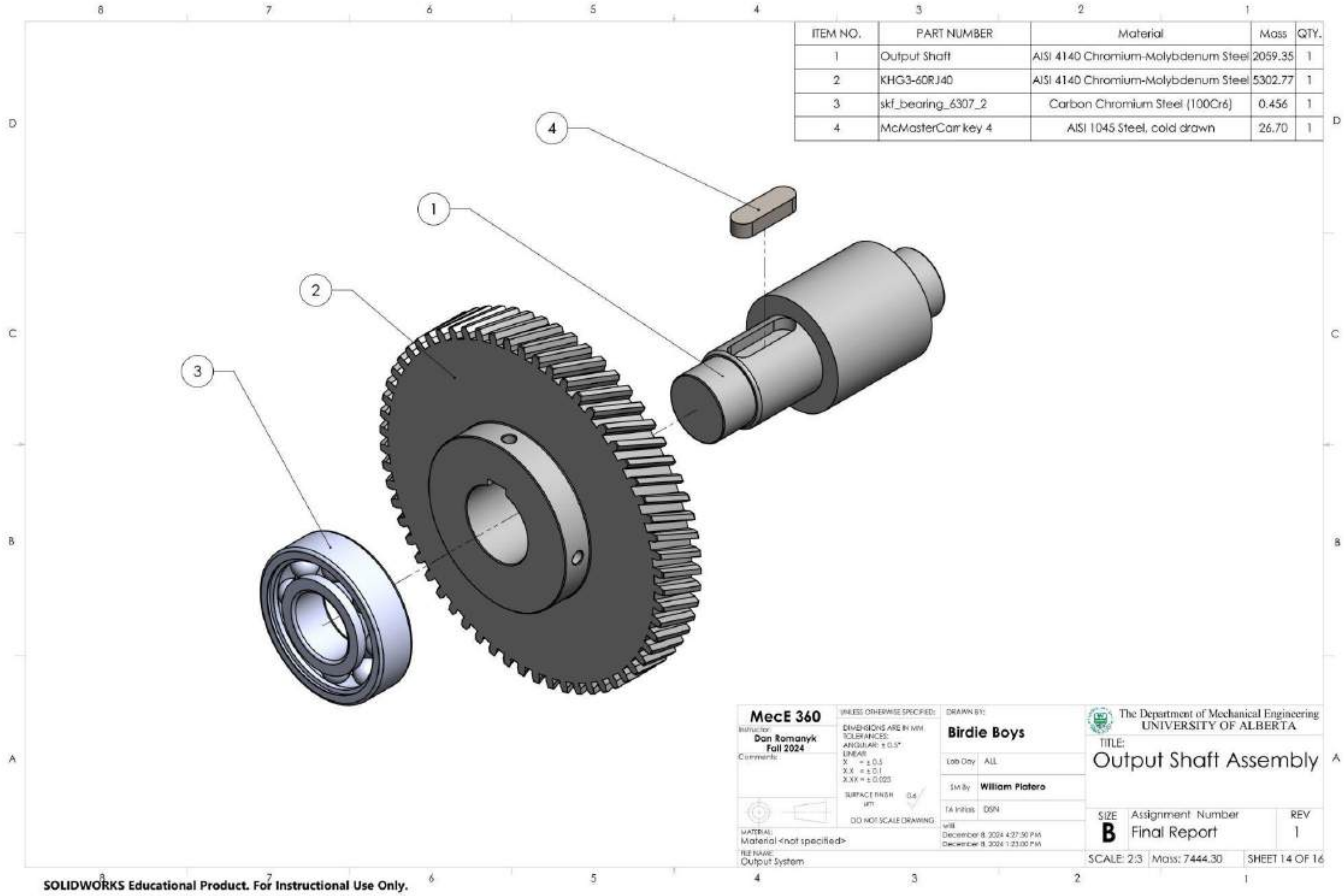


D14 - Output shaft assembly.

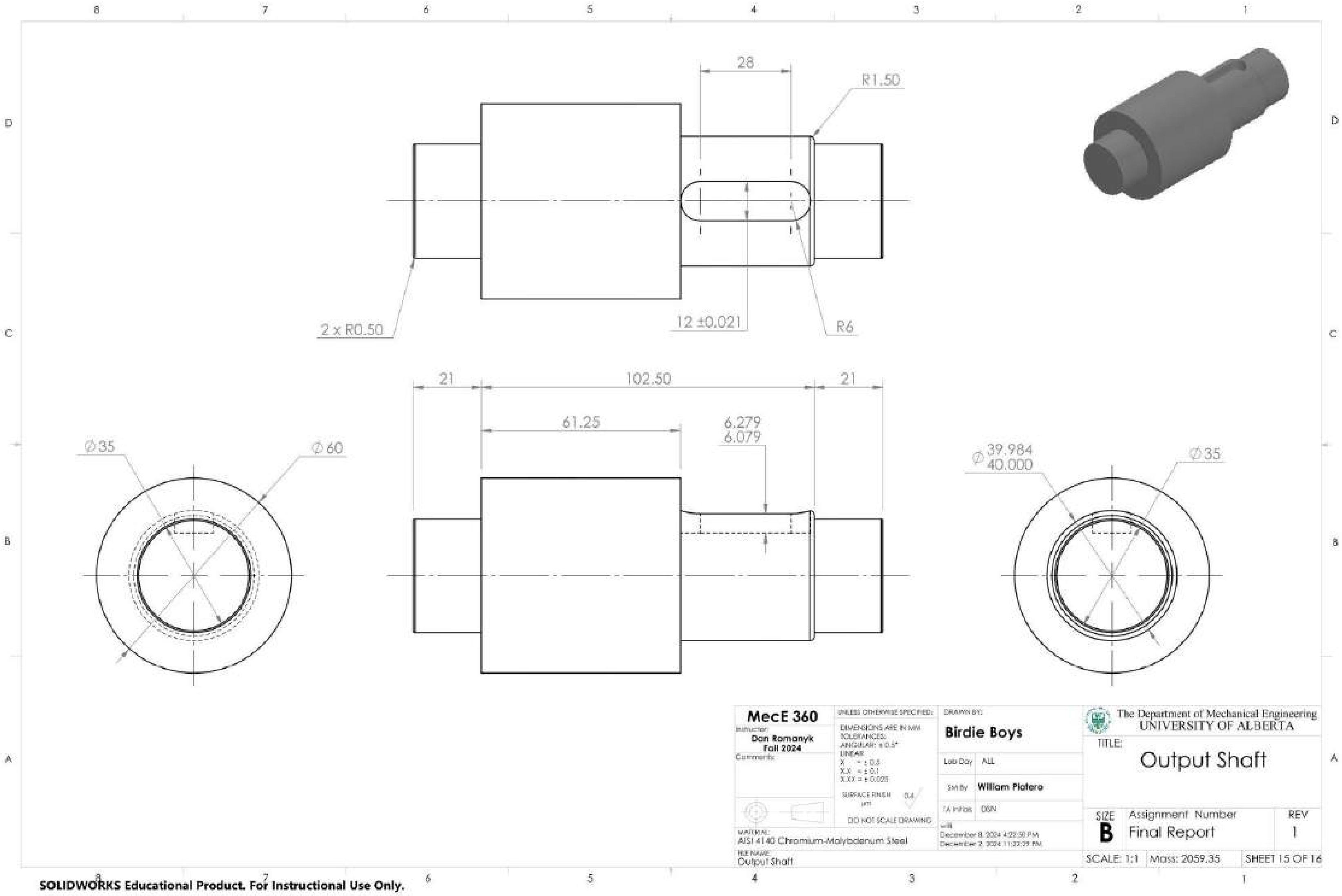
ITEM NO.	PART NUMBER	Material	Mass	QTY.
1	Output Shaft	AISI 4140 Chromium-Molybdenum Steel	2059.35	1
2	KHG3-60RJ40	AISI 4140 Chromium-Molybdenum Steel	5302.77	1
3	skf_bearing_6307_2	Carbon Chromium Steel (100Cr6)	0.456	1
4	McMasterCarr key 4	AISI 1045 Steel, cold drawn	26.70	1



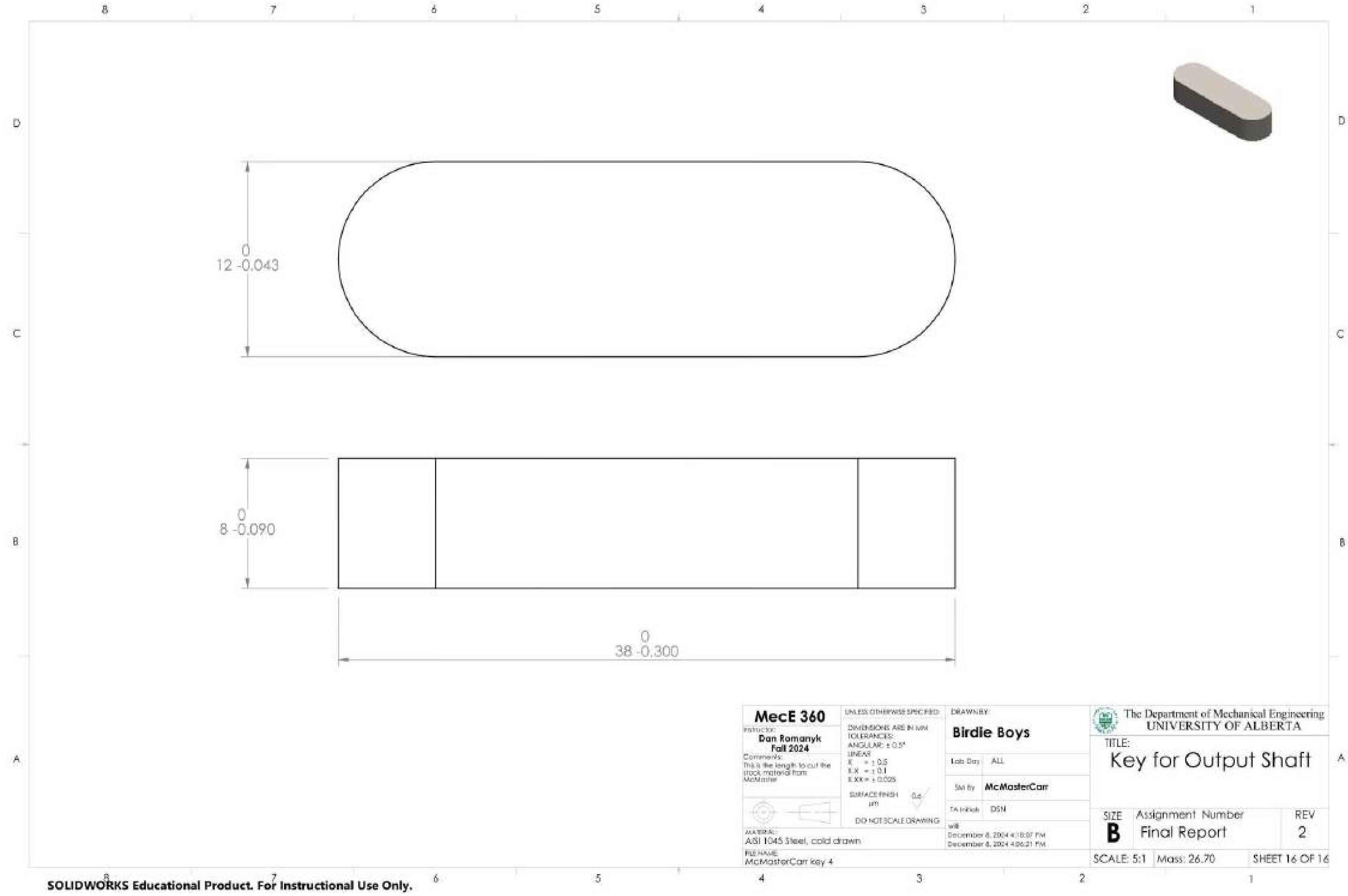
D14 - Output shaft exploded view.



D15 - Output shaft engineering drawing for manufacturing.



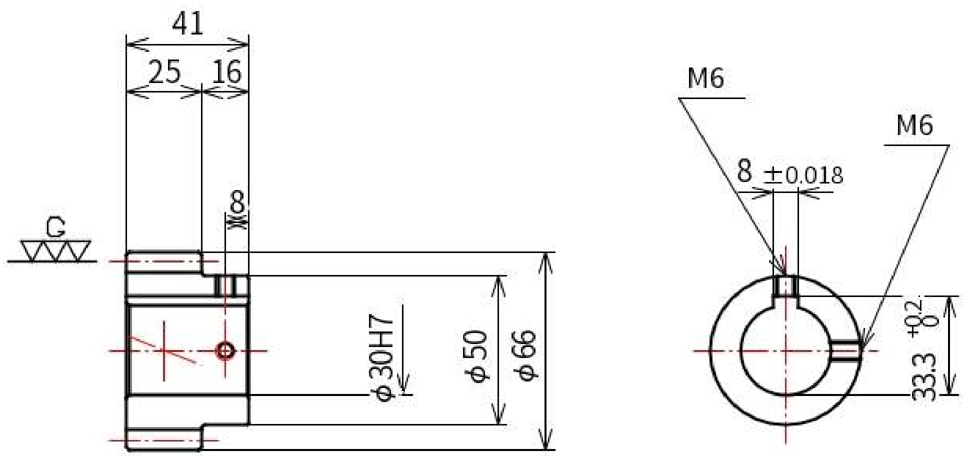
D16 - Key for output shaft engineering drawing, based on McMaster-Carr stock refined to an appropriate length.



D17 - 20 teeth helical engineering drawing, provided by KHK gears catalog (KHG3-20RJ30).

For J Series products, without black oxide repeatedly applied on secondary-operated parts.

G



Tolerance	
Dimension & Up ~ Max.	Tol. mm
0.5 ~ 6	±0.1
6 ~ 30	±0.2
30 ~ 120	±0.3
120 ~ 400	±0.5
400 ~ 1000	±0.8
1000 ~ 2000	±1.2
Angle	±0.5°

Ground Helical Gears Data	
Class	JIS B 1702-1 grade N6
Reference section of gear	Rotating plane
Tooth Form	Standard full depth
Module	3
Pressure Angle	20°
No. of Teeth	20
Helix Angle & Hand	21°30' R
Pitch Diameter	60
Profile shift coefficient	
Addendum	3
Tooth Depth	6.75
Outside Diameter	66
Thickness	
Backlash	
Mating Gear	
Heat Treatment	HB225~352
Induction Harden	HRC50~60
Remark	

Mark	Name	Matl.	Remarks
		SCM440	

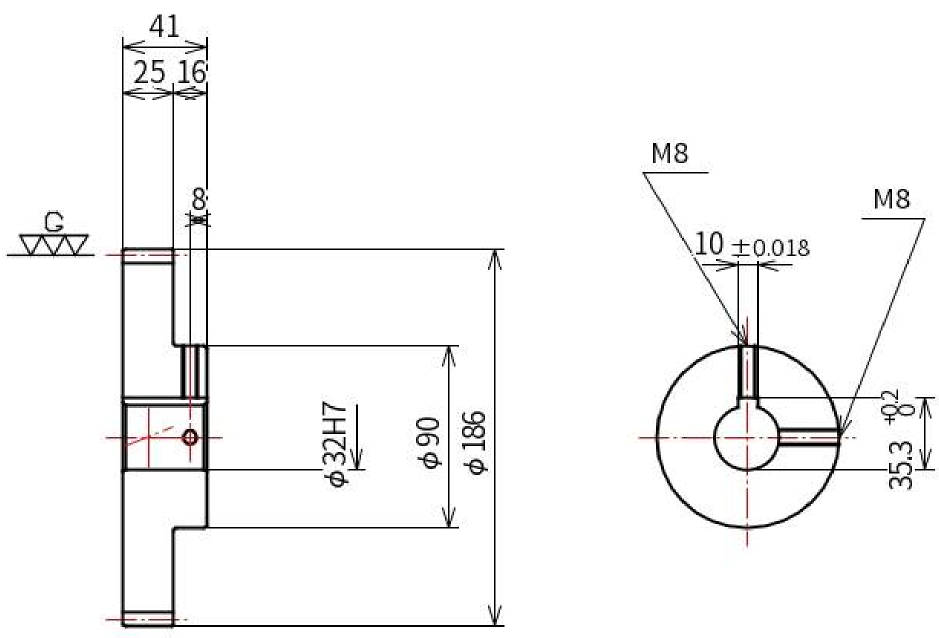
Drw. by	24.12.08	Title
Checked by		
Scale	N.T.S	
KHK for Web Catalog		Dwg.No. / Cust P/N

KHK KHG3-20RJ30

The precision grade of gears after secondary operations is equivalent but not identical to its original grade.

D18 - 60 teeth helical engineering drawing, provided by KHK gears catalog (KHG3-60LJ32).

For J Series products, without black oxide repeatedly applied on secondary-operated parts. ▽▽▽ (▽▽) G



Tolerance	
Dimension & Up ~ Max.	Tol. mm
0.5 ~ 6	±0.1
6 ~ 30	±0.2
30 ~ 120	±0.3
120 ~ 400	±0.5
400 ~ 1000	±0.8
1000 ~ 2000	±1.2
Angle	±0.5°

Ground Helical Gears Data	
Class	JIS B 1702-1 grade N6
Reference section of gear	Rotating plane
Tooth Form	Standard full depth
Module	3
Pressure Angle	20°
No. of Teeth	60
Helix Angle & Hand	21°30' L
Pitch Diameter	180
Profile shift coefficient	
Addendum	3
Tooth Depth	6.75
Outside Diameter	186
Thickness	
Backlash	
Mating Gear	
Heat Treatment	HB225~352
Induction Hardened	HRC50~60
Remark	

Mark	Name	Matl.	Remarks
		SCM440	

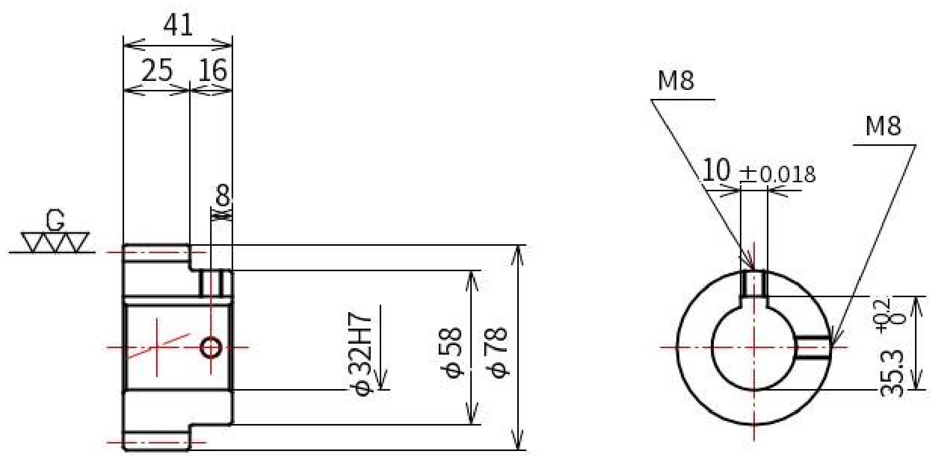
Drw. by	24.12.08	Title
Checked by		
Scale N.T.S		
KHK for Web Catalog		Dwg.No. / Cust P/N

KHK KHG3-60LJ32

The precision grade of gears after secondary operations is equivalent but not identical to its original grade.

D19 - 24 teeth helical engineering drawing, provided by KHK gears catalog (KHG3-24LJ32).

For J Series products, without black oxide repeatedly applied on secondary-operated parts.



Tolerance	
Dimension & Up ~ Max.	Tol. mm
0.5 ~ 6	±0.1
6 ~ 30	±0.2
30 ~ 120	±0.3
120 ~ 400	±0.5
400 ~ 1000	±0.8
1000 ~ 2000	±1.2
Angle	±0.5°

Ground Helical Gears Data	
Class	JIS B 1702-1 grade N6
Reference section of gear	Rotating plane
Tooth Form	Standard full depth
Module	3
Pressure Angle	20°
No. of Teeth	24
Helix Angle & Hand	21°30' L
Pitch Diameter	72
Profile shift coefficient	
Addendum	3
Tooth Depth	6.75
Outside Diameter	78
Thickness	
Backlash	
Mating Gear	
Heat Treatment	HB225~352
Induction Hardening	HRC50~60
Remark	

Mark	Name	Matl.	Remarks
		SCM440	

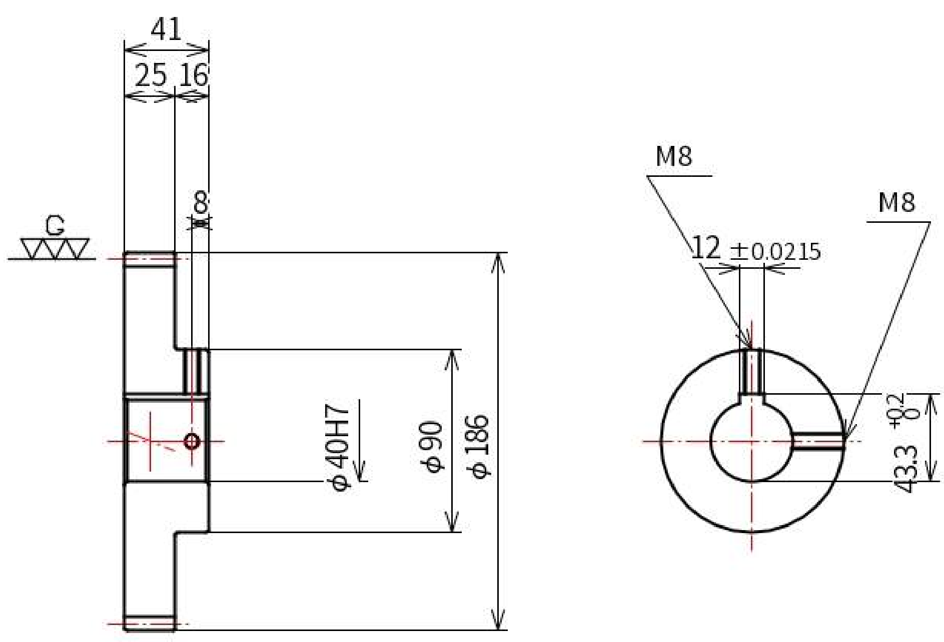
Drw. by	24.12.08	Title
Checked by		
Scale	N.T.S	
KHK for Web Catalog		Dwg.No. / Cust P/N

KHK KHG3-24LJ32

The precision grade of gears after secondary operations is equivalent but not identical to its original grade.

D20 - 60 teeth helical engineering drawing, provided by KHK gears catalog (KHG3-60RJ40).

For J Series products, without black oxide repeatedly applied on secondary-operated parts.



Tolerance	
Dimension & Up ~ Max.	Tol. mm
0.5 ~ 6	±0.1
6 ~ 30	±0.2
30 ~ 120	±0.3
120 ~ 400	±0.5
400 ~ 1000	±0.8
1000 ~ 2000	±1.2
Angle	±0.5°

Ground Helical Gears Data	
Class	JIS B 1702-1 grade N6
Reference section of gear	Rotating plane
Tooth Form	Standard full depth
Module	3
Pressure Angle	20°
No. of Teeth	60
Helix Angle & Hand	21°30' R
Pitch Diameter	180
Profile shift coefficient	
Addendum	3
Tooth Depth	6.75
Outside Diameter	186
Thickness	
Backlash	
Mating Gear	
Heat Treatment	HB225~352
Induction Hardened	HRC50~60
Remark	

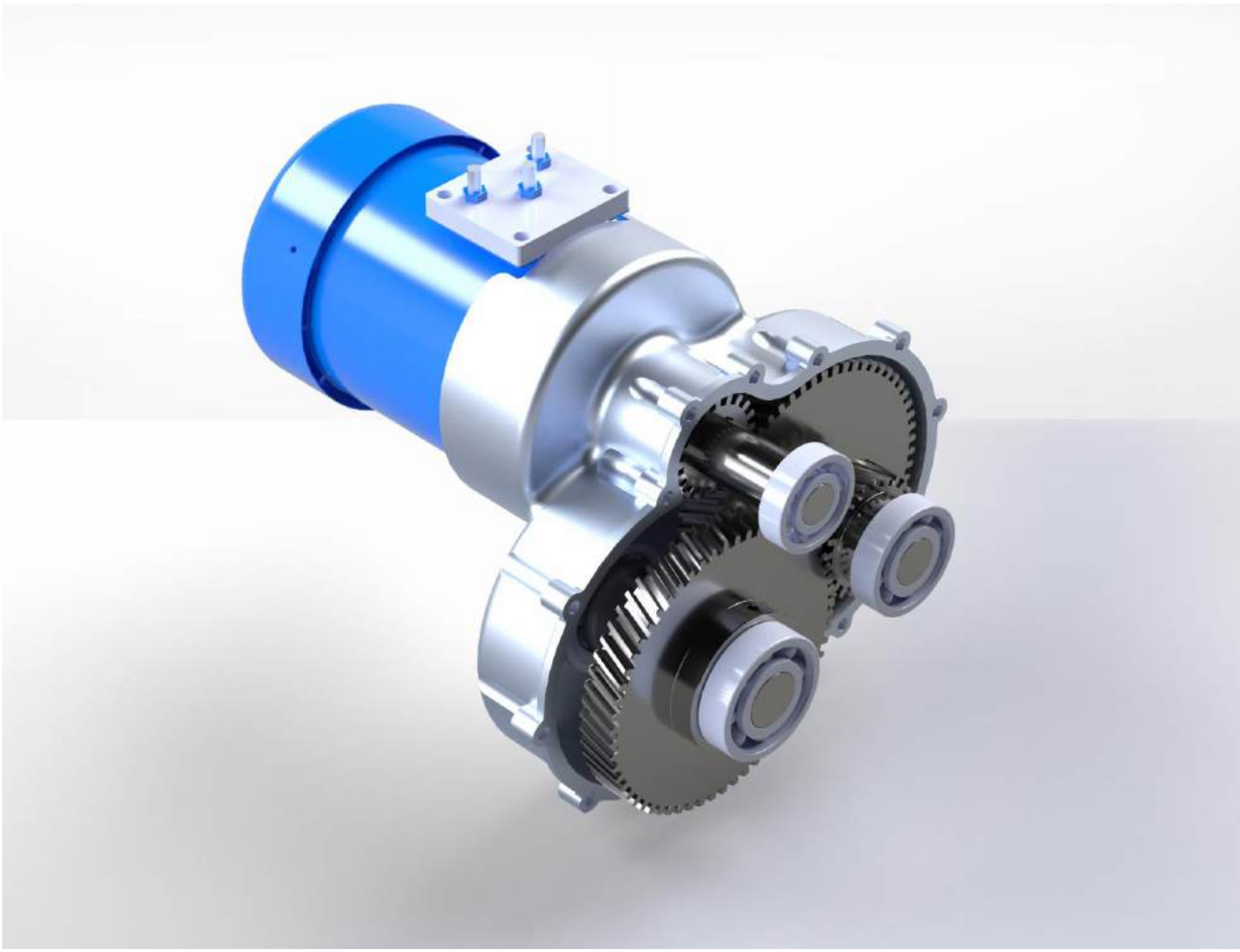
Mark	Name	Matl.	Remarks
		SCM440	

Drw. by	24.12.08	Title
Checked by		
Scale	N.T.S	
KHK for Web Catalog		Dwg.No. / Cust P/N

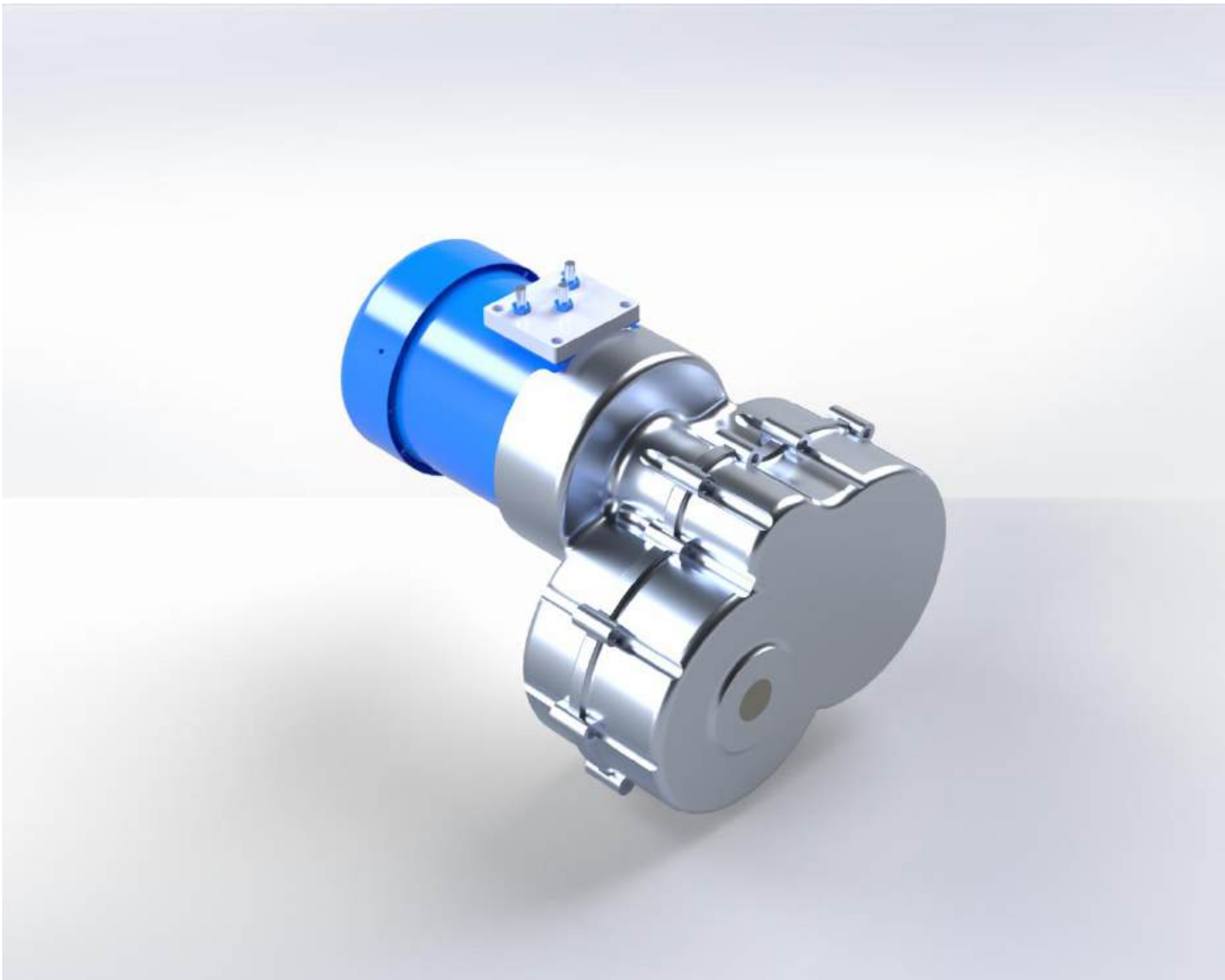
KHK KHG3-60RJ40

The precision grade of gears after secondary operations is equivalent but not identical to its original grade.

D21 - Render of the golf cart transmission.



D22 - Render of the golf cart transmission with housing.



Appendix E (Meeting Minutes)

Date: 10th SEP 2024
Time: 3:00 PM
Location: ECERF W2-90
Attendees:

- Pilgrim Cole Michael
- Ayala Platero William Jacob
- Cote Ben
- Jang Hyunseok
- Anjum Tahir Ahmad

Agenda:

1. Introduce Yourself
2. Project guideline review
3. LOI
4. Role/task assignment

Minutes:

1. **Welcome and Introductions**
    - [Summary of discussion]
2. **Approval of Previous Meeting Minutes**
    - N/A
3. **Agenda Item 1: Introduce Yourself**
    - **Discussion:** As members of the project group, we need to know each other. Introduce yourself (i.e., name, degree, hobby, etc.)
    - **Decisions Made:** N/A
    - **Action Items:**
        - N/A

4. **Agenda Item 2: Project meeting schedule**
    - **Discussion:**
        - Set a weekly meeting time
    - **Decisions Made:**
        - Wednesday 12-1PM (TBD)
    - **Action Items:**
        - Depending on availability, try to accommodate to everyone's schedule
5. **Agenda Item 3: LOI**
    - **Purpose: LOI background research task assignment**
    - **Discussion:**
        - Might want to do some more research on different options
        - Need to clarify specific purpose of the vehicle
    - **Decisions Made:**
        - Do more research on given options and decide tomorrow (11th SEP 2024)
    - **Action Items:**
        - Research transmission designs for different automobiles
            1. Ben, William - Golf Cart
            2. Hyunseok, Cole - Snowmobile
            3. Tahir - Something Complicated (motors)
        - Brainstorm purpose for building a vehicle transmission
6. **Agenda Item 4: Role/task assignment**
    - **Purpose: Decide roles and responsibilities of each member throughout the designing process. Review relevant designing tools and assign responsible members for the tools.**
    - **Discussion:**

- Do we need a team leader?
- **Decisions Made:**
  - No need a team leader, just assign roles tasks as we move along with the project
- **Action Items:**

7. **Other Business**
   - Might want to plan out event timeline
8. **Next Meeting**
   - **Date:** 11th SEP 2024
   - **Time:** 12PM
   - **Location: ECERF W2-90**

Date: 11th SEP 2024
Time: 12:00 PM
Location: ECERF W2-90
Attendees:

- Pilgrim Cole Michael
- Ayala Platero William Jacob
- Cote Ben
- Jang Hyunseok
- Anjum Tahir Ahmad

Agenda:

1. LOI

Minutes:

1. **Welcome and Introductions**
   - [Summary of discussion]

2. **Approval of Previous Meeting Minutes**
   - N/A
3. **Agenda Item 1: Complete LOI**
   - **Discussion:**
     - Team Name
     - Design Focus
     - Logo Design
   - **Decisions Made:**
     - Team Name: Birdie Boys
     - Design Focus: Reliability, Efficiency (Cost and Power)
     - Design Logo with AI
   - **Action Items:**
     - N/A
4. **Other Business**
   - Might want to plan out questions for first meeting with the professor
5. **Next Meeting**
   - **Date:** 18th SEP 2024
   - **Time:** 12PM
   - **Location: ECERF W2-90**

Adjournment:

- 12:50 PM

Date: 18th SEP 2024
Time: 12:00 PM

Location: ECERF W2-90
Attendees:

- Pilgrim Cole Michael
- Ayala Platero William Jacob
- Cote Ben
- Jang Hyunseok
- Anjum Tahir Ahmad

Agenda:

1. Preparing Progress Report I

Minutes:

1. **Agenda Item 1: Progress Report I**
    - **Discussion:**

    Please check the shared folder to input all the necessary initiation documents for the project

        - Mind Mapping and Gantt Chart
        - Design matrix Template
        - Design Scope
            1. Scenario and constraints for the design
                - Edmonton Summer (Temperature ~ 0C-40C)
                - Luxurious -> Smooth Driving
                - Speed -> upper limit 30km/h (2 speeds maybe)
                - Electric motor
                - Hills and bumps (weight distribution & clearance)
                - 2 person cart + 2 golf bags (300kg total)
                - flexible cost -> luxurious
                - Reliable (2-3 year warranty)
                - life

            2. Transmission design and connection to other components

- AGMA Standards

- Preliminary Design Specification
  1. Selecting Electric Motor
  2. Materials
  3. Loading
  4. Reliability
  5. Life Expectancy
  6. Size
  7. Weight
  8. Wear
  9. Safety

- **Decisions Made:**

  Research & Decisions to be made:

  - Motor → Tahir, Will
  - Gear → Hyunseok, Ben
  - Bearing → Cole

- **Action Items:**
  - N/A

2. **Other Business**
   -
3. **Next Meeting**
   - **Date:** 23rd SEP 2024
   - **Time:** 12:20PM
   - **Location:** MECE 4-31H

Adjournment:

- 12:50 PM

Date: 23rd SEP 2024
Time: 12:20 PM
Location: MEC 4-31H
Attendees:

- Pilgrim Cole Michael
- Ayala Platero William Jacob
- Cote Ben
- Jang Hyunseok
- Dan Romanyk

Agenda:

1. Progress Report I Design Meeting

Minutes:

1. **Agenda Item 1: Progress Report I**
    - **Discussion:**
        - Tips on Design Spec Matrix
        - Abstract tips
        - Motor spec - Where should we include this?
        - Gantt chart tips
        - Do we need laws although in private golf course?
    - **Decisions Made:**
        - Design Specification Matrix

---

1. Size: Fit within the golf cart → Refer to example club carts in the market
2. Terrain: Discuss maximum slope
3. Warranty: Discuss hours of operation for the vehicle
4. *** Keep the decision from decision matrix consistent throughout the report

- Abstract: Should contain critical information as well as the report outline
- Motor Spec: We can include this in the introduction instead of Design Specifications Section
- Gantt Chart: Should identify specific "tasks" that could be assigned to individuals
  1. Shaft Analysis
  2. Gear Analysis
  3. CAD Design
  4. etc.
- Laws: Though used in private golf course, still abide by the laws and restrictions of the province/city

- **Action Items:**
  - Based on this feedback, start putting together a Progress Report 1.

2. **Other Business**
   - N/A
3. **Next Meeting**
   - **Date:** 27th SEP 2024
   - **Time:** 10:00 AM
   - **Location:** ECERF W2-90

Adjournment:

- 12:35 PM

Date: 30th OCT 2024
Time: 12:00 PM
Location: ECERF
Attendees:

- Pilgrim Cole Michael
- Ayala Platero William Jacob
- Cote Ben
- Jang Hyunseok

Agenda:

1. Progress Report II Design Meeting

Minutes:

1. **Agenda Item 1: Progress Report I**
   - **Discussion:**
     - Gear and shaft selection
     - Schematic discussion
     - Forward and reverse ration
     - Role assignment
   - **Decisions Made:**
     - Gear and shaft
       1. 4 gears, 3 shafts: Helical gears, Regular shaft → 19 spline shaft connected to motor
          1. Reverse and forward ration = 1:1
       2. Assume Tire radius of 18 inch (aligns with club cart reference cart)
       3. Gear analysis: Hyunseok, Cole

4. Shaft Analysis: Ben, Tahir
5. Modelling & Motor spec review to see compatibility with shafts and gears: William

- **Action Items:**
  - Based on this feedback, start putting together a Progress Report 2 by end of Thursday, and review all on Friday

2. **Other Business**
   - N/A
3. **Next Meeting**
   - **Date:** 1st NOV 2024
   - **Time:** 12:00 PM
   - **Location:** ECERF W2-90

Adjournment:

- 12:50 PM

Date: 1st NOV 2024
Time: 12:00 PM
Location: ECERF
Attendees:

- Pilgrim Cole Michael
- Ayala Platero William Jacob
- Cote Ben
- Jang Hyunseok
- Tahir Anjum

Agenda:

1. Progress Report II Design Meeting

Minutes:

1. **Agenda Item 1: Progress Report I**
    - **Discussion:**
        - What is next for analysis?
            1. FBD of vehicle to find min. torque and schematic of gear box
            2. FBD of shafts and gears
            3. Based on decision matrix, FBD, and calculations, choose ideal gears and shafts
        - Who will work on what this weekend?
        - Gears info
    - **Decisions Made:**
        - Need to complete templates for gear and shaft analysis this weekend
        - Work on FBD for the whole vehicle to find min. torque to drive up the hill
    - **Action Items (over the weekend):**
        - Ben: Shaft analysis template on Smath
        - Tahir: Gear analysis template on Smath
        - Cole: Gear research and spec matrix
        - Will: Solidworks modeling of full layout (Shafts & Gears)
        - Hyunseok: Torque and incline calculations after help Cole
2. **Other Business**
    - N/A
3. **Next Meeting**
    - **Date:** 4th NOV 2024

- **Time:** 12:00 PM
- **Location:** ECERF W2-90

Adjournment:

- 12:50 PM

Date: 2nd DEC 2024
Time: 12:00 PM
Location: ECERF
Attendees:

- Pilgrim Cole Michael
- Cote Ben
- Jang Hyunseok

Agenda:

1. Final Report Design Meeting

Minutes:

1. **Agenda Item 1: Final Report**
    - **Discussion:**
        - PR2 Feedback Review
    - **Decisions Made:**
        - Have to change analysis on shaft and gear
            1. How can we improve the safety factor for gear?
            2. Include Axial Force for helical gear
            3. Go down to 99% reliability according to AGMA Standard
        - Start drafting Bearing Analysis
        - Attention to detail → Report Presentation

- **Action Items:**
  - Ben: Shaft analysis changes
  - Tahir: Review shaft (priority), gear, and bearing analysis
  - Cole:
    1. Start planning the presentation
    2. Support Gear, Shaft, and Bearing analysis review
  - Will: Solidworks modeling and drawing
  - Hyunseok:
    1. Bearing analysis set up
    2. Gear analysis review

2. **Other Business**
   - N/A
3. **Next Meeting**
   - **Date:** 4th DEC 2024
   - **Time:** 12:00 PM
   - **Location:** ECERF W2-90

Adjournment:

- 12:50 PM